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PHOTOPRODUCTION AND REGGE BEHAVIOUR

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A B S T R A C T

After a general discussion of the consistency of Regge behaviour in photoproduction we analyze the Gauge invariance constraints imposed on a Regge amplitude. We show in particular that the pion trajectory contribution is Gauge invariant by itself in spite of the fact that the elementary pion-exchange contribution is not. The near forward and forward amplitude is obtained from the Gauge invariant conditions in the evasive and conspiratorial cases.

1. INTRODUCTION

It has been pointed out in several recent articles¹⁾ that certain weak scattering amplitudes²⁾ could not obey an asymptotic Regge behaviour. To the contrary, such amplitudes could have fixed poles in the angular momentum plane and one of them would then determine the asymptotic energy dependence. The presence of a fixed pole does not violate any basic property of S-matrix theory, since the unitarity relation; satisfied by such amplitudes, is to be considered to lowest order in the coupling to the external fields. Unitarity is then a linear relation instead of a quadratic one for the scattering amplitude, and a fixed pole may therefore appear³⁾. This fixed pole is shown to affect one particular invariant amplitude which would otherwise in general satisfy a superconvergence relation. It should be stressed however that this result is a consequence of the existence of unsubtracted dispersion relations for the other invariant amplitudes, a property which is usually assumed to result from a particular Regge asymptotic behaviour which, could, in some cases not be satisfied.

It is interesting to analyse whether such properties can be extended to photoproduction. Even though it is tempting for several reasons⁴⁾ to describe high energy photoproduction in terms of Regge asymptotic amplitudes, there seems to be no compelling theoretical requirements to do so. Photoproduction is considered only to first order in the electromagnetic coupling and unitarity does not forbid the presence of fixed poles. However, if we apply the chain of reasoning so far applied to weak scattering amplitudes¹⁾ to photoproduction, it can be shown that fixed poles are not necessary. To the contrary, the condition, closely related to gauge invariance,

which appears to require a fixed pole, is actually the result of separating an invariant amplitude in a way which is, in this case, inconsistent with the Regge behaviour which it is assumed to follow. Even though a pure Regge behaviour may not hold for photoproduction, it at least leads to consistent results.

In introducing Regge poles in photoproduction, one has however a second question to face which is whether gauge invariance and Regge behaviour are compatible. Each trajectory corresponds to a well defined high energy behaviour, whether or not daughter trajectories are required for that purpose⁵⁾. Each Regge trajectory contribution must therefore be separately gauge invariant. This is particularly important in the case of the pion trajectory contribution which must satisfy gauge invariance by itself in spite of the fact that the one (elementary) pion exchange contribution is not separately gauge invariant. We find that this is indeed possible and obtain the corresponding relations among the residue functions.

In the second section of this paper we briefly show, using a fictitious but most simple system, how superconvergence may occur or may in turn be spoiled by the presence of a fixed pole. We show however that in the case of photoproduction the absence of subtraction in certain invariant amplitudes, which is necessary in order to derive the Fubini sum rule⁶⁾ leading to a fixed pole behaviour, should not hold due to the explicit separation of elementary particle pole terms. A fixed pole in the angular momentum plane is therefore not required.

In the third section we consider the realistic case of pion production from nucleons and show how Regge behaviour can be introduced in a gauge invariant way. We focus on the pion trajectory and show how the various constraints encountered determine the value of the residue functions on the pion pole and yield therefore an approximate expression for the pion trajectory contribution to the production amplitude near the forward direction. Imposing gauge invariance for the pion contribution alone, we first choose an evasive solution to the kinematical constraints which must also be satisfied⁷⁾. It seems however that if the sharp forward peak recently observed in π^+ photoproduction⁸⁾ is to

be explained within the Regge framework, a conspiratorial solution is necessary⁹⁾. The observed forward and near-forward production would then be mainly due to the combined effect of the pion and its conspirator with the A_2 trajectory dominating perhaps at higher value of t . In the fourth section we then turn to this "conspiratorial" solution and, using the gauge invariance and kinematical constraints, we obtain an approximate expression for the forward amplitude.

