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THE INFLUENCE OF REGGE ASYMPTOTICS
ON THE DETERMINATION OF RESONANCE PARAMETERS

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ABSTRACT

We suggest a new method of determining resonance parameters through a new form of the interference model.

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Recently there has been increasing interest in attempts to give a consistent treatment of scattering amplitudes at high and intermediate energies. In particular, superconvergence relations and finite energy sum rules have been used to correlate the successful Regge pole approach at high energies with data at lower energies.

The low and intermediate energy regions show resonance peaks, so it has been proposed ¹⁾ to add resonances, as determined from the Rosenfeld tables ²⁾, to the Regge pole expression. This is the so-called interference model which has recently been criticized as involving double-counting ³⁾. As an alternative, [e.g., Ref. 3], it has been suggested that one should use either the s channel resonances or the t channel Regge poles as (approximate) complete representations of the amplitude in the s channel physical region, in the belief that, from either one, one could obtain the other. This implies that the direct channel resonance description, i.e., $\sum_i g_i P_{l_i}(z_s)/(s-s_i)$, is able to generate the Regge pole form

$$A_t^R = \sum_j \beta_j(t) s^{\alpha_j(t)} \frac{e^{-i\pi\alpha_j(t)} \pm 1}{\sin \pi\alpha_j(t)} \quad (1)$$

at high energies, and conversely, the resonance contributions are, in some sense, already contained in the Regge pole form. We shall call this approach the "generating model".

An extreme form of this model has recently been proposed by Schmid ⁴⁾, who suggested that the above Regge pole form (where the sum runs over the limited number of Regge poles used in fitting high energy data) gives rise to resonances in the sense that it produces resonance-like Argand diagrams in the s -channel partial waves. However, the identification of these diagrams with resonances is very dubious, in particular :

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- a) the partial wave amplitudes that Eq. (1) gives rise to do not have the second sheet poles we would associate with genuine resonances ^{*});
- b) the amplitude A_t^R of Eq. (1) does not contain the experimentally observed resonance peaks;
- c) Eq. (1) leads to extra Argand circles for high partial waves ("ancestors") in positions that are very unlikely to correspond to resonances ⁶⁾.

Our point of view is that the Argand circles of the partial wave projections of Eq. (1) have, in general, nothing to do with resonances and that the appealing coincidence of their maxima with the actual position of resonances ^{4),6),7)} is a reflection of a dynamical coincidence, which can be quite fundamental, namely, the universality of nearly all known Regge trajectories ^{**)}.

^{*}) This inconsistency is even more striking if one considers a crossing symmetric example as $\pi\pi \rightarrow \pi\omega$. Then one starts with a ρ Regge pole which represents a pole of $a_\ell(t)$ for $\ell = 1$ and $t = m_\rho^2$ and one finds ⁵⁾ again the ρ as a maximum of $\text{Im } a_1(s)$ which is not a pole of the partial wave amplitude.

^{**)} The phase of the partial wave projection of A_t^R comes substantially from the $e^{-i\pi\alpha(t)}$ factors ^{6),8)} in the Regge pole terms, and this factor is approximately common to all Regge trajectories, except for the Pomeron, which, however, makes little contribution to the phase variation at low energies due to its small contribution and the flatness of the trajectory. This is the reason why the Argand maxima, lying on lines with the slope given by the crossed channel trajectories, may easily coincide with the position of the direct channel resonances.

Since A^R does not have the unphysical sheet poles of A , we propose, as a simple representation of the amplitude, to add them ^{*)}. We therefore write in the s channel physical region

$$A = A^R + \sum_i \frac{g_i P_i(z_s)}{s - s_i} \quad (2)$$

where $A^R = A_t^R + A_u^R$; A_t^R and A_u^R being the amplitudes for Regge pole exchange in the t and u channels, respectively. Although Eq. (2) superficially resembles the interference model ¹⁾, there is a crucial difference, which is that we will not identify the parameters of the poles with the phenomenological resonance parameters as obtained, for example, from the Rosenfeld tables ²⁾, but use this equation to define them. Since the partial wave projection of A^R has a rapidly varying phase, it gives a "background" which should not be neglected in the determination of the pole parameters, even in the limit of narrow width. For example, as mentioned above, the partial wave projection of A^R is in some cases in phase with a particular pole term. When this happens, it is clear that γ_1 is not simply related to the partial width of the resonance in question. Indeed, there is no reason why g_1 should not be negative or even complex. Although this violates accepted prejudices, at least for elastic scattering, one should realize that these prejudices are based on unitarity which, however, holds on the real axis and that the presence of a large background implies that the continuation to the pole is not straightforward, even in the narrow resonance limit. Note that, by construction, there is no double counting in our procedure.

