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CURRENT ALGEBRA AND THE DECAY $\eta \rightarrow 3\pi$

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A B S T R A C T

The process $\eta \rightarrow 3\pi$ is discussed by current algebra techniques and is found to be forbidden. Possible explanations of this alarming result are discussed briefly.

Recently very interesting results have been obtained for the decay $K \rightarrow 3\pi$ by Hara and Nambu ¹⁾ and Elias and Taylor ²⁾. These authors used the algebra of currents, PDDAC and a local current-current form for the effective Hamiltonian responsible for weak non-leptonic decay and, by assuming a linear dependence on the energy of the pions for the decay matrix element, were able to obtain extremely satisfying results for the rates and slope parameters in K decay by relating $K \rightarrow 3\pi$ to $K \rightarrow 2\pi$.

Bell ³⁾ showed that the assumption of locality could be relaxed so that the results would still apply in a theory with a vector boson. This suggests that one should try to obtain similar results in h decay where a current-current Hamiltonian arising from virtual photon exchange should be effective in producing the decay $h \rightarrow 3\pi$ ^{*}). In this note we show that this approach leads to the result that $h \rightarrow 3\pi$ is forbidden. We discuss possible explanations of this result briefly.

We take as our Hamiltonian

$$H = e^2 \int dy d_{\mu\nu}(-y) T(j_\mu(0), j_\nu(y))$$

where $j_\mu(z)$ is the electromagnetic current, and $d_{\mu\nu}(-y)$ is the photon propagator. Our calculation follows closely in notation that of Bell ³⁾; this allows us to keep track of the isospin of the pions easily. We write

$$j_\mu = \vec{s} \cdot \vec{J} + J^0$$

^{*}) This is particularly attractive in view of the similarity between the Dalitz plot of $K_2^0 \rightarrow \pi^+\pi^-\pi^0$ and $h \rightarrow \pi^+\pi^-\pi^0$; see, for instance, Veltman and Yellin ⁴⁾.

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where $\vec{s} = (0,0,1)$ is a numerical isovector which extracts the neutral component of the isovector $\vec{J}$ and J^0 is the isoscalar part. Now the G parity of $\vec{J}$ is positive and that of J^0 negative ^{*)}. Hence for the G violating decay $\eta \rightarrow 3\pi$, we need the amplitude,

$$\langle \pi_1 \pi_2 \pi_3 | T \int dy d_{\mu\nu}(-y) \vec{s} \cdot \vec{J}_\mu(0) J_\nu^0(y) | \eta \rangle$$

plus a similar term with 0 and y interchanged which can be treated similarly.

In applying the zero energy theorem for π_3 one requires two commutators whose conventional values are

$$\begin{aligned} \int d\vec{z} [\vec{\pi}_3 \cdot \vec{A}(\vec{z},0), \vec{s} \cdot \vec{J}_\mu(0)] &= \vec{\pi}_3 \times \vec{s} \cdot \vec{A}_\mu(0) \\ \int d\vec{z} [\vec{\pi}_3 \cdot \vec{A}(\vec{z},y_0), J_\mu^0(y)] &= 0, \end{aligned}$$

where, as in Ref. ³⁾, $\vec{\pi}_3$ is a numerical isovector labelling the isospin of π_3 and $\vec{A}(z)$ is the isovector axial vector current. Thus we obtain

$$\begin{aligned} &\langle \pi_1 \pi_2 \pi_3(p=0) | \int dy T(d_{\mu\nu}(-y) \vec{s} \cdot \vec{J}_\mu(0) J_\nu^0(y)) | \eta \rangle \\ &= \langle \pi_1 \pi_2 | T \left[\int d_{\mu\nu}(-y) (\vec{\pi}_3 \times \vec{s}) \cdot \vec{A}_\mu(0), J_\nu^0(y) \right] | \eta \rangle \end{aligned}$$

for which isospin dictates a form

$$(\vec{\pi}_3 \times \vec{s}) \cdot (\vec{\pi}_1 \times \vec{\pi}_2).$$

^{*)} We present this argument in terms of a C conserving electromagnetic theory, but it is easy to see that this is not essential.

