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THE PROCESS OF THE $\Sigma - \Lambda$ CONVERSION FOLLOWING THE NUCLEAR
ABSORPTION OF $\bar{K}$ WITH THE PRODUCTION OF A π^-

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A B S T R A C T

The second order process of the $\Sigma - \Lambda$ conversion following the nuclear absorption of a kaon from a $\bar{K}$ meso-atomic orbit with the production of a negative pion is discussed, and calculated from a second order Feynman diagram. The momentum distributions of the recoiling nucleus and of the pion are computed for the $\bar{K}$ absorption in ^4He and in deuterium. A semi-quantitative agreement with the data is obtained in all the cases. No attempt is made to calculate the absolute values of the reaction rates in question. The need of more and better data is stressed. The available data tend to favour the assumption of the 2P capture over that of the 1S capture in the case of ^4He , but it is insufficient to rule out any one of the two cases.

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The second order process of the production of a Σ particle and a pion in the nuclear absorption of a kaon followed by the conversion of the Σ into a Λ in a subsequent nuclear collision has been studied recently for some light nuclei, in particular, for helium and for deuterium ¹⁾⁻³⁾. The description of the reaction amplitude in Refs. ¹⁾⁻³⁾ has been based on the formalism of coupled channel distorted waves and essentially limited to the effective range approximation as, e.g., in the early papers by Karplus and Rodberg ⁴⁾, by Kotani and Ross ⁵⁾ or by Chand ⁶⁾. That treatment is certainly crude in the way of replacing the Σ - Λ vertex function together with the Σ propagator by kind of an effective transmission coefficient.

It is the aim of the present note to investigate the second order process by a second order impulse approximation with closure applied to the intermediate nuclear states. This kind of approximation has been applied to the study of the non-pionic nuclear absorption of kaons ⁷⁾⁻⁹⁾ and to an estimate of the double scattering (rescattering) corrections in the $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Lambda + {}^3\text{He}$ reaction ¹⁰⁾. For inelastic, in particular, for absorption and particle conversion processes the second order impulse approximation seems to be reasonable even for the initial meson at rest in contrast to the situation with the low energy elastic scattering of strongly interacting particles.

Our process can be represented by the diagram of Fig. 1. At the vertex Γ_2 the virtual nucleon N_2 absorbs the kaon producing the negative pion and the intermediate Σ' which is a Σ^0 or a Σ^+ according to whether N_2 is a neutron or a proton, respectively. At the upper vertex Γ_1 , the Σ' is reabsorbed in a collision with another nucleon N_1 , and converted into the final state outgoing Λ . The rest of the nucleus constitutes the spectator core C . We assume the propagator of Σ' to be unperturbed by C which means taking the (two) free nucleon-reaction amplitude to be the amplitude of virtual nucleons belonging to

the initial nucleus. The binding and other nuclear and hypernuclear field effects are represented by the nuclear and hypernuclear form factors, respectively. In view of the present experimental situation we shall concentrate in the following on the study of the most interesting momentum distributions of the outgoing pion and of the final recoil nucleus $fN = (C+N_f)$ which we call $R_{\pi}(p_{\pi})$ and $R_{fN}(P_{fN})$, respectively, rather than on a calculation of the extremely uncertain or unknown absolute values of the reaction rates or of the related cross-sections.

The amplitude of Fig. 1 can be calculated in a relativistic or a non-relativistic form. As the Σ' is a fermion, one should write the Feynman diagram formula with a Dirac propagator. It turns out that this is practically numerically equivalent to working with a simple Klein-Gordon propagator in so far as the momentum distributions $R_{\pi}(p_{\pi})$ and $R_{fN}(P_{fN})$ with an arbitrary normalization are concerned provided the effective vertex functions Γ_1 and Γ_2 (t matrix elements) are reasonably smooth functions of the momenta involved in the region of interest. This result corresponds to the fact that the details of the kaon-nuclear and nucleon-hypernuclear dynamics are generally less important for the shapes of the momentum distributions $R_{\pi}(p_{\pi})$ and $R_{fN}(P_{fN})$ than the kinematics involved. This fact remains roughly true even if one assumes the Y_0^* resonant component in the vertex function Γ_2 . The basic ingredients in $R_{\pi}(p_{\pi})$ and in $R_{fN}(P_{fN})$ are the energy denominator of the propagator, the energy-momentum conservation and the Σ - Λ mass difference.

