COMPOSITENESS AS A CLUE FOR THE UNDERSTANDING OF THE
ASYMPTOTIC BEHAVIOUR OF FORM FACTORS

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ABSTRACT

An inverse power behaviour of the form factors obtains when the hadrons are considered as composite systems described by a relativistic ladder. A "scale-law" for different form factors and the dependence on the spin of the hadron is established.

TH. 888

25 March 1968

The behavior of the nucleon form factors at large momentum transfers offers a picture of deceptive simplicity. With the last data included¹⁾, the "dipole fit" remains a remarkably good parameterization²⁾ but, on the other hand, its interpretation is still obscure.

In order to understand this behavior in terms of dispersion relations, in the mass of the current, one has to suppose that intermediate states contributions show an, up to now, ad hoc property of cancelling among themselves. This cancellation calls for a dynamical origin. We want to point out here that we can find it in our present understanding of the non-elementarity of hadrons.

We want our hadrons to lie on Regge trajectories or, if preferred, to be considered as composite systems. In order to calculate electromagnetic properties of such a hadron, we need a model and the simplest one (if not the only one) at our disposal is the ladder Bethe Salpeter (BS) model. In this scheme the wave function ϕ_J corresponding to a bound state of spin J is the solution of the corresponding homogeneous B.S. ladder equation or, equivalently, is the residue at the pole which comes from the divergence of the sum over infinity of ladder diagrams.

Let us consider first integer spin hadrons composed by spinless particles. The first calculation we discuss would correspond to a theory in which a charged and a neutral scalar field (ϕ^+ and ϕ^0 respectively) interact with a scalar field σ through an interaction Lagrangian

$$\mathcal{L}' = g_1(\phi^+)^* \phi^+ \sigma + g_2(\phi^0)^* \phi^0 \sigma.$$

The form factor for the transition between a state of spin J_1 and one of spin J_2 can be represented diagrammatically as in Fig. 1³⁾.

We can identify the vector current in Fig. 1 with the electro-magnetic one, due to the fact that the set of graphs considered is gauge invariant.

The asymptotic behavior for $k^2 \rightarrow \infty$ of the form factors has been obtained in two ways. One by summing up the asymptotic behavior of the perturbative diagrams of the right-hand side of Fig. 1, the other through the study of the asymptotic properties of the B.S. wave functions ϕ_J entering in the expression of the form factor³⁾ indicated in the left-hand side of Fig. 1.

In the first method we have applied the standard procedures to calculate asymptotic behavior of Feynman diagrams⁴⁾. Consider first the constituents (labelled with momenta p_1, p_2, p_3 and p_4 in Fig. 1) of our composite systems as incoming or outgoing particles in the Feynman diagrams.

We choose the following independent variables:

k^2 the square momentum transfer associated to the current

θ angle between $\vec{k}$ and $\vec{p}_2$ in the c.m. system of 1 and 2

θ' " " $\vec{k}$ and $\vec{p}_4$ " " " " " 3 and 4

ϕ azimuthal angle between the two planes $\vec{p}_1\vec{p}_2$ and $\vec{p}_3\vec{p}_4$

$$w_1 = (p_1 + p_2)^2$$

$$w_2 = (p_3 + p_4)^2.$$

We decompose the amplitude in the following invariant amplitudes

$$T_\mu = T_1 \frac{1}{4} (p_1 + p_2 - p_3 - p_4)_\mu + T_2 \frac{1}{2} (p_1 - p_2)_\mu + T_3 \frac{1}{2} (p_3 - p_4)_\mu \quad (1)$$

where T_1 , T_2 and T_3 are simple linear combinations of the amplitudes with definite helicity of the virtual photon.

The relevant t paths⁴⁾, in order to calculate the asymptotic behavior of T_1 , T_2 and T_3 for $k^2 \rightarrow \infty$ and fixed finite values of the other variables, are respectively: $\alpha_0 \delta_1$ and $\beta_0 \delta_2$ (cf., Fig. 1) for T_1 , $\alpha_0 \delta_1$ for T_3 and

finally $\beta_0 \delta_2$ for T_2 . We find:

$$T_1 \sim_{k^2 \rightarrow \infty} \frac{a_1}{(k^2 + b_1 \cos \phi)^2} \cdot \log k^2 \quad (2a)$$

$$T_1 \sim_{k^2 \rightarrow \infty} \frac{a_2}{(k^2 + b_2 \cos \phi)^2} \quad (2b)$$

$$T_3 \sim_{k^2 \rightarrow \infty} \frac{a_3}{(k^2 + b_3 \cos \phi)^2} \quad (2c)$$

where the a_i and b_i are functions of $\cos \theta$, $\cos \theta'$, w_1 and w_2 .