2. CONSISTENCY OF REGGE BEHAVIOUR

In order to illustrate the series of arguments leading to superconvergence on fixed pole behaviour, we consider the photoproduction of an isospin one scalar particle X from a scalar particle Y

$$\gamma(k) + Y(P) \rightarrow X(Q) + Y(P') \quad (2.1)$$

To this end, we introduce the Fourier transform of the retarded product of the isovector current density $V_\mu^i(x)$ and a scalar density $S^j(x)$, with isotopic spin index j . It reads

$$T_\mu^{ij} = i \int d^4x e^{-ikx} \langle Y(P') | R(V_\mu^i(x), S^j(0)) | Y(P) \rangle \quad (2.2)$$

with the retarded product defined as

$$R(A(x), B(y)) = i \theta(x_0 - y_0) [A(x), B(y)]$$

We also introduce the matrix element of the commutator of the two operators

$$C_\mu^{ij} = i \int d^4x e^{-ikx} \langle Y(P') | [V_\mu^i(x), S^j(0)] | Y(P) \rangle \quad (2.3)$$

4.

Following Fubini⁶⁾ we rewrite both T_μ and t_μ in terms of invariant amplitudes as follows:

$$\begin{aligned} T_\mu &= A(P+P')_\mu + B(P-P')_\mu + C K_\mu \\ t_\mu &= a(P+P')_\mu + b(P-P')_\mu + c K_\mu \end{aligned} \quad (2.4)$$

where A, B and C together with a, b and c are functions of the four independent variables

$$\begin{aligned} s &= -(P+K)^2 \quad t = -(P-P')^2, \quad K^2 \quad \text{and} \quad Q^2 \\ \text{with} \quad Q &= P+K-P' \end{aligned}$$

a, b and c are respectively the absorptive part of the three invariant amplitudes A, B and C.

Both (2.2) and (2.3) are then integrated by part. As a result of current conservation this gives

$$\begin{aligned} i K_\mu T_\mu^{ij} &= i \int e^{-iKx} \delta(x_0) d^4x \langle Y(P) | [V_4^i(x), S^j(0)] | Y(P) \rangle \\ i K_\mu t_\mu^{ij} &= 0 \end{aligned} \quad (2.5)$$

It follows from locality that the equal time commutator, on the right hand side of (2.5) has the origin as a support. It will therefore be most generally an entire function of K and a function of t. We will keep here only the part of the commutator proportional to $\delta(\vec{x})$ and rewrite (2.5) as

$$K_\mu T_\mu^{ij} = F^{ij}(t), \quad K_\mu t_\mu^{ij} = 0 \quad (2.6)$$

The next step is to assume that for certain fixed values of t, K^2 and Q^2 , A, B and C all satisfy unsubtracted dispersion relations in s. This then leads to the Fubini sum rule⁶⁾

$$\frac{1}{\pi} \int a(s', t, Q^2, k^2) ds' = -F(t) \quad (2.7)$$

which implies that A has an asymptotic $-[F(t)/s]$ behaviour. Questions now arise when one tries to justify the writing of unsubtracted dispersion relations on the ground of a possible Regge behaviour. In order to do so we follow the general procedure proposed by Trueman¹⁰⁾ and consider the crossed channel helicity amplitudes. Once properly combined and multiplied by known factors^{11,12)} they satisfy standard analyticity properties in s . It is readily seen that the parity conserving helicity flip amplitude¹⁰⁾ contributes only to A while the non flip amplitude contributes to both B and C. Reggeization then proceeds along standard lines¹¹⁾ and if $\alpha(t)$ stands for the leading trajectory, both B and C will have an $s^{\alpha(t)}$ asymptotic behaviour while A will have an $\alpha(t)s^{\alpha(t)-1}$ leading term. If the leading trajectory lies below zero for a particular value of t , A, B and C will then satisfy unsubtracted dispersion relation on the Fubini sum rule will hold. Nevertheless sA would also satisfy an unsubtracted dispersion relation and A should then satisfy a superconvergence relation rather than the Fubini sum rule. The way out of this difficulty¹⁾, which should appear in weak scattering amplitudes generally defined through an expression of the type (2.2), is to assume that A has a fixed pole in the angular momentum plane, (at $J = 0$), together with moving poles. The asymptotic behaviour implied by the Fubini sum rule is then determined by the fixed pole.