The basic t matrix element of the elementary process of the Feynman diagram of Fig. 1 in the absence of C , i.e., for the two nucleons actually free, can be written down as

$$\begin{aligned}
 \langle t \rangle &= \langle p'_1 p_{\Lambda} | R_1 | p_1 p_{\Sigma'} \rangle \frac{1}{D} \langle p_{\pi} p_{\Sigma'} | t_2 | p_K p_2 \rangle \equiv \\
 &\equiv \left(\Gamma_1 \frac{1}{D} \Gamma_2 \right) \delta^{(4)}(p'_1 + p_{\Lambda} - p_1 - p_{\Sigma'}) \delta^{(4)}(p_{\pi} + p_{\Sigma'} - p_K - p_2), \quad (1) \\
 \frac{1}{D} &\equiv (i \gamma \cdot p_{\Sigma'} - m_{\Sigma'}) (p_{\Sigma'}^2 + m_{\Sigma'}^2 + i\delta)^{-1}.
 \end{aligned}$$

where the four-momenta p_2 , p_1 and p_1' refer to the nucleons 2 and 1, $p_{\Sigma'}$, $p_{\Lambda'}$, p_K and p_{π} to the four-momenta of the remaining particles in obvious notation; the explicit spin dependence of $\Gamma_{1,2}$ has been suppressed in the notations. In the following calculations we suppress, for simplicity, the difference between Σ^0 and Σ^+ , and the corresponding isotopic spin effects, and we consider an "average" Σ' with an average mass $m_{\Sigma'}$. As in I and II (Refs. 1) and 2), respectively) we are interested here only in $R_{fN}(p_{fN})$ and $R_{\pi}(p_{\pi})$, and we keep, therefore, the normalization of our $\langle t \rangle$ of Eq. (1) arbitrary. The absolute values of the reaction rates remain to be determined from the relevant elementary coupling constants involved, but we feel that this part of the analysis is still premature both for experimental and for theoretical reasons.

II. THE REACTION $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Sigma' + {}^3\text{H} \rightarrow \pi^- + \Lambda + {}^3\text{He}$

As an example of a light nucleus let us consider the ${}^4\text{He}$. The corresponding non-relativistic second order impulse approximation formula is presented in Eq. (A1) of Appendix of I. Actually, all our kinematics in the following will be non-relativistic except that of the pion and except for the fact that we shall keep our Σ' propagator in its relativistic form; as to the latter point we should remark that it is practically a simplification relative to the Schroedinger form of Eq. (A1) of I, and permits to readily perform closure over the intermediate states of the three nucleon system $\{ \Psi_{3I, (1,3,4)} \}$.

With our $\langle t \rangle$ of Eq. (1) we can write in this case the $\Sigma - \Lambda$ conversion matrix element with an arbitrary normalization in the form :

$$\begin{aligned}
M_{\Sigma-\Lambda}^{(\text{He})} \cong & \int \frac{d\vec{p}_K}{(2\pi)^3} \int \frac{d\vec{p}_2 d\vec{p}_1 d\vec{p}_1'}{(2\pi)^9} \int d(1') d(2') d(3') d(4') \sum_{3I'} \\
& \cdot e^{-i\vec{p}_2 \vec{r}_2'} \psi_{3I'}^*(1', 3, 4') \psi_4(1', 2', 3', 4') \int d(3) d(4) \int d(1'') \\
& \cdot e^{-i\vec{p}_1 \vec{r}_1''} \psi_{3I'}(1'', 3, 4) \int d(1''') e^{i\vec{p}_1' \vec{r}_1'''} \psi_3^*(1''', 3, 4) \phi_K(\vec{p}_K). \quad (2)
\end{aligned}$$

