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CURRENT ALGEBRA AND SOME NON-STRONG MESONIC DECAYS

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ABSTRACT

A discussion is given of some radiative and isospin violating decays of mesons in the framework of current algebra. In particular $\pi^0 \rightarrow 2\gamma$, $\omega \rightarrow \pi^0 \gamma$, $\eta \rightarrow 2\gamma$ and $\eta \rightarrow 3\pi$ and considered and the possible implications of some recent theoretical proposals and experimental results for these processes are discussed.

In this note we should like to comment on some recent results concerning η decays. The most interesting of these seem to be the possible presence of $\Delta I=3$ transitions ¹⁾ as indicated by the low value of $R = (\Gamma_{3\pi^0}/\Gamma_{\pi^+\pi^-\pi^0})$ ²⁾, the large rate for $\eta \rightarrow \pi^0\gamma\gamma$ ²⁾, the large violation of SU(3) in the new measurement of $\Gamma_{\eta \rightarrow 2\gamma} = 1 \pm 0.5 \text{ keV}$ ³⁾, and the fact that, with a matrix element linear in the π^0 energy, current algebra forbids the $\eta \rightarrow 3\pi$ decay ⁴⁾, if a conventional second order electromagnetic integration is assumed to be responsible ^{*)}.

In connection with the last point, possible alternatives have been suggested. Veltman ⁵⁾ has suggested that one should modify the usual PCAC relation in the absence of weak and electromagnetic effects,

$$\partial_\mu A_\mu = c \phi_\pi$$

by adding terms involving gradients of the pion field. He has noted that not only $\eta \rightarrow 3\pi$ but also several other processes involving photons and π^0 's, such as $\pi^0 \rightarrow 2\gamma$, $\omega \rightarrow \pi\gamma$, $\eta \rightarrow \pi^0\gamma\gamma$ and $\omega \rightarrow \pi\pi\gamma$ ⁶⁾, are also forbidden in the usual approach but that these are allowed if one modifies PCAC as he suggests. However, we feel that when the weak interactions are also turned on the generalized gauge condition ⁷⁾ will also generate terms coming from these gradients which may upset the results for K decays ⁸⁾. In any event, we shall not discuss this further ^{**)}.

^{*)} In fact the η decays seem so anomalous and seem to show so many discrepancies that P. Csonka has suggested to the author that there may be strong, electromagnetic and weak decays and η decays. We shall not adopt this viewpoint here.

^{**)} We would like to point out that little progress has been made by us in the second way out suggested in Ref. ⁴⁾ for $\eta \rightarrow 3\pi$, namely that the long range nature of the photon field could spoil the results. However, we have a new, and clearer, argument that this should be unimportant. This is that for $\eta \rightarrow 3\pi^0$ the "dangerous diagrams" are clearly unimportant, and provided we assume an $I=1$, 3π final state, they must then be unimportant for $\eta \rightarrow \pi^+\pi^-\pi^0$ also.

Secondly, Adler ⁹⁾ has suggested modifying current algebra by adding a C violating term to the electromagnetic current. This term, K_μ , which contains not only $I=0$ and 1 , but also $I=2$ parts, is such that

$$[Q_\pm^5, K_\mu] = 0 \quad \text{but} \quad [Q_3^5, K_\mu] \neq 0$$

where Q_i^5 is the axial vector charge.

Assuming linearity for the $\eta \rightarrow 3\pi$ spectrum, Adler is then able to reproduce the correct slope and also to have some $\Delta I=3$ transitions, since his current contains $I=2$ parts. We shall mention briefly the possible implications for this theory of the above decay modes later.

