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PHENOMENOLOGICAL METHOD FOR KAON INTERACTIONS

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A B S T R A C T

The method of phenomenological fields and Lagrangians can be generalized to describe interactions involving strange particles. The examples of the strong interactions between pions and kaons and of the weak leptonic decays of the kaon are described in detail.

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The assumptions of (broken) chiral $SU(2) \times SU(2)$ and of total or partial conservation of currents are usually analyzed by the methods of the algebra of currents. Recently a different technique has been introduced, which makes use of phenomenological fields and Lagrangians ^{1),2),3)}. This technique has been applied so far only to the interactions of non-strange particles; in this note we show, on the example of the kaon, how the method of phenomenological fields can be generalized to strange particles and their interactions.

For simplicity we consider a system consisting of pions and kaons only. As explained in Ref. ²⁾, the isotriplet pion field transforms under a chiral transformation as

$$a \delta \vec{\Phi} = \alpha (1 - a^2 \vec{\Phi}^2) + 2 a^2 \vec{\Phi} \alpha \cdot \vec{\Phi} \quad (1)$$

where

$$a = \frac{1}{F_\pi}$$

and F_π is the usual pion decay constant. For the isodoublet field K of the kaon we assume the corresponding transformation law

$$\delta K = i a (\vec{\Phi} \times \alpha) \cdot \tau K \quad (2)$$

This is the same as the transformation law appropriate to the nucleon and leaves the expression $K^\dagger K$ invariant.

By using the methods of Ref. ²⁾ it is easy to construct a Lagrangian for the pion-kaon system which is invariant under $SU(2) \times SU(2)$. It is

$$\begin{aligned}
L = & -\frac{1}{2}(1+a^2\Phi^2)^{-2}(\partial_\mu\Phi)^2(1+bK^\dagger K) - \partial_\mu K^\dagger \partial_\mu K + \\
& +ia^2(1+a^2\Phi^2)^{-1}(\partial_\mu\Phi \times \Phi) \cdot (\partial_\mu K^\dagger \tau K - K^\dagger \tau \partial_\mu K) + \quad (3) \\
& -a^4(1+a^2\Phi^2)^{-2}\left\{\Phi^2(\partial_\mu\Phi)^2 - \frac{1}{4}(\partial_\mu\Phi^2)^2\right\}K^\dagger K - m_K^2 K^\dagger K
\end{aligned}$$

where the parameter b is arbitrary. This Lagrangian describes the strong interactions of pions and kaons. A symmetry breaking term, which includes a pion mass term, should be added to (3). Its possible form is discussed in Ref. ²⁾. We omit it here since it plays no role in the following. The scattering of two pions and two kaons is obtained by expanding (3) to second order in Φ

$$\begin{aligned}
L_{KK\pi\pi} = & -\frac{b}{2} K^\dagger K (\partial_\mu\Phi)^2 + \quad (4) \\
& +ia^2(\partial_\mu\Phi \times \Phi) \cdot (\partial_\mu K^\dagger \tau K - K^\dagger \tau \partial_\mu K) .
\end{aligned}$$

The strangeness changing vector and axial vector currents V_μ and A_μ are characterized by their transformation properties under $SU(2) \times SU(2)$. They are isodoublets and, under a chiral transformation

$$\begin{aligned}
\delta V_\mu &= i\alpha \cdot \tau A_\mu \\
\delta A_\mu &= i\alpha \cdot \tau V_\mu . \quad (5)
\end{aligned}$$

This transformation law, together with (1) and (2), determines the form of V_μ and A_μ as functions of Φ and K . Omitting terms which contain the tensor $\varepsilon_{\lambda\nu\rho}$, and which transform separately, we obtain

$$\begin{aligned}
V_\mu &= ia F_K \partial_\mu \left\{ (1+a^2 \Phi^2)^{-\frac{1}{2}} \Phi \cdot \tau K \right\} - ia G (1+a^2 \Phi^2)^{-\frac{3}{2}} \partial_\mu \Phi \cdot \tau K \\
A_\mu &= F_K \partial_\mu \left\{ (1+a^2 \Phi^2)^{-\frac{1}{2}} K \right\} + \\
&\quad + a^2 G (1+a^2 \Phi^2)^{-\frac{3}{2}} \left\{ \Phi \cdot \partial_\mu \Phi + i(\Phi \times \partial_\mu \Phi) \cdot \tau \right\} K
\end{aligned} \tag{6}$$

where F_K and G are two coefficients. Expanding in the pion field we have

$$\begin{aligned}
V_\mu &= ia F_K \partial_\mu (\Phi \cdot \tau K) - ia G \partial_\mu \Phi \cdot \tau K + [a^3 \Phi^3] \\
A_\mu &= F_K \partial_\mu K - \frac{1}{2} a^2 F_K \partial_\mu (\Phi^2 K) + \\
&\quad + a^2 G \left\{ \Phi \cdot \partial_\mu \Phi + i(\Phi \times \partial_\mu \Phi) \cdot \tau \right\} K + [a^4 \Phi^4] .
\end{aligned} \tag{7}$$

These formulas give immediately the matrix elements which enter in the leptonic decays of the kaon. Clearly, one must identify F_K with the kaon decay constant; the other coefficients are seen to satisfy the Callan-Treiman relations⁴⁾. For the matrix element of A_μ between a one-kaon and a two-pion state it is necessary to evaluate also the contribution of the term $F_K \partial_\mu K$. This arises through the interaction (4) by exchange of a virtual kaon. The calculation is elementary and gives a result in agreement with that obtained with soft pion techniques⁵⁾.

The results obtained above owe their simplicity to the particular choice of the field K as specified by the particular transformation law (2). We could have chosen, for instance, the field

$$K' = K (1 + a^2 \Phi^2)^{\frac{1}{2}}$$

which transforms as

$$\delta K' = a \{ \Phi \cdot \alpha + i(\Phi \times \alpha) \cdot \tau \} K' \quad (8)$$

If one expresses the Lagrangian and the currents in terms of K' , they have a more complicated form. The above results can still be obtained, but only as a consequence of cancellations. This provides a simple illustration of the importance of choosing the right phenomenological field to describe a given particle. Note that the "wrong" transformation law (8) is the transformation law which one would obtain if one specializes to $SU(2) \times SU(2)$ an $SU(3) \times SU(3)$ transformation obtained by a naive generalization of the methods of Ref. ²⁾.

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R E F E R E N C E S

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