$$\begin{aligned}
& \int \frac{d^4 p_{\Sigma'}}{(2\pi)^4} \delta(\varepsilon_{\Sigma'} + E_{\pi}^0 - \varepsilon_2 - m_K) \delta(\vec{p}_{\Sigma'} + \vec{p}_{\pi} - \vec{p}_2) (\Gamma_1 \frac{1}{D} \Gamma_2) \\
& \cdot \delta(\vec{p}_1' + \vec{p}_{\Lambda} - \vec{p}_1 - \vec{p}_{\Sigma'}) \delta(\varepsilon_{\Lambda} + E_{\pi}^0 + T_3 - M - m_K).
\end{aligned}$$

Here ψ_4 and ψ_3 are the ${}^4\text{He}$ and ${}^3\text{He}$ ground state wave functions, respectively; $\phi_K(\vec{p}_K)$ is a Fourier transform of the initial $\bar{K}$ meson Bohr orbit wave function. For an nS orbit $\phi_K(\vec{p}_K) \cong N_{nS} \delta(\vec{p}_K)$, and for an mP case $\phi_K(\vec{p}_K) \cong N_{mP} i(\vec{e} \cdot \vec{\nabla}_{\vec{p}_K}) \delta(\vec{p}_K)$; $\vec{e}$ is the P orbit polarization vector. T_i represents the kinetic energy of the particle "i", and ε_i is the free particle energy of the same; only E_{π}^0 of the pion is relativistic. The external energy conservation delta with the argument $\varepsilon_{\Lambda} + E_{\pi}^0 - m_K + T_3 - M = T_{\Lambda} + T_3 + E_{\pi}^0 - Q_{\text{He}}$ where $Q_{\text{He}} = M_4 - M_3 + m_K - m_{\pi}^*$ represents the total energy conservation in our non-relativistic form^{*}. In the formula of Eq. (2) we have assumed the

^{*}) This follows formally from Eq. (1), etc., if one replaces $\varepsilon_1' - \varepsilon_1 - \varepsilon_2$ by $T_3 - M$. In other words, this non-relativistic form is the ansatz of replacing in our $\langle t \rangle$ of Eq. (1) one of the two energy conservation delta functions by the total energy conservation delta. This is quite consistent with our non-relativistic treatment and analogous to the ansätze of Refs. 7)-10). The only difference between this and Eq. (A1) of I is the replacement of " $\langle t_2 \rangle (E_f - E_{\Sigma'} + 3I_1)^{-1} \langle t_1 \rangle$ " by our $(\Gamma_1 D^{-1} \Gamma_2)$ with $\vec{p}_{\Sigma'} = \vec{p}_2 - \vec{p}_{\pi}$ and $\varepsilon_{\Sigma'} = \varepsilon_2 - E_{\pi}^0 + m_K$.

initial mesic atom and the ${}^4\text{He}$ nucleus at rest ($\vec{p}_1 = 0 = \vec{p}_4$), and we have tacitly exploited the fact that $p_K = 0$. Finally, no final state ${}^3\text{He}$ interaction (distortion) has been assumed implying that the corresponding hypernuclear form factor degenerates to a delta function (plane waves). This latter assumption is justified by the results of I and by Said ¹¹⁾.

Introducing now, as in the Appendix of I, the intrinsic spatial co-ordinates of the three nucleon system (1,3,4) :

$$\begin{aligned}\vec{V}' &= \frac{1}{6}(\vec{r}_3' + \vec{r}_4') - \frac{1}{3}\vec{r}_1', \quad \vec{W}' = \vec{r}_3' - \vec{r}_4', \quad \vec{V} = \frac{1}{6}(\vec{r}_3 + \vec{r}_4) - \frac{1}{3}\vec{r}_1'', \\ \vec{W} &= \vec{r}_3 - \vec{r}_4 \quad \text{and} \quad \vec{V}'' = \frac{1}{6}(\vec{r}_3 + \vec{r}_4) - \frac{1}{3}\vec{r}_1''',\end{aligned}$$

