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S MESON DOMINANCE AND CURRENT ALGEBRA

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A B S T R A C T

We combine the ideas of vector meson dominance as applied to three and four-point functions with those of current algebra in a simple situation. We find in particular that the slow variation assumption of three-point functions can be justified.

1. INTRODUCTION

The ideas of vector meson dominance and universality have received a lot of attention in high energy physics ¹⁾. A characteristic feature of most calculations using the vector meson dominance model (VDM) is the neglect of off-mass shell corrections ²⁾ since the universality principle refers to vector mesons having zero momentum and zero total energy. In this article we combine such ideas with those of current algebra (current commutation relations and PDDAC) using the techniques of nearby singularities as described, for example, by Jacob ³⁾. We discuss the slow variation assumption and find that it can be justified for the three-point function. We cannot make a definite statement, however, for the four-point function. In the next Section we briefly recapitulate the formulation of VDM and then proceed to establish our results.

2. DERIVATION

We take the viewpoint of general quantum field theory. To simplify the analysis we treat the vector mesons as though they were stable. Problems arising from instability are not very severe and Gell-Mann and Zachariasen ¹⁾ have indicated how one should cope with them. An important result of general field theory ⁴⁾ is that any field $A(x)$ which carries the quantum numbers of a particle may be used to describe the scattering process of such a particle. The essence then of VDM is that one can write the electromagnetic current as a superposition of renormalized field operators for the neutral vector mesons ρ^0, ω, ϕ ^{5), 6)} [see Ref. 5) for details]. I.e.,

$$j_{\mu}^{em}(x) = - \left(\frac{m_{\rho}^2}{2\gamma_{\rho}} S_{\mu}^0(x) + \frac{m_{\phi}^2}{2\gamma_{\phi}} \phi_{\mu}(x) + \frac{m_{\omega}^2}{2\gamma_{\omega}} \omega_{\mu}(x) \right) \quad (1)$$

2.

Here m_ρ , m_ω , m_ω are the masses of the vector mesons and γ_ρ , γ_ω are constants. Such an identification has some practical consequences. From the asymptotic condition in field theory one can write for the T matrix element describing the process $V+A \rightarrow B+C$

$$\langle p_B, p_C | T | q, \lambda; p_A \rangle = \langle p_B, p_C | j_\mu^V(0) | p_A \rangle e_\mu^\lambda(q) \delta^4(p_B + p_C - q - p_A) \quad (2)$$

with

$$j_\mu^V(x) = (\square + m_V^2) V_\mu(x)$$

$e_\mu^\lambda(q)$ is the polarization vector of the vector meson. We have suppressed the spin indices of particles A, B, C.

Similarly, the T matrix element for the photoproduction process $\gamma + A \rightarrow B + C$ reads

$$\langle p_B, p_C | T | q, \lambda'; p_A \rangle = e \langle p_B, p_C | j_\mu^{e.m.}(0) | p_A \rangle e_\mu^{\lambda'}(q) \delta^4(p_B + p_C - q - p_A) \quad (3)$$

where $\square A_\mu(x) = e j_\mu^{e.m.}(x)$. $A_\mu(x)$ is the vector potential and $j_\mu^{e.m.}(x)$ is the electromagnetic current. $e_\mu^{\lambda'}(q)$ is the polarization vector of the photon with momentum q and helicity λ' . With the identification given by Eq. (1), the Eqs. (2) and (3) seem to differ only by a factor

$$\frac{e}{2\gamma_V} \frac{m_V^2}{q^2 - m_V^2}$$

in the different pieces of the electromagnetic current. However, there is a very important difference because in Eq. (2) we have

$$q^2 = (p_B + p_C - p_A)^2 = m_v^2 \quad (4)$$

in the scattering matrix element for a physical φ meson while in Eq. (3)

$$q^2 = (p_B + p_C - p_A)^2 = 0 \quad (5)$$

The assumption is then made ⁵⁾ that the matrix element of $j_\mu^v(0)$ is slowly varying in q^2 for fixed m_A^2 , m_B^2 , m_C^2 and fixed $t_{AC} = (p_A - p_C)^2$ or fixed $t_{AB} = (p_A - p_B)^2$ or under any other suitable kinematical condition. Of course, such an extrapolation procedure would make sense only provided no kinematical singularities are encountered when varying q^2 from m_v^2 to zero. Neglecting the variation of $\langle p_A, p_B | j_\mu^v(0) | p_C \rangle$ in calculating cross-sections, one then gets ⁵⁾