Our proposal is to use Eq. (2) in order to understand which of the Argand circles phenomenologically found are actually resonances. By a suitable parametrization of A^R , one can try to fit experimental data with a minimum number of pole terms. It is conceivable therefore that

*) The proposal that what is to be added to A^R has only the pole structure is perhaps an (inessential) oversimplification, but an appealing one, at least to begin with. It is clear that Eq. (2) is just a specific model for the Regge background integral in which its poles are displayed.

a large number of the Argand maxima found phenomenologically would arise in this model simply from A^R and that rather few resonance poles would be required. Indeed, something of this sort is already suggested in the comparison of the phenomenological π -N partial wave amplitudes⁹⁾ with the partial wave projection of A^R as obtained in Ref. 7). It is striking that in this calculation this projection contains approximately all the Argand circles (with roughly the correct elasticity) except the F_{15} , P_{33} and F_{37} partial waves. Thus, it would appear that the only genuine resonances (unphysical sheet poles) required are the N, the Δ and their Regge recurrences.

Of course, at this stage, this conclusion should not be taken very seriously since we have not made a detailed comparison of the partial wave amplitudes coming from A^R with those which fit the experimental data, and in any case the form of A^R to be used in the intermediate and low energy regions is by no means unambiguous. Indeed, it may be that the most practical suggestion would be to use Eq. (2), with a rather flexible parametrization of A^R , to fit high, intermediate, and eventually low, energy data. It has not escaped our attention that this procedure may be useful in resolving ambiguities in partial wave analyses.

We discuss now some general considerations of the relative advantages of our "modified interference model" [Eq. (2)] with the "generating model" (i.e., only s channel resonances or only Regge poles). The first remark is that Eq. (2) is readily suited for crossing and, therefore, for bootstrap-like calculations. For example, if we consider narrow resonances lying on straight trajectories, then by continuing to the t channel physical region, the last term in the right-hand side of (2) would just give rise¹⁰⁾ to the Regge term

$$A_s^R \approx \sum_l \rho_l(s) t^{\alpha_l(s)} \frac{e^{-i\pi\alpha_l(s)} \pm 1}{\sin \pi\alpha_l(s)}$$

while A_t^R is just a representation of the t channel resonances which would appear as a convergent sum in the t channel. Actually, we could write our amplitude in the form

$$A \approx \sum_i \frac{g_i P_{\ell_i}(z_s)}{s - s_i} + \sum_j \frac{g_j P_{\ell_j}(z_t)}{t - t_j} + \sum_k \frac{g_k P_{\ell_k}(z_u)}{u - u_k}$$

where the g 's are supposed sufficiently well-behaved, so that :

- a) the sum over Legendre polynomials of physical argument ($-1 \leq z \leq 1$) converges in such a way that a limited number of resonance poles should be considered;
- b) the sum over Legendre polynomials of unphysical argument (z_x) should be summed through the Sommerfeld-Watson transform [à la Van Hove ¹⁰⁾] to yield the corresponding Regge pole amplitude A_x^R plus, eventually, a rather remote background which should not contribute appreciably, at least for not too low energies.

Therefore we can have the situation in which a limited number (or even one) trajectory could reproduce itself, while an infinite number would be required if Regge poles in the s channel must be generated by Regge poles in the t or u channels.

Let us see which are the requirements that our amplitude must satisfy in order to obey finite energy sum rules ³⁾, or superconvergence relations. At fixed t these relations imply conditions over the integrals of momenta of $\text{Im}A$ in the s and u channels. In this case it is easily seen that the sum à la Van Hove over t channel poles yields A_t^R , and therefore the remaining sums over s and u channel poles must be superconvergent by themselves. Obviously, this needs an infinite number of resonances, but, as proved by Atkinson ¹¹⁾, a single tower of poles (i.e., a single Regge trajectory) could be sufficient.

Moreover, although it is possible for a sum of narrow resonances to be superconvergent, as needed in our approach, it has been shown ¹²⁾ that it is not possible for them to generate an $s^{\alpha(t)}$ behaviour unless an infinite number of Regge trajectories is involved. This is another manifestation of the simplicity of our approach, as compared to the "generating model".

Finally, we return to the phenomenological implications of our approach which, in essence, is just a theoretical model for the background. As said before, the major consequences of a strong rapidly varying background (as comes out from the model) implies that the phenomenological parameters of the "resonances", as appear in the Rosenfeld tables, are not directly a reflection of the pole position and residues. It is only through a dynamical analysis (as proposed before) that these pole parameters would emerge. Of course, when these "resonances" are phenomenologically used (e.g., in isobar models), we expect them to be correct due to the fact that this is only a convenient parametrization for physical energies which are those in which experiments are made. On the other hand, symmetry considerations or quark models, for example, refer to properties of pole positions and residues, and would therefore be modified. In the very preliminary considerations made before for π^-N , for instance, we would guess the residue of the $\Delta(1236)$ in π^-N scattering to be only $\sim 70\%$ of the width, instead of the total width which would be expected if no background would be present.

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