and performing a number of simple integrations we can reduce Eq. (2) to the form ^{*)}

$$\begin{aligned}M_{\Sigma\Lambda}^{(\text{He})} &= \text{const} \delta(E_{\pi}^0 + T_1 + T_3 - Q_{\text{He}}) \delta(\vec{p}_{\pi} + \vec{p}_1 + \vec{p}_3) \sum_{\substack{3I' \\ \text{I}'}} \int \frac{d\vec{p}_2 d\vec{p}_1 d\vec{p}_1'}{(2\pi)^9} \delta(\vec{p}_1' - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) I(\vec{p}_2) \int d\vec{v}' d\vec{w}' \phi_{3I'}^*(\vec{v}, \vec{w}') \\ &\cdot \phi_4(\vec{v}', \vec{w}') \int d\vec{v} d\vec{w} e^{i(3\vec{p}_1' + \vec{p}_2) \cdot \vec{v}} \phi_{3I'}(\vec{v}, \vec{w}) \int d\vec{v}'' e^{-i(3\vec{p}_1' + 3\vec{p}_2 + 2\vec{p}_3) \cdot \vec{v}''} \\ &\cdot \phi_3^*(\vec{v}'', \vec{w}) \int \frac{d\vec{p}_K}{(2\pi)^3} \phi_K(\vec{p}_K) \int d^4 p_{\Sigma} \delta(\varepsilon_{\Sigma} + E_{\pi}^0 - \varepsilon_2 - m_K) \\ &\cdot \delta(\vec{p}_{\Sigma} + \vec{p}_{\pi} - \vec{p}_2) \left(\Gamma_1 \frac{1}{D} \Gamma_2 \right).\end{aligned}\tag{3}$$

^{*)} Equation (A2) of Appendix of I contains a couple of misprints.

where $I(\vec{p}_2) \equiv \int \exp(i\vec{p}_2 \cdot \vec{u}) \chi_{(n-^3\text{He})}(\vec{u}) d\vec{u}$ is the Fourier transform of the factor of ψ_4 relative to the intrinsic co-ordinate $\vec{u}$, the vector distance from the I' centre-of-mass to the nucleon 2; in deriving Eq. (3) we have introduced the simplification of assuming the separability of the intrinsic part of ψ_4 in the form

$$\psi_4 \equiv \chi_{(n-^3\text{He})}(\vec{u}) \phi_4(\vec{v}, \vec{w})$$

which is the case for several well-known model wave functions of ^4He .

In view of the form of our $\langle t \rangle$ of Eq. (1) we immediately perform the sum over $^3I'$ (closure).

It is already known from I and II that the main features of our momenta distributions depend rather little on the details of Γ_1 and Γ_2 ; on the other hand, such details are essentially known. If we assume Γ_1 and Γ_2 approximately independent of p_1 and p_1' , we can finally reduce Eq. (3) to the form

$$M_{\Sigma-\Lambda}^{(\text{He})} \cong \text{const} \delta(\Sigma \vec{p}_i) \delta(en) \int \frac{d\vec{p}_2}{(2\pi)^3} I(\vec{p}_2) F_{(34)}(2\vec{p}_2 + 2\vec{p}_3) \cdot \int \frac{d\vec{p}_k}{(2\pi)^3} \phi_k(\vec{p}_k) \int d^4 p_{\Sigma'} \delta(\epsilon_{\Sigma'} + E_{\pi}^0 - \epsilon_2 - m_k) \delta(\vec{p}_{\Sigma'} + \vec{p}_{\pi} - \vec{p}_2) \cdot (\Gamma_1 \frac{1}{D} \Gamma_2) \equiv \delta(\Sigma \vec{p}_i) \delta(en) \bar{M}_{\Sigma-\Lambda}^{(\text{He})}, \quad (4)$$

where

$$F_{(34)}(\vec{\kappa}) \equiv \int e^{-i\vec{\kappa} \cdot \vec{v}} \phi_3^*(\vec{v}, \vec{w}) \phi_4(\vec{v}, \vec{w}) d\vec{v} d\vec{w}.$$

In the case of ^4He this extra form factor introduces a considerable "damping" as it decreases rather sharply with increasing $|\vec{\kappa}|$.