$$\sigma(\gamma + A \rightarrow B + C) = \frac{\alpha \pi}{\gamma_v^2} \sigma_{tr}(V + A \rightarrow B + C) \quad (6)$$

in the special case when only one piece of the electromagnetic current [Eq. (1)] contributes. On the right-hand side of Eq. (6) we have the cross-section for transversely polarized vector mesons. Note here, however, that the slow variation assumption has been extended to a four-point function. Originally this was done only for three-point function, i.e., electromagnetic form factors and calculations based on single particle exchange models ²⁾. This extension is similar in spirit to PCAC when extended to apply to scattering amplitudes as well as to form factors ⁷⁾. In the case of PCAC one expects corrections of order

m_π^2/m_N^2 which are negligible because of the smallness of the pion mass. Here, however, it is not a priori clear why such corrections should be negligible as the difference in masses is not great. The slow variation assumption applied to the electromagnetic form factor of the pion, for example, leads one to replace the ϱ - π - π form factor by a coupling constant $g_{\varrho\pi\pi}$ and the result is the same as would be obtained if one writes an unsubtracted dispersion relation for the pion electromagnetic form factor and keeps only the ϱ term. The derivation of Eq. (6) is given by Joos⁵⁾ who also discusses several other applications.

Now define an amplitude $T_{\mu\nu}^{ij}$ as the Fourier transform of the retarded product of a vector current and an axial vector current taken between nucleon states of momenta p, p'

$$T_{\mu\nu}^{ij}(p', q'; p, q) = \int d^4x e^{-iq \cdot x} \langle p' | R(j_\mu^i(0) j_\nu^{Aj}(x)) | p \rangle \quad (7)$$

Contract with q^ν

$$T_{\mu\nu}^{ij} q^\nu = t_\mu^{ij} + F_\mu^{ij} \quad (8)$$

where

$$\begin{aligned} t_\mu^{ij} &= -i \int d^4x e^{-iq \cdot x} \langle p' | R(j_\mu^i(0) \partial^\nu j_\nu^{Aj}(x)) | p \rangle \\ F_\mu^{ij} &= - \int d^4x e^{-iq \cdot x} \langle p' | \delta(x_0) [j_\mu^i(0), j_0^{Aj}(x)] | p \rangle \end{aligned} \quad (9)$$

choosing as variables

$$\begin{aligned} s &= (p+q)^2, \\ t &= (p'-p)^2, \\ u &= (p'-q)^2, \end{aligned}$$

we then have

$$T_{\mu\nu}^{ij}(s,t;q^2,q'^2) - t_{\mu}^{ij}(s,t;q^2,q'^2) = F_{\mu}^{ij}(t). \quad (10)$$

We make a graphical decomposition of $T_{\mu\nu}^{ij}$ as shown in Fig. 1. t_{μ}^{ij} is also decomposed in an analogous fashion as shown in Fig. 2. In these graphs the symbol $\times^{ij\dots}$ represents the part of the axial current j_{ν}^{Aj} which is coupled to the pion field, while the symbol $\times$ represents the rest of j_{ν}^{Aj} . The symbols $\overset{j}{\circ}\dots\dots$ and $\circ$ have the same significance for the divergence $\partial^{\nu} j_{\nu}^{Aj}$. Similarly, the symbols $\times^{i\mu}\dots\dots$ denote the part of the vector current j_{μ}^i coupled to the vector meson field (see below) while $\times$ denotes the rest of j_{μ}^i . (No confusion should arise from using the same symbol $\times$ for both vector and axial vector currents.) The meaning of these graphical decompositions will be transparent in the following analysis.