The sums over the infinity of graphs of Fig. 1b will produce the poles in w_1 and w_2 at the position of the bound states (Fig. 1b). After extracting the residue, the projection over the helicity states or equivalently over the different form factors can be achieved⁵⁾ with the help of recently developed propagators techniques⁶⁾ to give the following results: Define $(J_2 \geq J_1)$:

$$\begin{aligned} \langle J_1, q_1, m_1; J_2, q_2, m_2 | \mathcal{J}_\lambda(0) | 0 \rangle = & \\ & \sum_{i=1}^{J_1+1} G_i(k^2) e_{\mu_1 \mu_2 \dots \mu_{J_1}}^{(m_1)} e_{\nu_1 \nu_2 \dots \nu_{J_2}}^{(m_2)} O_{\mu_1 \dots \mu_{J_1}, \nu_1 \dots \nu_{J_2}}^{(i)} \cdot (q_1 - q_2)_\lambda + \\ & + \sum_{i=1}^{J_1} H_i^{(1)}(k^2) e_{\lambda \mu_2 \dots \mu_{J_1}}^{(m_1)} e_{\nu_1 \nu_2 \dots \nu_{J_2}}^{(m_2)} O_{\mu_2 \dots \mu_{J_1}, \nu_1 \dots \nu_{J_2}}^{(i)} + \\ & + \sum_{i=1}^{m+1} H_i^{(2)}(k^2) e_{\mu_1 \dots \mu_{J_1}}^{(m_1)} e_{\lambda \nu_2 \dots \nu_{J_2}}^{(m_2)} O_{\mu_1 \dots \mu_{J_1}, \nu_2 \dots \nu_{J_2}}^{(i)} \end{aligned} \quad (3)$$

where $q_1(q_2)$ and $e_\mu^{(m_1)}(e_\nu^{(m_2)})$ are the momenta and polarization tensors of the hadron of spin $J_1(J_2)$ respectively, and $m = \min(J_1, J_2 - 1)$. The tensors $O^{(i)}$ are defined as

$$O_{\rho_1 \dots \rho_m, \sigma_1 \dots \sigma_n}^{(i)} = k_{\sigma_{m+1}} \dots k_{\sigma_n} (k_{\rho_1} k_{\sigma_1} k_{\rho_2} k_{\sigma_2} \dots k_{\rho_{i-1}} k_{\sigma_{i-1}} \dots \delta_{\rho_i \sigma_i} \dots \delta_{\rho_m \sigma_m}) \quad (4)$$

The asymptotic behavior of the form factors is predicted to be:

$$G_i(k^2) \approx (k^2)^{-J_2-i+1} \cdot T_1 \sim (k^2)^{-J_2-i-1} \log k^2 \quad (5a)$$

$$H_i^{(1)}(k^2) \approx (k^2)^{-J_2-i+1} \cdot T_2 \sim (k^2)^{-J_2-i-1} \quad (5b)$$

$$H_i^{(2)}(k^2) \approx (k^2)^{-J_2-i+2-\delta_{J_1 J_2 T_3}} \sim (k^2)^{-J_2-i-\delta_{J_1 J_2}} \quad (5c)$$

Notice that the most "convergent" (decreasing) form factor goes as

$$\left(\frac{1}{k^2}\right)^{J_1+J_2+2} \log k^2, \text{ and that there is a definite scaling (by steps of } 1/k^2) \text{ for the others.}$$

Consider for instance the form factors of the ρ -meson defined as

$$\begin{aligned} \langle J_1=1, J_2=1 | \hat{O}_\lambda(0) | 0 \rangle &= (q_1 - q_2)_\lambda [\epsilon^{(1)} \cdot \epsilon^{(2)} G_1(k^2) + \epsilon^{(1)} \cdot k \epsilon^{(2)} \cdot k G_2(k^2)] + \\ &+ (\epsilon_\lambda^{(1)} \epsilon^{(2)} \cdot k - \epsilon_\lambda^{(2)} \epsilon^{(1)} \cdot k) H_1(k^2) \end{aligned} \quad (6)$$

We predict that for $k^2 \rightarrow \infty$

$$G_1 \sim \frac{\log k^2}{(k^2)^3}; \quad G_2 \sim \frac{\log k^2}{(k^2)^4}; \quad H_1 \sim \frac{1}{(k^2)^3} \quad (7)$$

In the other method, whose details will be published elsewhere, the asymptotic behavior of the B.S. wave function ϕ_J is first studied. Then, in order to perform the integration implied in Fig. 1³⁾ it is shown that a suitable path can be chosen in such a way that on it even for $k^2 \rightarrow \infty$ both wave functions ϕ_{J_1} and ϕ_{J_2} can be bounded and a maximization of the whole integral can be performed.