In the case of deuterium we obtain a formula quite similar to that of Eq. (4), but just without the factor $F(\vec{\kappa})$ (in this case $\vec{p}_2 = -\vec{p}_1$). It is just the obvious reason for why the $\Sigma - \Lambda$ conversion amplitude is much more important for our momentum distributions in the case of deuterium than it is in the case of ${}^3\text{He}$ at high values of the recoiling nucleus momentum P_{fN} in $R_{fN}(P_{fN})$ (of the outgoing proton in the case of deuterium) and for a region of p_π in the distribution $R_\pi(p_\pi)$.

The importance of this form factor in general is obvious as $\Delta p_1 = |\vec{p}_1 - \vec{p}_1'| = |\vec{p}_2 - \vec{p}_3|$ is typically large in view of the $\Sigma - \Lambda$ mass difference.

In computing the transition probability of our reaction one would have to perform a trace involving the Dirac positive energy projectors. Two examples of this have been studied under the assumption of Γ_1 and Γ_2 constant: one with the complete expression and the other with keeping only the γ_4 parts. It turns out that provided Γ_1 and Γ_2 are approximately constant the shapes of our momentum distributions in these cases are almost identical (not so the absolute values; we refer to an arbitrary common normalization); the same applies to the case where one leaves out the term $i \vec{\gamma} \cdot \vec{p}_{\Sigma'}$ in the numerator in Eq. (1) altogether. The last case means using essentially a simple Klein-Gordon propagator for the Σ' in our $\langle t \rangle$. All the numerical results presented below refer just to this simplest case.

The $\Sigma - \Lambda$ reaction rate can be written as:

$$R_{3\pi} = d\vec{P}_3 d\vec{p}_\pi \int d\vec{p}_\Lambda \delta(en) \delta(\vec{p}_\Lambda + \vec{p}_\pi + \vec{P}_3) \frac{1}{4} \text{Tr} \left(\overline{M}_{\Sigma-\Lambda}^{(\text{He})} M_{\Sigma-\Lambda}^{(\text{He})\dagger} \right) \quad (5)$$

$$\equiv d\vec{P}_3 d\vec{p}_\pi \delta(en) \varphi(\vec{P}_3, \vec{p}_\pi).$$

8.

With our approximations we have $\varphi = \varphi(P_3, p_\pi)$, i.e., φ does not depend on the angle between $\vec{P}_3$ and $\vec{p}_\pi$. In this case the momentum distributions (with an arbitrary normalization) are

$$R_3(P_3) = P_3 \int_{P_{\pi \min}(P_3)}^{P_{\pi \max}(P_3)} \varphi(P_3, p_\pi) p_\pi dp_\pi \quad (6)$$

and

$$R_\pi(p_\pi) = p_\pi \int_{P_{3 \min}(p_\pi)}^{P_{3 \max}(p_\pi)} \varphi(P_3, p_\pi) P_3 dP_3 \quad (7)$$

The lower (min) and the upper (max) limits of the respective integrations are determined from the energy conservation as functions of the respective remaining variables.

It has been shown in I and II for the case of ${}^4\text{He}$ that the only reasonable explanation of the far-off (high P_3) tail of the distribution $R_3(P_3)$ of our reaction lies in the $\Sigma - \Lambda$ conversion process. Similarly, it seems rather easy to understand the small peak of the pion distribution $R_\pi(p_\pi)$ at about $p_\pi \cong 175 \text{ MeV}/c$. In spite of considerable effort and of various different models employed the fits to the data of Refs. ¹²⁾ obtained in I and II are only semi-quantitative. On the other hand, one should stress the poorness of the data available and the importance of improving the statistics. The main aim of the present numerical calculations is to study the numerical differences between our present approach and the two-channel distorted wave method of I and II.