This reasoning does not apply readily to a photoproduction amplitude. Instead of defining it through an expression similar to (2.2), one may introduce it as the residue of a one particle pole term (at $Q^2 = -m_x^2$ here) present in (2.2). Since F has no singularity in Q^2 , one reaches a superconvergence relation rather than a Fubini sum rule and photoproduction is therefore found to be consistently described in terms of pure Regge behaviour. In order to reach anyway a Fubini sum rule, one may however take as the scalar operator S written in (2.2) the current operator associated to particle X. This leads readily to an

expression of the type (2.6). The right hand side however translates current conservation and corresponds to the fact that single particle exchange term; such as the contribution from the graph of Fig. 1, are not included in (2.2). The amplitude T_μ , which does not have the single particle exchange contribution, is not by itself gauge invariant and the presence of a non zero right hand side simply translates this fact. One might then be lead to conclude that a fixed pole is required in the photoproduction amplitude, as it has been defined. This would however take for granted that the invariant amplitudes appearing in the decomposition of T satisfy unsubtracted dispersion relations, a property which is now extremely unlikely. Regge asymptotic behaviour is in part the result of assuming that exchanged particles lie on Regge trajectories and, hence, the separation of a fixed elementary hadron pole contribution, as done in treating the photoproduction amplitude as the sum of the one-particle exchange term and a remainder, will destroy the Regge behaviour of this remainder. Even if the leading trajectory lies below zero B and C , as now defined, should not satisfy unsubtracted dispersion relations and no Fubini sum rule will result. One should not infer the presence of a fixed pole as the result of separating out an elementary particle pole. Here again pure Regge behaviour is consistent.

3. PION PRODUCTION ON NUCLEONS

a) Gauge-invariance conditions

The reasoning sketched in the previous section readily applies to a physical process such as production of pions off nucleons which we now consider in some detail. In the absence of gauge invariance, there are eight independent reaction amplitudes which can be chosen as to satisfy the Mandelstam representation^{1,3}). The decomposition, written here for the crossed channel amplitudes $\gamma + \pi \rightarrow N\bar{N}$, is as follows

$$T_\mu = \sum_{i=1}^8 B_i(s, t, k^2) N_i(p, p', k, \varepsilon, \gamma) \quad (3.1)$$

where

$$\begin{aligned}
 N_1 &= i \gamma_5 (\gamma, \varepsilon) (\gamma, K) \\
 N_2 &= i \gamma_5 (P - P') \cdot \varepsilon \\
 N_3 &= -2i \gamma_5 (Q, \varepsilon) \\
 N_4 &= 2i \gamma_5 (K, \varepsilon) \\
 N_5 &= \gamma_5 (\gamma, \varepsilon) \\
 N_6 &= \gamma_5 (\gamma, K) \left(\frac{P - P'}{2} \cdot \varepsilon \right) \\
 N_7 &= \gamma_5 (\gamma, K) (K, \varepsilon) \\
 N_8 &= -\gamma_5 (\gamma, K) (Q, \varepsilon)
 \end{aligned}
 \tag{3.2}$$