The coupling of the axial current j_{μ}^{Ai} with the pion is by definition

$$\langle 0 | j_{\mu}^{Ai}(0) | \pi^i(q) \rangle = i F_{\pi}(q^2) q_{\mu} \quad (11)$$

Therefore,

$$j_{\mu}^{Ai}(x) = -F_{\pi} \partial_{\mu} \phi_{\pi}^i(x) + \tilde{j}_{\mu}^{Ai}(x) \quad , \quad (12)$$

where $\phi_{\pi}^i(x)$ is the pion field (we restrict the indices i, j to the isospin values only) and $\tilde{j}_{\mu}^{Ai}(x)$ is the part of the current not coupled to the pion. We write the isovector current as

$$j_{\mu}^i(x) = \Gamma \phi_{\mu}^i(x) + \tilde{j}_{\mu}^i(x) \quad , \quad (13)$$

where $\phi_{\mu}^i(x)$ is the ϱ meson field operator and Γ is a constant. $\tilde{j}_{\mu}^i$ is the ϱ - pole-free part of j_{μ}^i . In $T_{\mu\nu}^{ij}$ we effect the substitution for j_{μ}^i as given by Eq. (13) as well as

$$j_{\nu}^{Aj}(x) \rightarrow -F_{\pi} i q_{\nu} \phi_{\pi}^j(x) + \tilde{j}_{\nu}^{Aj}(x)$$

Hence, we decompose $T_{\mu\nu}^{ij}$ into four terms where the ϱ pole in q'^2 and the pion pole in q^2 appear explicitly

$$\begin{aligned} T_{\mu\nu}^{ij} = & -i F_{\pi} q_{\nu} \frac{\Gamma}{m_{\pi}^2 - q^2} \frac{1}{m_{\varrho}^2 - q'^2} M_{\mu}^{ij}(s, t; q^2, q'^2) \\ & - i \Gamma \frac{1}{m_{\varrho}^2 - q'^2} \Lambda_{\mu\nu}^{ij}(s, t; q^2, q'^2) \\ & + F_{\pi} q_{\nu} \frac{1}{m_{\pi}^2 - q^2} \bar{\Lambda}_{\mu}^{ij}(s, t; q^2, q'^2) \\ & + R_{\mu\nu}^{ij}(s, t; q^2, q'^2) \quad , \end{aligned} \quad (14)$$

where we have the following definitions

$$\begin{aligned}
 M_{\mu}^{ij}(s,t;q_L^2,q'^2) &= (m_{\pi}^2 - q_L^2)(m_{\pi}^2 - q'^2) \int d^4x e^{-iq_L \cdot x} \langle p' | R(\phi_{\mu}^i(0) \phi_{\pi}^j(x)) | p \rangle, \\
 \Lambda_{\mu\nu}^{ij}(s,t;q_L^2,q'^2) &= i(m_{\pi}^2 - q'^2) \int d^4x e^{-iq_L \cdot x} \langle p' | R(\phi_{\mu}^i(0) \tilde{j}_{\nu}^{Aj}(x)) | p \rangle, \\
 \bar{\Lambda}_{\mu}^{ij}(s,t;q_L^2,q'^2) &= -i(m_{\pi}^2 - q_L^2) \int d^4x e^{-iq_L \cdot x} \langle p' | R(\tilde{j}_{\mu}^i(0) \phi_{\pi}^j(x)) | p \rangle, \\
 R_{\mu\nu}^{ij}(s,t;q_L^2,q'^2) &= \int d^4x e^{-iq_L \cdot x} \langle p' | R(\tilde{j}_{\mu}^i(0) \tilde{j}_{\nu}^{Aj}(x)) | p \rangle.
 \end{aligned}
 \tag{15}$$

In M_{μ}^{ij} we recognize the scattering amplitude for the process $\pi^j + N \rightarrow \pi^i + N$ defined for off mass-shell pion and π meson. $\Lambda_{\mu\nu}^{ij}$ represents the amplitude for π production by scattering a nucleon in an external axial field coupled with the current $\tilde{j}^A$. $\bar{\Lambda}_{\mu}^{ij}$ and $R_{\mu\nu}^{ij}$ do not have such simple interpretations.

We recognize in the four successive terms of Eq. (14) the contributions from the graphs in Fig. 1 (a and b), (c and d), (e and f) and (g and h) respectively. Similarly

$$\begin{aligned}
 t_{\mu}^{ij} &= -i m_{\pi}^2 F_{\pi} \frac{1}{m_{\pi}^2 - q_L^2} \frac{1}{m_{\pi}^2 - q'^2} M_{\mu}^{ij}(s,t;q_L^2,q'^2) \\
 &\quad + m_{\pi}^2 F_{\pi} \frac{1}{m_{\pi}^2 - q_L^2} \bar{\Lambda}_{\mu}^{ij}(s,t;q_L^2,q'^2),
 \end{aligned}
 \tag{16}$$

8.