Every bound state enters in the calculation with its appropriate polarization tensor, so that the projection over helicity states can be easily achieved.

In so doing, we find results like those stated in eq. (5) ⁷⁾.

Next we extend our considerations to half-integer spin hadrons, thought of as bound states of a spin 0 and a spin 1/2 particle. (In Fig. (1b) we can consider the p_1, p_3 line (the one with which the current interacts) as either that of the spin 0 or that of the spin 1/2 constituent of the baryon). The calculations are slightly more complicated than in the integer spin case, but the results are as simple: With a minimal interaction of the current with the constituents we find for the two-composite nucleon

$$F_1(k^2) , F_2(k^2) \underset{k^2 \rightarrow \infty}{\approx} (k^2)^{-2} \log k^2 \quad (8)$$

where F_1 and F_2 are the usual Dirac and Pauli form factors. Eq. (8) is compatible with present experimental data ^(1,2), in fact predicts the famous dipole behavior for the magnetic Sachs form factor of the nucleon.

It is also amusing to notice that projecting out a spin 3/2 and a spin 1/2 in the two ladders of Fig. 1 we obtain predictions for the $N, N^*(1238)$ electromagnetic form factors. It turns out that all of them have at least the behavior of eq. (8) and some get an extra $1/k^2$ convergence factor.

How model dependent is the calculation? First we remark that the addition of other diagrams for the two-particle bound state would not change the result. In particular, the one depicted in Fig. 2, which would correspond to "exchanged charged particles" gives the same behavior for T_2 and T_3 as that of eq. (5) and a different, non-leading one, for T_1 . It is interesting that no "pinch" contribution to the asymptotic behavior in k^2 can be found even including non-planar diagrams.

What would happen if instead of considering the hadron as a two-particle bound state we should have thought it to be composed of three or more particles? The analysis of n particle ladder diagrams (which give rise to Regge trajectories $\alpha(t)$ receding to $-2n+3$ for $t \rightarrow -\infty$) gives for the form factor an additional convergence factor $1/(k^2)^{2(n-2)}$ disregarding $\log k^2$ factors.

In the limit of infinitely composed particles, which should correspond to a bootstrap situation, we find infinitely decreasing form factors in agreement with droplet models⁸⁾ or Stack's⁹⁾ result in potential models. An examination of other possible diagrams beside the ladder ones (in particular the non-planar ones, which do not appear in potential theory) of many-particle bound states seem to indicate that the rapid decrease of form factors would not be altered. In this sense, the persistence of the dipole form factor for very high transfer momenta would give an indication of a definite compositeness (quarks?) of reggeized particles. This indication has to be correlated with how far the corresponding Regge trajectory recedes for $t \rightarrow -\infty$ (spacelike).

Other results are independent of the number of constituents, so, for instance, the scale law of different form factors, or the different power between transverse and longitudinal photons, having a purely kinematical origin, remain also unchanged. We have not investigated here other effects, as, e.g., the spin of the constituents of the hadrons.

Let us discuss a moment the relation between our result and the usual dispersive approach as applied, for instance, to the nucleon form factors. Evidently, the $1/k^4$ behavior of the form factors coming from our model implies that the many intermediate states which appear show cancellation among them. This implies superconvergence relations of the form

$$\int \text{Im } F_i(k^2) dk^2 = 0 \quad (9)$$

It is interesting to note that the ladder model which gives reggeization, and, therefore, superconvergence of scattering amplitudes, for external spinning particles⁵⁾, also gives superconvergence of the form factors of the hadrons it generates. Also amusing is the similar role of the spin of the hadrons in producing additional convergence factors both in scattering amplitudes and in form factors.

Of course, the intermediate states we have considered are not all the possible ones. It can be thought, for instance, that the ρ could mediate the current, even if it has nothing to do with the bound state structure of the hadrons. The ρ can, of course, be coupled to our hadrons but to take it into account properly for the asymptotic behavior one

has to keep all other states (including many particle states) which will compensate its contribution in Eq. 9.

In terms of graphs, this would mean, that one has to add to the diagrams considered in Fig. 1b those of Fig. 3. The compositeness of the hadron would remain essential in order to understand the asymptotic properties of the form factors.

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Figure Captions

- Fig. 1. a) The general production (five legs) amplitudes.
 b) Family of leading diagrams for $k^2 \rightarrow \infty$.
 c) Connection with the form factors for the $\gamma \rightarrow J_1 + J_2$ transition.
- Fig. 2. Diagrams corresponding to the interaction of the current with
 "exchanged charged" particles,
- Fig. 3. Generation of the vector meson pole in e.m. form factors.

fig. 1

fig. 2

fig. 3