The polarization of the photon (actual or virtual) is denoted by ϵ . P , P' , K and Q now respectively stand for the nucleon, antinucleon, photon and pion four-momenta. s and t are the Mandelstam invariants: $s = -(P' - K)^2$, $t = -(Q + K)^2$. The photon mass will be denoted by ν , with $\nu^2 = -K^2$, and the virtual photon is described as a combination of a vector and a scalar one particle state. Even though we are presently interested in photoproduction, one should not dissociate it from pion production induced by virtual photons. The set of invariant amplitudes defined by (3.1) and (3.2) applies to both and so does the gauge invariance condition. Gauge invariance reduces the number of independent amplitudes from eight to six as it follows from the relations

$$\nu^2 \left(\frac{B_1}{2} + B_4 \right) + \frac{1}{2} B_2 \left(s + \frac{t - 2m^2 - \mu^2 - \nu^2}{2} \right) - \frac{1}{2} (t - \mu^2 - \nu^2) B_3 = 0 \tag{3.3a}$$

$$B_5 - v^2 B_7 + \frac{1}{2} B_6 \left(s + \frac{t - 2m^2 - \mu^2 - v^2}{2} \right) - \frac{1}{2} (t - \mu^2 - v^2) B_8 = 0 \quad (3-3b)$$

The nucleon and pion masses have been denoted by m and μ .

It is preferable not to use immediately (3.3) in order to eliminate two amplitudes from (3.1). In so doing one would introduce kinematical factors which would make the analysis of kinematical constraints a more involved one. For the same reasons the eight amplitudes are also used when studying the application of the Mandelstam representation to photoproduction¹³⁾. In this paper we will first obtain Regge asymptotic amplitudes for the eight B amplitudes and then impose relations (3.3) as constraints on the residue functions, together with so-called kinematical constraints which have to be imposed to remove poles from the B amplitudes at some particular points. It is also clear from (3.3) that, even though the B_4 and B_7 amplitudes do not affect the physical production process, as it follows from current conservation, they should also be analysed for kinematical constraints. They enter the gauge invariance relations as soon as one does not restrict oneself to zero photon mass.

Before writing Regge expansions for the B amplitudes we first examine the single particle pole in the t -channel, which, as seen from the previous discussion, play a very special role. We therefore single out in particular the one-pion exchange contribution which can be obtained from perturbation theory. This pole occurs only in the $(-)$ amplitude¹³⁾, that is the amplitude corresponding to the isovector photon component and isovector exchange, to which the ρ meson does not contribute. At $t = \mu^2$ one readily finds

$$(t - \mu^2) B_3 = e g \quad (t - \mu^2) B_4 = - \frac{1}{2} e g \quad (3.4)$$

where e.g., the product of the pion nucleon and the electromagnetic coupling constant, scales the pion pole residues. The contribution from these pole terms, as they appear in (3.3a) cancel, for s in the neighbourhood of m^2 , against the nucleon pole contribution to B_1 and B_2 , that is

$$(s - m^2) B_1 = \frac{1}{2} eg \quad (s - m^2) B_2 = eg \quad (3.5)$$

It might appear tempting to separate out the pole terms explicitly and apply relation (3.3) to the remainders. However, since the Regge pole hypothesis is applied to the B 's directly, any attempting to separate out fixed poles will destroy the asymptotic behaviour of the remainder. One is certainly at liberty to examine the cancellation of poles in the neighbourhood of $t = \mu^2$ and $s = m^2$, but any attempt to, at the same time, consider the asymptotic behaviour in s is extremely dangerous. We insist on this since it may seem at this stage that the pion and the nucleon contributions are linked together by the gauge invariance conditions as well known in perturbation theory. These constraints may at first appear as incompatible with Reggeization applicable to the asymptotic behaviour in a particular variable in which only one of these poles enters. For example for s large and t small, the pion trajectory is relevant while the nucleon pole term is ignored as such. Introducing s channel pole terms together with the contribution from Regge trajectories would lead to double counting. The Regge asymptotic amplitude should also summarize all the s channel pole or resonance contributions. One should then verify that a Reggeized pion, in particular, satisfies gauge invariance as given by (3.3a), by itself, in spite of the fact that a fixed pion pole would not satisfy it alone. In the following we will focus on the pion trajectory. The analysis may however be readily extended to any trajectory in order to obtain Regge asymptotic formulas for pion production. The Regge pole approach to photoproduction has already been discussed in the literature¹⁴⁾, in particular in connection with the presence of kinematical constraints. Nevertheless, no attempt has been made so far to show how the further constraints introduced by gauge invariance could be satisfied at the same time.

