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REMARKS ON DIFFRACTION SCATTERING OF HADRONS
AS SHADOW OF INELASTIC COLLISIONS

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ABSTRACT

A convergence difficulty in the uncorrelated jet model is pointed out. Remarks are made on some modifications of the model which may be introduced to remove the difficulty.

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I. INTRODUCTION

Unitarity relates the amplitude for elastic scattering to the amplitudes for inelastic processes. Consequently it is possible to use the available information about elastic scattering to eliminate or support models for inelastic final states. A study of this type was performed by one of us ¹⁾ (further quoted I) who found a class of models for the inelastic final states which yield an exponential overlap function. He proved further that the exponential overlap function corresponds to the well-known experimental shape for elastic scattering at small momentum transfer $\sqrt{-t}$

$$d\sigma/dt = (d\sigma/dt)_0 e^{at+bt^2}; \quad 0 < b \ll a^2. \quad (1)$$

The models considered in I have the following properties :

- 1) at high energies final states with large numbers of particles dominate the inelastic scattering ;
- 2) the particles are correlated by energy-momentum conservation, but there are no other correlations ;
- 3) the one-particle amplitudes are real, or more generally have all the same constant phase ;
- 4) the one-particle momentum distributions belong to a certain class specified in I ;
- 5) the steepest descent method, or equivalently the central limit theorem of the theory of probabilities, is applicable to the evaluation of the overlap function.

In the present paper we point out that assumptions 1)-5) are not compatible with the simultaneous requirement that the overlap function is exponential and that the parameter a in formula (1) does not decrease with increasing energy. In the next Section we expose the difficulty. In Section III we mention possible modifications of the models which might be used in order to restore consistency, and we present a few concluding remarks.

2.

II. THE CONVERGENCE DIFFICULTY ^{*)}

In I the crucial point of the proof that the overlap function is exponential is the proof of formula (3.34) ^{**)} :

$$\prod_j \bar{f}_j(\bar{u}_\mu, \theta) = \text{Const} \cdot e^{-A\theta^2} \quad (2)$$

where (3.33 and 3.34 of I)

$$\bar{f}_j(\bar{u}_\mu, \theta) = \bar{f}(0,0) (1 - a_j \theta^2 + b_j \theta^4 + \dots) \quad (3)$$

$$A = \sum a_j. \quad (4)$$

In order to obtain (2) it is necessary to neglect in the exponent the quartic term

$$\sum b_j \theta^4 - \frac{1}{2} \sum a_j^2 \theta^4. \quad (5)$$

It was shown in I that the second sum in (5) is indeed negligible compared to $A\theta^2$. Here we show that the first sum, which was not considered in I [see point a) in Section III below], is in general of the same order as the quadratic term and cannot be neglected without special assumptions. Our analysis is only a slight extension of the analysis given in I.

The c.m. energy of each final state particle has some value in the interval $[0, k]$, where k is the c.m. momentum of one of the incident particles. The interval is divided into $\mathcal{N}$ sub-intervals and the probability of finding a particle in the j -th sub-interval is denoted by g_j . The simplest possibility is to choose the intervals equiprobable and so numerous that the probability of finding two particles in one sub-interval is negligible. Then

^{*)} We present here a simplified analysis.

^{**)} This and other similar equation numbers refer to I.

$$\bar{g}_j = N/\mathcal{N} \quad (6)$$

where $N = \sum \bar{g}_j$ is the average number of particles. According to I

$$a_j \sim \bar{g}_j q_{oj}^2 \quad (7)$$

where $\sim$ means proportional with a constant proportionality factor, and q_{oj} is the energy of a particle in the j -th sub-interval ^{*}). The same argument yields