But this form is excluded by Bose symmetry for the conventional assignment of the final 3 pions to an $I = 1$, totally symmetric state. Then if we assume a linear dependence of the matrix element with the energy of the odd pion as in Refs. ¹⁾⁻³⁾, we find η decay to 3 pions is forbidden.

Clearly this result is very worrying. We have at least four possible ideas which we list in increasing order of unattractiveness to us personally.

The first possibility is the old-fashioned idea that the slopes in η and K decay which appear similar ⁴⁾ are really due to final state interactions such as those due to a σ ^{5) *)}. Roughly one might then parametrize the amplitudes as proportional to $\sin \delta(k)/k$ ⁷⁾ where $\delta(k)$ is the S wave $\pi\pi$ phase shift in the $I = 0$ state of $\pi^+\pi^-$ and k is the momentum of the $\pi^+\pi^-$ in their centre-of-mass. Now strictly one only shows that the amplitude vanishes at $m_{\pi^+\pi^-} = m_\pi$ and $m_{\pi^+\pi^-} = m_\eta$, while in K decay we know the amplitude is non-zero at $m_{\pi^+\pi^-} = m_K$. Now, if $\delta(k)$ were to tend rapidly to 180° near $m_{\pi^+\pi^-} = m_\eta$ while still being $\sim 130^\circ$, say, at $m_{\pi^+\pi^-} = m_K$, we might find an explanation for the suppression of decay at these values of $m_{\pi^+\pi^-}$. The trouble with this explanation is that it is rather hard to see how the method of Refs. ¹⁾⁻³⁾ works so well. We are pursuing this question by looking for non-linear terms in the Dalitz plot with the CERN η decay group and with the authors of Ref. ⁵⁾.

A second possibility lies in the long-range nature of the photon interaction, which might lead to differences compared with the short ranged W . As pointed out by Amati, this might lead to the continuation in q^2 from 0 to m_π^2 being dubious. For instance, the diagram

*) The σ model in η decay has long been advanced by L.M. Brown and his collaborators ; see, for instance, Ref. ⁶⁾.

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could, for soft photons, lead to cut terms analogous to the pole terms found in P wave non-leptonic decays²⁾. Unfortunately, the above diagram, and all other similarly dangerous ones we have found so far, do not contribute to a final state where only S waves are present. We are pursuing this point further.

A third possibility is that the η does not decay electromagnetically but by some new interaction with $\Delta I = 1$ and $\Delta I = 3$. The $\Delta I = 3$ term has been suggested by Veltman and Yellin⁴⁾ to explain difficulties with the ratio $\eta \rightarrow 3\pi^0 / \eta \rightarrow \pi^+\pi^-\pi^0$ which, depending on the $\eta \rightarrow \pi^0\gamma\gamma / \eta$ total^{*)} ratio assumed, (may be) in very bad agreement with the conventionally assumed $I = 1$ final state. In this connection it may also be that there are $I = 2$ parts in the electromagnetic current as suggested by Kabir and Dombey¹⁰⁾. However, we find all these possibilities extremely unaesthetic.

Fourthly, there is the possibility that the results obtained for K decay, beautiful though they seem, are accidental. This we, in common with most other theorists, are very reluctant to accept.

*) The CERN decay parameter table, updated after Berkeley, gives $\eta \rightarrow \pi^+\pi^-\pi^0 = 22.4 \pm 1.8\%$ and $\eta \rightarrow 3\pi^0 = 20.7 \pm 5\%$. The discrepancy with the result of the conventional model, as discussed in Ref. ⁴⁾, $\eta \rightarrow 3\pi^0 / \eta \rightarrow \pi^+\pi^-\pi^0 = 1.7$ is thus about 3 standard deviations⁹⁾.

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