We have performed numerical computations based on Eqs. (4)-(7) for the cases of an S capture and of a P capture and for several ^4He model wave functions. As for the S Bohr orbit kaon wave function we simply have to set $p_K = 0$ in the formula of Eq. (4). According to a recent experiment by Burleson et al. ¹³⁾ the P capture is much more important (about 80% of events). In the P wave case, provided we suppress any possible p_K dependence of Γ_1 and Γ_2 , we have in the place of the factor $I(\vec{p}_2) F(2\vec{p}_2 + 2\vec{p}_3)$ in the matrix element of Eq. (4) the expression :

$$-i \left[\vec{e} \cdot \vec{\nabla}_{\vec{\omega}} I(\vec{\omega}) \right]_{\vec{\omega} = \vec{p}_2} \cdot F(2\vec{p}_2 + 2\vec{p}_3) - i 2 I(\vec{p}_2) \left[\vec{e} \cdot \vec{\nabla}_{\vec{\omega}'} F(\vec{\omega}') \right]_{\vec{\omega}' = 2\vec{p}_2 + 2\vec{p}_3}$$

This then requires the angular averaging over the directions of $\vec{e}$ in calculating the corresponding rates R , and leads to relatively complex expressions.

In order to determine the form factor $I(p_2)$ we choose for the wave function $\chi(u)$ the simple Hulthén form as in I and II, and we have :

$$I(p_2) = N \left[(p_2^2 + \mu^2)^{-1} - (p_2^2 + \nu^2)^{-1} \right] \quad (8)$$

with the parameter values as in I and II : $\mu = 0.8 \text{ fm}^{-1}$, $\nu = 1.25 \text{ fm}^{-1}$. This form of $I(p_2)$ has been explained and justified in I. As for the intrinsic functions of the three nucleon system, $\phi_3(v,w)$ and $\phi_4(v,w)$, we at first chose a simple Gaussian form. In this case we have

$$F(\vec{x}) = \text{const} e^{-(x/2)^2} [24\alpha_4^2 + 18\alpha_3^2]^{-1} \quad (9)$$

where according to Bruno ¹⁴⁾ $\alpha_4^2 = 0.054 \text{ fm}^{-2}$, and according to Newns ¹⁵⁾ $\alpha_3^2 = 0.0625 \text{ fm}^{-2}$. Unfortunately, the asymptotic behaviour (tail) of

such a simple Gaussian $\phi_3(v, w)$ is rather unrealistic, and it is also responsible for too sharp a suppression of the distribution $R_3(P_3)$ at high P_3 . For this reason we have employed also another form of $\phi_3(v, w)$ as recently determined by Eichmann¹⁶⁾

$$\phi_3(v, w) = N_{Eich} \sum_{i=1,2} a_i e^{-c_i (9v^2 + \frac{3}{4}w^2)} \quad (10)$$

where $c_1 = 0.3 \text{ fm}^{-2}$, $c_2 = 0.07 \text{ fm}^{-2}$ and $a_1 = 1$, $a_2 = 0.15$. This latter form of $\phi_3(v, w)$ gives much better results here as it does also in the photodisintegration problem¹⁶⁾.

In Fig. 2 we compare the $R_3(P_3)$ corresponding to the two forms of ϕ_3 assumed and to the cases of the 1S and the 2P captures ; only the $\Sigma - \Lambda$ conversion amplitude is included.

In Fig. 3 the same comparison is made for the distribution $R_{\pi}(p_{\pi})$.

As the quantities Γ_1 and Γ_2 remain undertermined, so does the relative phase of our $\Sigma - \Lambda$ conversion amplitude and the direct (π^-, Λ) production amplitude. On the other hand, the two hardly overlap in an important way ; their approximate "separability" could be seen already from the results of I and II. For this reason we have chosen to neglect the corresponding interference terms altogether. As for the "direct" (π^-, Λ) production part, we take the values of the parameters involved, the phases and the relative normalization of the Y_1^* resonant component and the non- Y_1^* component directly over from I and II for the 1S and the 2P captures respectively. The combined effect of the respective non- $\Sigma - \Lambda$ components and of the $\Sigma - \Lambda$ conversion amplitudes is illustrated for $R_3(P_3)$ in Fig. 4 with our best choice of $\phi_3(v, w)$, and, similarly, for the distribution $R_{\pi}(p_{\pi})$ in Fig. 5. Obviously, the same proportions of the $\Sigma - \Lambda$ conversion and the non- $\Sigma - \Lambda$ components are preserved for both $R_3(P_3)$ and $R_{\pi}(p_{\pi})$ in each case (1S and 2P).