b) Reggeized pion contribution

The Reggeization procedure follows standard lines, that is one proceeds through the crossed-channel helicity amplitudes¹⁰⁾. In effect we parallel calculations already presented in details in the literature¹⁴⁾. Nevertheless, instead of going directly to the gauge invariant pion photoproduction amplitudes and using only four amplitudes as already done, we first obtain a Regge pole expansion for the eight B amplitudes and, only at the end, will we impose the condition of gauge invariance and eventually consider zero photon mass. We will therefore have to write explicitly here extended relations even though parts of them are already known¹⁴⁾. They are grouped together in tables.

In the t-channel there are eight independent helicity amplitudes which are written as $f_{\lambda\lambda';\mu}$ where λ and λ' respectively stand for the nucleon and anti-nucleon helicities, and where μ stands for the photon helicity. We use +1, -1 and 0 for the two transverse vector components and the longitudinal component. We also use s to denote a pure scalar time component, the virtual photon being described as a combination of vector and scalar¹⁵⁾. These helicity amplitudes are calculated in terms of the B's amplitudes. The corresponding expressions are given in Table 1. We denote by p and k the nucleon and photon momentum in the crossed-channel centre of mass frame. θ is the angle between the incident photon and the outgoing nucleon; E, k_0 and q_0 stand for the nucleon, photon and pion energies. All other amplitudes are obtained from parity invariance. The amplitudes as given are normalized in such a way that the differential cross section for a particular spin configuration would read:

$$\frac{d\sigma}{d\Omega} = \frac{p}{k} \left| \frac{f_{\lambda\lambda';\mu}(\theta)}{4\pi W} \right|^2 \quad (3.6)$$

where $W(t = W^2)$ is the centre of mass energy.

The next step is to introduce definite parity amplitudes which are free of kinematical singularities in s . This is done as follows^{11,12)}

$$\begin{aligned}
 F_1 &= \frac{1}{\sin \theta} \left(f_{\frac{1}{2}\frac{1}{2};1} + f_{-\frac{1}{2}-\frac{1}{2};1} \right) \\
 F_2 &= \frac{1}{\sin \theta} \left(f_{\frac{1}{2}\frac{1}{2};1} - f_{-\frac{1}{2}-\frac{1}{2};1} \right) \\
 F_3 &= \frac{1}{1+\cos \theta} f_{\frac{1}{2}-\frac{1}{2};1} + \frac{1}{1-\cos \theta} f_{-\frac{1}{2}\frac{1}{2};1} \\
 F_4 &= \frac{1}{1+\cos \theta} f_{\frac{1}{2}-\frac{1}{2};1} - \frac{1}{1-\cos \theta} f_{-\frac{1}{2}\frac{1}{2};1} \\
 F_5 &= f_{\frac{1}{2}\frac{1}{2};0} & F_6 &= f_{\frac{1}{2}-\frac{1}{2};0} \\
 F_7 &= f_{\frac{1}{2}\frac{1}{2};\Delta} & F_8 &= f_{\frac{1}{2}-\frac{1}{2};\Delta}
 \end{aligned}$$

(3.7)

We then express the B 's in terms of the F 's and the corresponding relations are given in Table 2. The next step is to introduce partial wave expansions for the F 's. This set of relations, given in Table 3, shows to which amplitudes a particular Regge trajectory will contribute. One finds that F_1^J and F_3^J are triplet amplitudes with parity $(-1)^J$, F_2^J , F_5^J and F_7^J are singlet amplitudes with parity $(-1)^{J+1}$ while F_4^J , F_6^J and F_8^J are triplet amplitudes with parity $(-1)^{J+1}$. For isovector exchange, as obtained in the $(-)$ amplitude¹³⁾, one further gets G -parity = -1 for F_1 , F_2 , F_3 , F_5 and F_7 while the remaining amplitudes have $G = +1$. In particular the pion trajectory will contribute only to F_2 , F_5 and F_7 , with a priori independent residue functions, while a scalar trajectory, which could be called for as a pion conspirator, would contribute only to F_1 and F_3 .