Thirdly, several authors ¹⁰⁾⁻¹³⁾ have suggested using a scalar density $S_3(x)$ for the $\eta \rightarrow 3\pi$ decays. By reducing two pions to zero momentum, the first three groups of authors find :

$$\langle \pi^0 \pi^- \pi^+ | T | \eta \rangle = -\frac{1}{3} \frac{\mu^4}{c^2} \langle \pi^0 | \delta^{(e.m.)} m^2 | \eta \rangle$$

The value of the electromagnetic transition operator they then obtain as :

$$\langle \pi^0 | \delta^{(e.m.)} m^2 | \eta \rangle = \frac{1}{\sqrt{3}} (m_{K^+}^2 - m_{K^0}^2)$$

from $SU(3)$, which gives them a total η width of about 300 eV. This seems far too low to agree with the new measurement of $\Gamma_{\eta \rightarrow 2\pi}$ ^{*)}. Furthermore, if no low mass σ is assumed, by analogy with other low energy treatments ⁸⁾ in current algebra, one finds that R is too large. On the other hand, as shown in Ref. ¹³⁾, a good value of the slope is found, if only one pion is reduced as for K decay ¹⁴⁾. Thus this proposal has at least one good feature.

*) It must be cautioned that most of the error in the DESY result is systematic, not statistical, so that 0.5 keV should constitute a reasonable lower bound on $\Gamma_{\eta \rightarrow 2\pi}$.

We should like to make the somewhat heretical proposal that this interaction be interpreted not as an electromagnetic one, since the conventional electromagnetic interaction gives zero in the linear approximation, but as a new isospin violating interaction. We are tempted to suggest this since we have been dismayed to find no strong evidence that isospin violating effects are due only to second order effects. Indeed, last year Hamilton¹⁵⁾, in low energy πN phenomena, was discussing violations of isospin of several per cent in amplitude. We feel that this proposal may at least spur experimentalists to improve the limits on violations of isospin, and theoreticians to estimate more reliably that these are really of electromagnetic origin.

Once one accepts that S_3 is not an electromagnetic transition operator, it becomes possible to remedy the discrepancy in the $\eta \rightarrow 3\pi$ rates, since we should no longer identify the matrix element

$$\langle \pi^0 | \delta m^2 | \eta \rangle = \frac{1}{\sqrt{3}} (m_{K^+}^2 - m_{K^0}^2)$$

with $\langle \pi^0 | S_3 | \eta \rangle$, but instead should write

$$\delta m^2 = d S_3 + \delta^{(e.m.)} m^2.$$

where d is some constant.

One might then identify the contribution to the $K^+ - K^0$ mass difference from $\delta^{(e.m.)} m^2$ with the value calculated from low lying resonances, and $d S_3$ with the tadpole contribution of Coleman and Glashow¹⁶⁾. If one takes the phenomenological value for $d S_3$ obtained in this way rather than the universality value of Ref.¹⁶⁾, one finds an eta width of some 900 eV, which is still rather low. However, Harari¹⁷⁾ has recently argued that the electromagnetic interaction itself requires a subtraction in the calculation of the mass differences, so that the estimate of $\delta^{(e.m.)} m^2$, may be very unreliable, in which case one cannot easily

separate out the value of $dS_3^{*})$. Then it would not be possible to use the argument of rates either for or against this model $^{**})$.

If we wish to incorporate a $\Delta I=3$ interaction in this model without losing the correct value of the slope, this can be done very easily. Since the essential feature in obtaining the correct slope is the relation

$$[Q_+^5, H] = 0$$

supplemented by linearity, as can easily be seen from Refs. 9) and 13), this result will still be valid if $H \propto S_3^n$, where n is any integer, since

$$[Q_+^5, S_3] = 0.$$

Although such an interaction as $(S_3)^3$ is very unaesthetic, we feel that any $\Delta I=3$ interaction is rather ugly, so that if no $I=2$ part is found in the electromagnetic current, this will be a possible model.

*) It should be pointed out that if we believe that the $SU(3)$ prediction for $\langle \bar{\pi}^0 | \delta^{(e.m.)} m^2 | \eta \rangle$ is correct, and that it is suppressed by the current algebra and PCAC, we would conclude that $\langle \bar{\pi}^0 | dS_3 | \eta \rangle$ is still close to $1/\sqrt{3}(m_{K^+}^2 - m_{K^0}^2)$ and the scalar interaction model would still be in trouble. However, in the same way as we shall discuss below for the two photon decays, we may argue that the first assumption is not reasonable.