The experimental histograms of Ref. ¹²⁾ both for $R_3(P_3)$ and for $R_{\pi}(p_{\pi})$ seem to favour the 2P capture over the 1S one. However, the data are definitely too scarce, and the differences between the two cases too small to permit ruling out any one of these two assumptions. The treatment of the $\Sigma - \Lambda$ components in the present paper is definitely crude, and a calculation avoiding the closure and other approximations will be necessary when more and better data become available. In particular, the first peak of $R_{\pi}(p_{\pi})$ is too sharp a spike in our approximation. This peak corresponds to the $\Sigma - \Lambda$ conversion amplitude near the $\Sigma - \Lambda$ conversion threshold (production of real Σ hyperons). It should be noted that the high P_3 tail of $R_3(P_3)$ appears to be explained much better by the present calculation than by those of I and II.

III. THE REACTION $\bar{K} + D \rightarrow \pi^- + \Sigma' + N' \rightarrow \pi^- + \Lambda + P$

In the case of deuterium no spectator nucleons are involved, and we have no extra form factor F ($F=1$). The recoiling nucleus in the final state is the proton P .

This case has been studied rather extensively by Letessier and Vinh Mau ³⁾. They calculated the corresponding $\Sigma - \Lambda$ conversion amplitude using the coupled channel distorted wave method as that of I.

In the present paper we calculate the same quantity using the second order Feynman diagram as that of Fig. 1 only without the spectator "core" C .

According to Day, Snow and Sucher ¹⁷⁾ and to Ref. ³⁾ we assume that only the orbits S are relevant in this case. Again we set the initial deuteron momentum $P_D = 0$ (capture at rest), and $p_K = 0$.

In analogy to Eq. (4) we obtain in this case

$$M_{\Sigma-\Lambda}^{(D)} \approx \text{const.} \delta(E_{\pi}^0 + T_{\Lambda} + T_P - Q_D) \delta(\vec{p}_{\pi} + \vec{p}_{\Lambda} + \vec{P}) \int \frac{d^3 p_2 d^3 \vec{p}_{\Sigma'}}{(2\pi)^6}, \quad (11)$$

$$I_D(\vec{p}_2) \delta(\varepsilon_{\Sigma'} + E_{\pi}^0 - \varepsilon_2 - m_K) \delta(\vec{p}_{\Sigma'} + \vec{p}_{\pi} - \vec{p}_2) \left(\Gamma_1 \frac{1}{D} \Gamma_2 \right),$$

where $Q_D = M_D - M + m_K - m_{\Lambda}$; $\vec{P}$ and T_P are the outgoing proton momentum and energy, respectively. The intrinsic deuteron wave function is taken to be of the Hulthén form, and

$$I_D(p_2) = N_D \left[(M_D^2 + p_2^2)^{-1} - (\nu_D^2 + p_2^2)^{-1} \right] \quad (12)$$

where, after Ref. ¹⁸⁾, we take $\mu_D \cong 0.252 \text{ fm}^{-1}$, $\nu_D \cong 1.763 \text{ fm}^{-1}$. This function differs rather little from the ones employed in Ref. ³⁾.

The corresponding distributions $R_P(P)$ and $R_{\pi}(p_{\pi})$ are presented in Figs. 6 and 7, respectively. Obviously, one common normalization of our amplitude (ratio of the $\Sigma-\Lambda$ conversion and the non- $\Sigma-\Lambda$ components is used for both $R_P(P)$ and $R_{\pi}(p_{\pi})$).