It is well known that with unequal masses questions arise as to how the asymptotic behaviour is reached since one cannot simply rely on the leading power in $\cos \Theta$. Daughter trajectories may be required at this stage⁵⁾ in order to restore a typical $s^{\alpha(t)}$ high energy behaviour. We will not face this difficult question here but simply introduce standard Regge asymptotic expressions in the particular amplitudes to which a trajectory is known to contribute, that is expressions of the form

$$\beta(t) \frac{1 \pm e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} s^{\alpha(t)}$$

for zero helicity flip amplitudes,

$$\beta(t) \frac{1 \pm e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} \alpha(t) s^{\alpha(t)-1}$$

for the amplitudes with one unit of helicity flip, etc. Such expressions satisfy the Mandelstam representation¹⁶⁾, and should approximate helicity amplitudes which are free of kinematical singularities in t as well as in s ¹²⁾. This is obviously not the case of the F 's. Nevertheless, since we happen to have a set of invariant amplitudes free of kinematical singularities with the B amplitudes, it is clear how helicity amplitudes which are free of kinematical singularities can be constructed. A set of such amplitudes denoted as $\bar{F}$ is given in Table 4.

We will first specialise to the pion trajectory alone which appears only in the F_2 , F_5 and F_7 amplitudes. Due to the particular relation between B_2 and F_2 we are lead to define

$$\bar{F}_2 = \frac{1}{4E_p} F_2$$

as an amplitude free of kinematical singularities. $\bar{F}_2$ is then approximated as

$$\bar{F}_2 \sim \beta_2(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t)} \alpha(t) s^{\alpha(t)-1}$$

so that:

$$F_2 \sim 4 E p \beta_2(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t)} \alpha(t) s^{\alpha(t)-1} \quad (3.8)$$

where $\alpha(t)$ is the pion trajectory ($\alpha(\mu^2) = 0$). F_2 then vanishes as $\sqrt{t}$ when t goes to zero. This is the only way to have B_2 finite at this point with only the pion trajectory having a particular value of $\alpha(0)$. This is an "evasive" solution to the problem of constraints on helicity amplitudes, as opposed to a "conspiratorial" one, which one may also have⁷⁾. The pion conspirator would contribute to F_3 and the F_2 and F_3 amplitudes could then both be singular and cancel out among each other their singular term at $t = 0$, which would otherwise appear in B_2 . This "conspiratorial" solution, while possible and perhaps necessary in order to explain present data within the Regge framework⁹⁾, is not particularly relevant to our attempt to Reggeize the pion contribution while imposing gauge invariance. We will therefore first follow the simplest "evasive" approach and turn in the next section to the "conspiratorial" solution.

We next define $\bar{F}_5 = EkF_5$ which is seen from Table 2 to be free of kinematical singularities. F_7 satisfies the Mandelstam representation without modification. We therefore complete (3.8) with:

$$F_5 \sim \frac{1}{Ek} \beta_5(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t)} s^{\alpha(t)} \quad (3.9)$$

$$F_7 \sim \beta_7(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t)} s^{\alpha(t)}$$

both amplitudes being finite at $t = 0$. We have written explicitly only the t dependence in the residue functions. They also apply to production with virtual photons and are also functions of the photons mass squared. In the Regge framework this dependence is required to be smooth at $\nu^2 = 0$.