**) One should observe that the argument of Das et al. 18) that Schwinger terms may be important in η decay, apart from contradicting the widely held view that Schwinger terms should be unimportant for zero energy theorems 19), implies as with Refs. 10)-13), that $\Gamma_{\eta \rightarrow 2\gamma}$ should be near its $SU(3)$ value, but they do not have the freedom to take a different value for $\langle \pi^0 | \delta m^2 | \eta \rangle$, since their interaction is purely electromagnetic. Furthermore, as has been stressed by J.S. Bell to the author, Das et al. use the divergence equations 7), so that they are tacitly committed to ignoring Schwinger terms.

Before passing to the radiative processes, let us emphasize that both the bad results of current algebra and the low value of R can be explained without new assumptions if we consider a more complicated energy dependence. It has been claimed by Silvestrini et al.²⁰⁾ and by Chiu et al.¹³⁾, that the presence of a low mass σ , and in the former paper also of some p wave, can produce agreement with both R and the spectrum^{*)}. Woo²²⁾ has produced similar results by allowing an "intrinsic" variation of the matrix element without a σ . Further, Itzykson et al.²³⁾ have shown that an equally good fit can be obtained either with a linear matrix element or with a non-linear one satisfying the constraints of conventional current algebra. We have been informed by the CERN experimental group working on η decays that it is difficult in their experiment also to rule out quadratic terms large enough to ruin the extrapolation to the unphysical region. It seems that about 100,000 bias-free $\eta \rightarrow \pi^+ \pi^- \pi^0$ events are needed to guarantee the adequacy of the linear approximation reasonably well for our purposes.

A more serious objection to the use of S_3 is that it will not help in ensuring that the other forbidden processes are allowed. Let us now examine these proposals. For $\eta \rightarrow \pi^0 \gamma \gamma$ we can only say that the amplitude will be zero at $E_{\pi^0} = 0$, $E_{\gamma_1} = E_{\gamma_2} = m_\eta/2$. If we have a simple matrix element of the form

$$g F_{\mu\nu} F^{\mu\nu},$$

with g constant, then this will imply that g is zero. If one allows more complicated energy dependence, then one can only say that there should be some tendency for the Dalitz plot to extrapolate to zero outside

*) Note that the mass of the σ in Silvestrini et al.'s paper is too low to agree with the recent data on σ of Carroll et al.²¹⁾

the physical region at the above point. The difficulty with having more complicated energy dependence is, firstly, that one cannot easily see why, in view of the fact that no strong $\pi\eta$ interaction at low energies is known, these more complicated terms should appear, and secondly that, being of order $(m_\eta/3m)^2$, where m is some scaling mass, one might expect them to be small, especially since, as shown by Singer²⁴⁾, for instance, the observed $\eta \rightarrow \pi^0 \gamma\gamma$ rate is very large. Perhaps in view of the fact that the $\eta \rightarrow 3\pi$ amplitude seems experimentally to vanish at $E_{\pi^\pm}=0$, if one extrapolates the Dalitz plot as done in Ref. 8) for K decays, and also to be very large in the physical region¹⁾, one should regard the prediction of current algebra only as that one should see a tendency to extrapolate to zero at $E_{\pi^0}=0$, or else that one should have a surprisingly rapid variation with energy, above that given by the simplest matrix element^{*)}.

For the process $\bar{\pi}^0 \rightarrow 2\gamma$, one also obtains that conventional current algebra gives a suppression. At first sight it seems that this amplitude is such as to satisfy the conditions of Kellett²⁵⁾ for a kinematic zero at $q_{\pi^0}=0$, so that no conclusions can be drawn in current algebra. However, one can see as follows that this is not so because of the conditions of gauge invariance²⁶⁾. Let us write the amplitude for $\bar{\pi}^0 \rightarrow 2\gamma$ as :

$$= \epsilon_{\alpha\beta\gamma\delta} \epsilon_{1\alpha} \epsilon_{2\beta} K_{1\gamma} K_{2\delta} = \epsilon_{1\alpha} \epsilon_{2\beta} \int \langle 0 | T [j_{1\alpha}(x) j_{2\beta}(0)] | \bar{\pi}_q^0 \rangle e^{-ik_1 x} d^4x$$