where we have assumed PDDAC which amounts to neglecting graphs c, d, g, h, in Fig. 2 in which the divergence $\partial^\nu j_\nu^{Aj}$ is not coupled through the pion field and have set

$$\partial^\nu j_\nu^{Aj}(x) = F_\pi m_\pi^2 \phi_\pi^j(x) \quad (17)$$

Substituting these expressions for $T_{\mu\nu}^{ij}$ and t_μ^{ij} into Eq. (10) we find

$$\frac{i\Gamma}{m_g^2 - q'^2} \left[F_\pi M_\mu^{ij} - \Lambda_{\mu\nu}^{ij} q^\nu \right] - F_\pi \bar{\Lambda}_\mu^{ij} + R_{\mu\nu}^{ij} q^\nu - F_\mu^{ij} = 0 \quad (18)$$

The pole in q'^2 has disappeared because of PDDAC, Eq. (17). The residue of the q pole must vanish and therefore

$$F_\pi M_\mu^{ij}(s, t; q_L^2, m_g^2) = \Lambda_{\mu\nu}^{ij}(s, t; q_L^2, m_g^2) q^\nu \quad (19)$$

This relation is written at the point $q'^2 = m_g^2$. As discussed earlier the extended VDM as applied to four point functions tells us that M_μ^{ij} and $\Lambda_{\mu\nu}^{ij}$ are slowly varying in q'^2 . Assume now that the relation (19) is valid in some neighbourhood to the left of m_g^2 extending as far as $q'^2 = 0$. I.e., we set

$$F_\pi M_\mu^{ij}(s, t; q_L^2, q'^2) = \Lambda_{\mu\nu}^{ij}(s, t; q_L^2, q'^2) q^\nu, \quad 0 \leq q'^2 \leq m_g^2 \quad (20)$$

With this we are able to deduce another relation from Eq. (18), namely

$$R_{\mu\nu}^{ij} q^\nu = F_{\pi} \bar{\Lambda}_{\mu}^{ij} + F_{\mu}^{ij} \quad (21)$$

which is also valid in the same region. If we now set $\tilde{j}_{\mu}^i$ equal to zero, i.e., assume that $j_{\mu}^i = \gamma \delta_{\mu}^i$ only, we immediately run into trouble because from definition (15) we see that both $R_{\mu\nu}^{ij}$ and $\bar{\Lambda}_{\mu}^{ij}$ vanish identically and hence by Eq. (21) F_{μ}^{ij} will vanish. Using the current commutation relations of Gell-Mann

$$[j_{\mu}^i(0), j_0^A(x)] \delta(x_0) = i \epsilon^{ijk} j_{\mu}^A(x) \delta^4(x), \quad (22)$$

we see that

$$F_{\mu}^{ij}(t) = -i \epsilon^{ijk} \langle p' | j_{\mu}^A(0) | p \rangle, \quad (23)$$

is the matrix element of the axial current between nucleon states which is clearly different from zero. There are at least two ways out of this difficulty. For while it is true that the extended VDM leads us to expect that M_{μ}^{ij} and $\Lambda_{\mu\nu}^{ij}$, which have well defined physical meaning, are slowly varying in q'^2 , the same might not be true of $\Lambda_{\mu\nu}^{ij} q^\nu$. i.e., when $\Lambda_{\mu\nu}^{ij}$ is expanded on a tensor basis contraction with q^ν could introduce a dependence on q'^2 for which our initial assumptions are not tailored. We cannot exclude such a possibility, but even if it turns out that contracting with q^ν does not generate a term proportional to q'^2 we still have a way out by keeping non g pole terms in the isovector current. To see this more clearly we look at the example of radiative μ capture treated by Adler and Dothan⁹⁾.

In the reaction

$$\mu^- + p \rightarrow \nu + \gamma + n$$

the matrix element $\langle n\gamma | j_\mu | p \rangle$ occurs in the expression for the T matrix element describing the process. j_μ is the weak current. Contracting the photon from the "out" state this can be written as

$$\begin{aligned} \langle n\gamma | j_\mu(0) | p \rangle &= i\epsilon^\lambda \int d^4x e^{iq \cdot x} \langle n | R(j_\lambda^{em}(x) j_\mu(0)) | p \rangle \\ &\quad + i\epsilon^\lambda S_{\lambda\mu} \end{aligned} \quad (24)$$

where

$$S_{\lambda\mu} = \int d^4x e^{iq \cdot x} \delta(x_0) \langle n | [\partial A_\lambda(x) / \partial x_0, j_\mu(0)] | p \rangle \quad (25)$$