$$b_j \sim \bar{g}_j q_{oj}^4 \quad (8)$$

Using the distribution for q_o given in I (5.53) we obtain

$$\overline{q_o^n} \sim N^{-1} k^n. \quad (9)$$

Therefore

$$A\theta^2 = \mathcal{N} \overline{a_j} \theta^2 \sim N \overline{q_o^2} \theta^2 \sim k^2 \theta^2 \quad \text{and} \quad a^{-1} \sim N^{-1} k^2 / \overline{q_o^2} \sim N \overline{q_o^2} / \overline{q_o^2} \quad (10)$$

$$\sum a_j^2 \theta^4 = \mathcal{N} \overline{a_j^2} \theta^4 \sim \mathcal{N}^{-1} N^2 \overline{q_o^4} \theta^4 \sim \frac{N}{\mathcal{N}} k^4 \theta^4 \quad (11)$$

$$\sum b_j \theta^4 = \mathcal{N} \overline{b_j} \theta^4 \sim N \overline{q_o^4} \theta^4 \sim k^4 \theta^4. \quad (12)$$

For physically interesting values of θ ($k\theta$ of order 1) the correction (11) can be made arbitrarily small by choosing $\mathcal{N}$ very large. Correction (12), however, is seen to be of the same order as the main term.

^{*}) It is easy to generalize the model so that the coefficient in (7) becomes an arbitrary bounded function of q_{oj} , but for the moment we keep all the assumptions made in I.

4.

The results (10)-(12) were derived for a very special energy distribution for the final state particles, see Eq. (5.53). We shall show that it is impossible to remove the difficulty by changing this distribution. Indeed we would need

$$\sum b_j / (\sum a_j)^2 \xrightarrow{N \rightarrow \infty} 0. \quad (13)$$

In view of (10) and (12) this is equivalent to

$$N^{-1} (\overline{q_0^4} / \overline{q_0^2})^2 \xrightarrow{N \rightarrow \infty} 0. \quad (13.a)$$

Let us substitute the condition

$$N = O(\overline{q_0^2} / \overline{q_0^2}) \quad (14)$$

which is necessary in order to make the diffraction peak width a^{-1} in formula (10) a non-increasing function of energy [compare (5.13)]. We obtain

$$X = \overline{q_0^4} / \overline{q_0^2} \xrightarrow{N \rightarrow \infty} 0.$$

In order to see that this is impossible it is enough to notice that

$$(\overline{q_0^2} - 2)^{-1} (\overline{q_0^2} - 2)^2 = X - 2 + O(1/N) \gg 0. \quad (15)$$

III. DISCUSSION

The preceding analysis implies that the models presented in I can only predict a diffraction peak of shape (1) if inequality (14) is reversed, i.e., if the width a^{-1} increases with energy. A modification is necessary to reach consistency with the general experimental indication that diffraction peaks approach constant width at high energy. It should be noted at this point that other investigations have independently shed doubt on the possibility to obtain a peak width as narrow as the observed one by means of an overlap function obeying assumptions 1), 2), 3) of Section I. This was found first by Fukuda and Iso ²⁾ using simple models for very high energy inelastic collisions. These authors maintain assumption 2) but depart from 3) by introducing a phase for the single pion wave function; they conclude that this phase must be appreciable to make a^{-1} as small as observed. Quite recently, more convincing evidence ³⁾ is being obtained from actual measurements of inelastic π^+p collisions at 8 GeV/c, which allow to estimate the overlap function under assumptions 1), 2), 3). Again the resulting peak width is much larger than the observed one. A strengthening of these experimental indications will be of the greatest interest.