On extracting the pion and using $\partial_\mu A_\mu = c \varphi_{\bar{\pi}}$ we find

$$\begin{aligned} & \frac{(q^2 + \mu^2)}{c} \epsilon_{1\alpha} \epsilon_{2\beta} \int \langle 0 | T [j_{1\alpha}(x) j_{2\beta}(0) \partial_\mu j_\mu^5(z)] | 0 \rangle e^{-ik_1 x + i q z} d^4x d^4z = \\ & = -i \frac{(q^2 + \mu^2)}{c} \epsilon_{1\alpha} \epsilon_{2\beta} q_\mu \int \langle 0 | T [j_{1\alpha}(x) j_{2\beta}(0) j_\mu^5(z)] | 0 \rangle e^{-ik_1 x + i q z} d^4x d^4z \\ & \quad + [\text{E.T.C.} = 0] \end{aligned}$$

*) I am grateful to Dr. S.L. Adler for a useful correspondence on this process, and on the decay of the $\bar{\pi}^0$ in his theory.

Now gauge invariance requires that

$$\int \langle 0 | T \bar{J}_{1\alpha}(x) j_{2\beta}(0) j_{\mu}^5(z) | 0 \rangle e^{-ik_1 x + i q z} d^4 x d^4 z$$

be not $\epsilon_{\alpha\beta\mu\sigma} k_{1\sigma}$, but instead be of the form

$$\epsilon_{\alpha\beta\nu\sigma} k_{1\nu} k_{2\sigma} q_{\mu}$$

Thus we obtain, for $q^2 \rightarrow 0$, that $g \rightarrow 0$. Thus one finds that, although the coefficient of $g \rightarrow 0$ when $q_{\mu} \rightarrow 0$, one still can reach the conclusion that g should be small by using gauge invariance.

Traditionally, it has been argued that π^0 decay is very slow²⁷⁾. Though we are not too convinced of this, since experimentally $g(m_{\pi}^2) \approx \frac{\alpha}{m_N}$, which does not seem very small, it may be argued that, in comparison to other second order electromagnetic processes involving strongly interacting particles^{1), 24)} it is rather small. The decay $\omega \rightarrow \pi^0 \gamma$ is similarly forbidden, provided one assumes ω is coupled to a conserved current²⁸⁾, in order to circumvent the theorem of Kellest²⁵⁾ again, but, since a reasonable model²⁹⁾ exists to relate $\pi^0 \rightarrow 2\gamma$ to $\omega \rightarrow \pi^0 \gamma$, our attitude to the forbiddenness of $\omega \rightarrow \pi^0 \gamma$ is closely related to that on $\pi^0 \rightarrow 2\gamma$.

We now discuss the large violation of $SU(3)$ implied by the recent measurement at DESY of $\Gamma_{\eta \rightarrow 2\gamma}$. $SU(3)$ predicts for this rate $(165 \pm 33) \text{ eV}$, if we use the latest value for $\Gamma_{\pi^0 \rightarrow 2\gamma}$ ²⁾. With mixing with the $X^0(959)$, a quadratic mass formula, and a quark model prediction for $X_1^0 \rightarrow 2\gamma$, one obtains $(400 \pm 80) \text{ eV}$ ³⁰⁾. With a linear mass formula, one obtains $\Gamma_{\eta \rightarrow 2\gamma} = 730 \pm 150 \text{ eV}$. While a mixing angle of -21° , as implied by the linear mass formula, would not contradict high energy quark model predictions³¹⁾, it is hard to understand on this basis why the partial width for $A_2 \rightarrow \pi \eta$ is smaller²⁾ than the $SU(3)$ prediction of 9 MeV, rather than much larger as predicted by Glashow and Socolow³²⁾ for

this sign of the mixing angle. If one uses a mixing angle of -10.8° , then there seems some indication from the DESY result that other symmetry breaking effects are at work ^{*)}.