In arriving at Eq. (24) one assumes that the weak current j_μ commutes with the vector potential A_λ at equal times. $S_{\lambda\mu}$ is the so-called "sea-gull" term and describes the coupling of the weak and electromagnetic current at the same point. Now the physical principle of vector meson dominance allows us to replace the outgoing photon by an outgoing meson. This should be done in the covariant matrix element. Clearly, such a replacement is not meaningful in $S_{\lambda\mu}$ which cannot be dropped in general when calculating the covariant matrix element $\langle n\gamma | j_\mu | p \rangle$ as emphasized by Adler and Dothan⁹⁾. Since our basic amplitude $T_{\mu\nu}^{ij}$ does not directly describe a physical process this circumstance may be taken to indicate that the non ρ pole term should not be dropped.

A relation like $j_\mu^i = \int \phi_\mu^i$ is not very meaningful in general because one does not know what the ξ meson field looks like. This concept has its place in a Lagrangian field theory where ϕ_μ is characterized as a solution of specific field equations (6).

However, one can extend Eq. (19) in a different way. Namely, we can separate the Born terms and make the slow variation assumption only for them, i.e., we write

$$\begin{aligned} F_\pi M_{B\mu}^{ij}(s, t; q^2, q'^2) + F_\pi \tilde{M}_\mu^{ij}(s, t; q^2, m_\xi^2) \\ = \Lambda_{B\mu\nu}^{ij}(s, t; q^2, q'^2) q^\nu + \tilde{\Lambda}_{\mu\nu}^{ij}(s, t; q^2, m_\xi^2) q^\nu, \quad (26) \\ 0 \leq q'^2 \leq m_\xi^2, \end{aligned}$$

where $\tilde{M}_\mu$ and $\tilde{\Lambda}_{\mu\nu}$ indicate the Born term free amplitudes. In this way the slow variation assumption is made only for the three point function. Moreover, we can test its validity by passing to the soft pion limit ($q \rightarrow 0$) and comparing the residues of the nucleon pole in s on both sides of the equation. The nucleon pole term in M_μ^{ij} is given by diagram a of Fig. 1. First, let us take definite charge states: choose $i = \frac{1}{\sqrt{2}}(1+i2)$, $j = \frac{1}{\sqrt{2}}(1-i2)$ and take the external nucleon states to be proton states. Then M_μ^{+-} is the amplitude for the scattering process $\pi^- + p \rightarrow \xi^- + p$.

$$\begin{aligned} M_\mu^{+-}(s, t; q^2, m_\xi^2) = \bar{u}(p') \left[\gamma_{\xi NN} F_1^3(q^2) \gamma_\mu + \mu_{\xi NN} F_2^3(q^2) i \sigma_{\mu\nu} q^\nu \right] \\ \times \frac{\not{p} + \not{q} + m_N}{s - m_N^2} K(q^2) g \sqrt{2} i \gamma_5 u(p) \quad (27) \\ + \tilde{M}_\mu^{+-}(s, t; q^2, m_\xi^2) \end{aligned}$$

$\gamma_{\pi NN}$, $\mu_{\pi NN}$ are the "electric" and "magnetic" coupling constants of the π meson and $F_{1,2}^{\pi}$ are the "charge" and "moment" form factors respectively ¹⁾. $g\sqrt{2}i\gamma_5$ is the charged pion-nucleon coupling and $K(q^2)$ is the pion-nucleon form factor

$$\langle N(p') | j_{\pi}^i(0) | N(p) \rangle = g K(t) \bar{u}(p') i\gamma_5 u(p) \langle \tau^i \rangle \quad (28)$$

$\tilde{M}_{\mu}^{+-}$ is the contribution regular at $s=m_N^2$ of intermediate states other than the single nucleon state.