In the present discussion we shall list some possibilities which suggest themselves when attempting to circumvent the negative conclusion reached in Section II.

a) A partial cancellation mechanism

As mentioned in the preceding Section the coefficients in formulae (7) and (8) could depend on j . It was proved in I that all the a_j are non-negative, therefore we expect no cancellations in the sum (4). The coefficients in (8) might, however, have different signs and make the sum of b_j small because of cancellations. It was in fact for this reason that the first sum in expression (5)

was not considered in I. Using formula (3) we may formulate the corresponding condition as follows. $\bar{f}_j$ averaged over j is a linear function of θ^2 . This is a very special assumption, but if it is done consistency is recovered between assumptions 1) to 5) and energy independence of a . The experimental indications ^{2),3)} that the resulting value of a is too small to fit the observed peak width remain nevertheless valid.

b) Phase factors

The desired exponential overlap function is easily obtained if it is not assumed that the one-particle amplitudes are real. Fukuda and Iso ²⁾ stressed that in absence of correlations phase factors are necessary to reproduce the experimental value of the slope of the overlap function. We tried a simple one-particle distribution

$$c(\hat{p}) = |c(\hat{p})| \exp(-i \frac{\alpha}{2} p_T^2) \quad (16)$$

and obtained a reasonable fit to the experimental data for pp scattering with

$$\alpha^2 \approx 2N m_{\pi}^{-2}. \quad (17)$$

In the calculation we used a rigorous steepest descent method and we were able to show that all the neglected terms are negligible for N tending to infinity.

The remarkable feature of amplitude (16)-(17) is that it gives an interaction radius (as seen by the out-going particles) which increases indefinitely with increasing energy. This can be seen by using the operator formula

$$r^2 = \partial^2 / \partial p_T^2 \quad (18)$$

and taking the expectation value for the wave function (16)-(17). According to the optical model picture one expects the final radius to be equal to the initial, and then our model breaks down very soon. There are however other pictures. We quote one which corresponds to the hydrodynamical model. In this model the system immediately after the collision cannot be described as a system of N non-interacting particles. It is visualized as a drop of something, where the number of particles is not a good quantum number. This drop expands adiabatically and the energy density in it decreases. Only after it falls below a certain critical value, the independent particle description makes sense. Using this picture it is very natural to expect that the final radius is an increasing function of the average multiplicity.

In discussing the consequences of assumption (16), the assumption was made that all the one-particle amplitudes in the final state have comparable phase factors and correspond to comparable energy distributions. Actually in an inelastic final state one often sees the so-called leading particles, i.e., a pair of particles which take away about half of the energy available in the c.m. system. When calculating the overlap function with the assumption that the amplitudes are real, one finds that the contribution of the leading particles is comparable to the contribution of all the remaining particles taken together ^{5),3)}. Therefore if the leading particles are included and if it is assumed that their amplitudes are real, our conclusions remain valid. If, however, the leading particles have phase factors it might happen that their contribution is dominant. In the most drastic case the behaviour of the "non-leading" secondaries would be essentially irrelevant for the overlap function, in particular their amplitudes might be consistent with conditions 1) to 5) from Section I, and elastic scattering would impose conditions only on the amplitudes of the leading particles.

c) Correlations

Condition (14), which is at the root of the convergence difficulty, is necessary only if there are no correlations. Even weak correlations may change completely the result. A model with weak correlations, where an elastic peak with increasing a was obtained as a shadow of inelastic scattering, was described by Amati, Cini and Stanghellini⁴⁾. Another form of correlation occurs when an appreciable fraction of inelastic collisions are of two-body type (where a body can be a particle or a resonance), or more generally go through low multiplicity intermediate states.

While possibility a) is unlikely to be compatible with experiment, possibilities b) and c) allow to construct overlap functions agreeing with the observed diffraction peak shapes and with our limited experimental knowledge of inelastic final states. They unfortunately involve too many free parameters and functions, but it may be hoped that this difficulty can be gradually overcome as we gain more insight in the structure of dominant inelastic collisions. Our qualitative conclusion at present is that the observed properties of the elastic diffraction peak are incompatible with the simple assumptions made in I and repeated above for the dominant inelastic collisions, and that phase factors of single particle wave functions and/or correlations between particles offer natural possibilities to restore agreement. Since phase factors are not directly accessible to measurement, the study of correlations between particles produced in inelastic collisions deserves particular attention.

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