If we accept that $\pi^0 \rightarrow 2\gamma$ is suppressed by current algebra and PCAC, an amusing possibility seems to us to arise for explaining this large violation of SU(3). If we consider the calculation for $\eta \rightarrow 2\gamma$, with the use of generalized PCAC for the η meson, one again concludes that g_η should tend to zero as $q^2 \rightarrow 0$. In this case, however, the extrapolation from $q_\eta^2 = 0$ to $q_\eta^2 = m_\eta^2$ is much more dubious, so that one may expect less suppression than for π^0 decay. Thus one can expect a large violation of SU(3), which arises from the mass difference between π^0 and η causing a breakdown of PCAC in the latter case. At the least we feel this possibility should be considered seriously in discussing the implications of the DESY result.

Let us now discuss the implications of Adler's proposal ⁹⁾ for the processes we have discussed. For $\eta \rightarrow \pi^0 \gamma \gamma$ one would not expect a zero as $E_{\pi^0} \rightarrow 0$. In view of the large rate this may be an advantage. In any case it would seem worth while to try to obtain a Dalitz plot for this process to find if there is any tendency for the amplitude to extrapolate to zero as $E_{\pi^0} \rightarrow 0$, as this would provide some test of Adler's algebra unless the matrix element turned out to be very complicated which in itself would be very interesting. For the $\pi^0 \rightarrow 2\gamma$ decay, although the E.T.C. is no longer zero, its contribution to the decay still vanishes, as has been emphasized to us in private communication from Adler. This arises because the matrix element coming from the equal time commutator has the form

^{*)} It is interesting to note that Weisskopf and Van Royen ³³⁾ have obtained $\Gamma_{\eta \rightarrow 2\gamma} = 1.3 \text{ keV}$ in a version of the static quark model. If X- η mixing is included in this calculation, with the specific mass dependent breaking of SU(6) which they suggest and a mixing angle of -10.8° , a width of 2.9 keV results, in poor agreement with experimental results.

$$\varepsilon_{\mu 1} \varepsilon_{\nu 2} \left[\int e^{ik_1 x} \langle 0 | T(K_\mu(x) L_\nu(0)) | 0 \rangle d^4 x \right]$$

where L_ν is the pseudovector current arising from the commutator

$$[Q_3^5, K_\mu] = L_\mu.$$

However, we have now only one momentum K_1 and from this it is impossible to form a pseudotensor; thus the term in square brackets is zero and the result follows. Whether this is to be regarded as good or bad for Adler's theory depends on one's attitude to the phenomenological rate and, as stated above, it is not clear to us what attitude one should adopt. As in the C conserving theory, $\omega \rightarrow \pi^0 \gamma$ is also forbidden, as in any case only the J_μ current can contribute in this case. Note that in the Gell-Mann-Sharp-Wagner model²⁹⁾ the $\omega \rightarrow \pi^0 \gamma$ rate is related to that of $\pi^0 \rightarrow 2\gamma$, while in Adler's theory the $\pi^0 \rightarrow 2\gamma$ decay receives contributions both from JJ and KK (although, of course, in the limit of exact PCAC, both are zero), so that this relation would be lost. The prediction of the Gell-Mann-Sharp-Wagner model are not in sufficiently good accord with experiment to allow this to be a serious flaw of Adler's theory [see, for example, Ref. ¹⁾]. Our argument that $\eta \rightarrow 2\gamma$ should not be given very well by $SU(3)$ due to the breakdown of generalized PCAC, remains true in the case of Adler's theory.

Finally, we wish to emphasize the interest of the decay $\omega \rightarrow \pi^+ \pi^- \gamma$ for these theories. This has been discussed by Conway⁶⁾, who showed that for soft pions this process should be inhibited. Unfortunately, the data which he quotes have since been withdrawn and there is now no evidence for this decay³⁴⁾. In Adler's theory, this suppression should still be present since only the J_μ part of the current contributes to the decay when the two pions are in s waves. A measurement of the spectrum and angular correlations would be very valuable, though difficult, since one could detect C violation by observing the p wave states, while separating out the s wave would give a test of the conventional current algebra while revealing the possible presence of a $\mathcal{G}$ when the pions are not very soft, as has been discussed phenomenologically by Yellin³⁴⁾.

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