In an analogous way we get for $\Lambda_{\mu\nu}^{+-}$ (diagrams c and d of Fig. 1)

$$\begin{aligned} \Lambda_{\mu\nu}^{+-}(s, t; q^2, m_{\pi}^2) = & \bar{u}(p') \left[\gamma_{\pi NN} F_1^{\pi}(q^2) \gamma_{\mu} + \mu_{\pi NN} F_2^{\pi}(q^2) i\sigma_{\mu\nu} q^{\nu} \right] \\ & \times \frac{\not{p}' + \not{q}' + m_N}{s - m_N^2} i \frac{G_A(q^2)}{\sqrt{2}} \gamma_{\nu} \gamma_5 u(p) \quad (29) \\ & + \tilde{\Lambda}_{\mu\nu}^{+-}(s, t; q^2, m_{\pi}^2) \end{aligned}$$

We now substitute Eqs. (27) and (29) into Eq. (26) and demand the equality of the residues of $(s-m_N^2)^{-1}$ on both sides. We compare the coefficients of the covariants on both sides. Thus, for example, the coefficient of $\gamma_{\mu} \not{q}$ reads for $q'^2=0$

$$ig\sqrt{2} F_{\pi} K(q^2) \left[\gamma_5 - 2m_N \mu_{\pi} \right] \gamma_{\mu} \not{q} = i \frac{G_A(q^2)}{\sqrt{2}} \left[2m_N \gamma_5 - 4m_N^2 \mu_{\pi} \right] \gamma_{\mu} \not{q}, \quad (30)$$

where we have defined ¹⁾

$$\begin{aligned}\gamma_g &= \gamma_{gNN} F_1^g(0) , \\ \mu_g &= \mu_{gNN} F_2^g(0) .\end{aligned}\tag{31}$$

Similar equations hold for the rest of the covariants appearing in the Born term. Thus, Eq. (30) yields for $q^2=0$

$$F_{\pi} = \frac{m_N G_A(0)}{g K(0)} ,\tag{32}$$

which is nothing but the famous Goldberger-Treiman relation. It emerges in the way it does in our analysis as a consequence of the slow variation hypothesis as applied to the three-point function. We take the emergence of Eq. (32) as a justification for that hypothesis.

Finally, in Eq. (26) $\tilde{\Lambda}_{\mu\nu}^{+-}$ is regular at $s=m_N^2$ and $\tilde{\Lambda}_{\mu\nu}^{+-} q^\nu \rightarrow 0$ as $q^\nu \rightarrow 0$. Thus $\tilde{M}_\mu^{+-} \rightarrow 0$ as $q^\nu \rightarrow 0$. This result is in accord with the Kroll-Ruderman theorem ¹⁰⁾ which states that in the limit of zero pion four-momentum the photoproduction amplitude is given by the Born term only. Thus, the same result should hold for M_μ^{+-} and this is what we find since in the Born term only the form factors enter and for these VDM and slow variation make sense. The fact that M_μ^{+-} is given in the limit of vanishing pion four-momentum by the Born term alone is already apparent from Eq. (19) since again $\tilde{\Lambda}_{\mu\nu}^{+-} q^\nu$ tends to zero in that limit.

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$$T_{\mu\nu}^{ij} =$$

(a) (b) (c)

(d) (e) (f)

(g) (h)

FIG.1

$$t_{\mu}^{ij} =$$

The figure shows seven Feynman diagrams labeled (a) through (g), which are summed to give the expression for t_{μ}^{ij} .

- (a)** A diagram with a horizontal line. The left end is a wavy line labeled $i \mu$ and q' , with a cross on the wavy line. The right end is a dashed line labeled j and q , with a dot on the dashed line. The horizontal line has arrows pointing left at p' and right at p .
- (b)** A diagram with a central circle. A wavy line labeled $i \mu$ with a cross enters from the top-left. A dashed line labeled j with a dot enters from the top-right. Two solid lines exit from the bottom.
- (c)** A diagram with a horizontal line. The left end is a wavy line labeled $i \mu$ with a cross. The right end is a solid line with a dot.
- (d)** A diagram with a central circle. A wavy line labeled $i \mu$ with a cross enters from the top-left. A solid line with a dot enters from the top-right. Two solid lines exit from the bottom.
- (e)** A diagram with a horizontal line. The left end is a solid line with a cross. The right end is a dashed line labeled j with a dot.
- (f)** A diagram with a central circle. A solid line with a cross enters from the top-left. A dashed line labeled j with a dot enters from the top-right. Two solid lines exit from the bottom.
- (g)** A diagram with a horizontal line. The left end is a solid line with a cross. The right end is a solid line with a dot.

The diagrams are arranged in three rows, separated by plus signs, indicating they are summed together.

FIG.2