Similarly as in Ref. ³⁾ a semi-quantitative agreement with the data of Dahl et al. ¹⁹⁾ is obtained for both the distributions studied. Here again we find a better agreement for the distribution $R_P(P)$. The $\Sigma-\Lambda$ conversion spike in $R_{\pi}(p_{\pi})$ seems to be too sharp. This peak is much more important in the case of deuterium than in that of helium because of the absence of the "spectator" form factor F . Here, as in the case of helium, more data and more accurate calculations avoiding our approximations will be interesting. They will be necessary for the most important determination of the absolute values of the reaction rates and the cross-sections. The relevant calculations will provide an important test of the elementary coupling constants involved.

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FIGURE CAPTIONS

Figure 1 Diagram for the process : $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Sigma' + {}^3\text{I}' \rightarrow \pi^- + \Lambda + {}^3\text{He}$.

Figure 2 Momentum distribution of the ${}^3\text{He}$ from the reaction $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Sigma' + {}^3\text{I}' \rightarrow \pi^- + \Lambda + {}^3\text{He}$:

- a) 1S capture, ϕ_3 of Ref. 15) ;
- b) 1S capture, ϕ_3 of Ref. 16) ;
- c) 2P capture, ϕ_3 of Ref. 15) ;
- d) 2P capture, ϕ_3 of Ref. 16) ;

all the cases have an arbitrary common normalization.

Figure 3 Momentum distribution of the pion from the reaction $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Sigma' + {}^3\text{I}' \rightarrow \pi^- + \Lambda + {}^3\text{He}$:

- a) 1S capture, ϕ_3 of Ref. 15) ;
- b) 1S capture, ϕ_3 of Ref. 16) ;
- c) 2P capture, ϕ_3 of Ref. 15) ;
- d) 2P capture, ϕ_3 of Ref. 16) ;

all the cases have an arbitrary common normalization.

Figure 4 Total momentum distribution of the ${}^3\text{He}$ from the reaction $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Lambda + {}^3\text{He}$:

1S: case of the 1S capture, ϕ_3 of Ref. 16), the non- $\Sigma - \Lambda$ component is taken from Ref. 1) ;

$(1\text{S})^{\text{conv}}$ - the corresponding $\Sigma - \Lambda$ conversion component ;

2P: case of the 2P capture, ϕ_3 of Ref. 16), the non- $\Sigma - \Lambda$ component is taken from Ref. 2) ;

$(2\text{P})^{\text{conv}}$ - the corresponding $\Sigma - \Lambda$ conversion component.

The theoretical curves are arbitrarily normalized. The experimental histogram is taken from Ref. 12).

Figure 5

Total momentum distribution of the pion from the reaction $\bar{K} + {}^4\text{He} \rightarrow \pi^- + \Lambda + {}^3\text{He}$:

1S: case of the 1S capture, ϕ_3 of Ref. 16) and with a) the non- $\Sigma - \Lambda$ component of Ref. 1) ;

2P: case of the 2P capture, ϕ_3 of Ref. 16) and with b) the non- $\Sigma - \Lambda$ component of Ref. 2). The theoretical curves are arbitrarily normalized. The experimental histogram is taken from Ref. 12).

Figure 6

Kinetic energy (T_p) distribution of the proton from the reaction $\bar{K} + D \rightarrow \pi^- + \Lambda + p$:

a) our $\Sigma - \Lambda$ conversion component ;

b) the non- $\Sigma - \Lambda$ component of Ref. 3) ;

c) total.

The experimental histogram is of Ref. 19).

Figure 7

Kinetic energy ($T_\pi = E_\pi^0 - m_\pi$) distribution of the pion from the reaction $\bar{K} + D \rightarrow \pi^- + \Lambda + p$:

a) our $\Sigma - \Lambda$ conversion component ;

b) the non- $\Sigma - \Lambda$ component of Ref. 3) ;

c) total.

The experimental histogram is of Ref. 19).

FIG. 1

FIG.2

FIG.3

FIG.4

FIG. 5

FIG. 6

FIG.7