The next step is to substitute these asymptotic expressions, written for the F 's, in the relations for the B amplitudes (Table 2). The pion trajectory contributes to only three of them all and one gets, up to a common

$$\frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} s^{\alpha(t)}$$

factor,

$$B_2 = -im\sqrt{2} \beta_2(t) \frac{\alpha(t)}{s}$$

$$\begin{aligned} B_3 &= \frac{im}{t k^2} \left\{ -\sqrt{2} \alpha(t) \beta_2(t) E k_0 + \frac{4 E k_0}{t} \beta_5(t) + k^2 \beta_7(t) \right\} \\ B_4 &= -\frac{im}{t k^2} \left\{ \sqrt{2} \alpha(t) \beta_2(t) E q_0 - \frac{4 E q_0}{t} \beta_5(t) + k^2 \beta_7(t) \right\} \end{aligned} \quad (3.10)$$

which does not involve any kinematical singularities as follows from the relations

$$\begin{aligned} E k_0 &= \frac{1}{4} (t - \mu^2 - \nu^2) \\ E q_0 &= \frac{1}{4} (t + \mu^2 - \nu^2) \\ k^2 &= \frac{t}{4} - \frac{\mu^2 + \nu^2}{2} + \frac{(\mu^2 - \nu^2)^2}{4t} \end{aligned} \quad (3.11)$$

The zeros of the denominator in B_3 and B_4 should be cancelled by the numerator vanishing at the corresponding points. This in turn implies relations among the residue functions.

The contributions to the invariant amplitudes, explicitly written in (3.10) correspond to a well defined asymptotic behaviour. They should then be themselves satisfy the gauge invariance condition (3.3a). Combining (3.3a) and (3.10) one obtains no constraint on β_2 as its coefficient vanishes identically, more generally gauge invariance is readily satisfied for natural parity $((-1)^J)$ trajectories¹⁷⁾. A relation holds however here between β_5 and β_7 . It reads:

$$\beta_5(t, \nu^2) = -\frac{1}{4}(t - \mu^2 + \nu^2) \beta_7(t, \nu^2) \quad (3.12)$$

This relation, implies in particular that:

$$\beta_5(0, \nu^2) = \frac{1}{4}(\mu^2 - \nu^2) \beta_7(0, \nu^2)$$

which removes the poles from B_3 and B_4 at $t = 0$. One may also write explicitly:

$$\beta_5(\mu^2, \nu^2) = -\frac{1}{4}\nu^2 \beta_7(\mu^2, \nu^2)$$

which forces a ratio of -2 between B_3 and B_4 at $t = \mu^2$, where $\alpha\beta_2$ does not contribute. This is, as it should be, the ratio which may be readily obtained from perturbation theory. Since the Regge pole should reproduce the perturbation theory result at $t = \mu^2$.

It remains to impose that both B_3 and B_4 remain finite at $k^2 = 0$, that is $t = (\mu \pm \nu)^2$. Such a threshold condition as opposed to a condition at $t = 0$, which may require conspiracy, should be satisfied through a relation among the residue functions of the same pole. In the pion case here considered it reads:

$$\beta_5(t, \nu^2) = \frac{1}{2\sqrt{2}} t \alpha(t) \beta_2(t, \nu^2) \quad \text{at } t = (\mu \pm \nu)^2 \quad (3.13)$$

It can be used to obtain a relation at $t = \mu^2$ and $\nu^2 = 0$, namely:

$$\mu^2 \frac{d\alpha}{dt} \beta_2(t, \nu^2) = 2\sqrt{2} \frac{d}{dt} \beta_5(t, \nu^2)$$

which, combined with (3.12) gives

$$\sqrt{2} \mu^2 \frac{d\alpha}{dt} \beta_2 = -\beta_7 \quad \text{at } t = \mu^2, \nu^2 = 0 \quad (3.14)$$

going back to (3.10) with $\nu^2 = 0$, the kinematics simplifies as it follows from (3.11). If condition (3.13) is now not required to cancel the double pole term in B_3 it should hold to make B_4 finite at $t = \mu^2$. The pion pole term, present in both B_3 and B_4 , comes from the extra common factor not explicitly written (3.10). Gauge invariance therefore leads to constraints which are not explicitly required when restricting the analysis from the start to actual photons.

The further interest of including the β_5 and β_7 residue functions in the analysis, even though only β_2 contributes to photoproduction, is that these two residue functions can be obtained from perturbation theory at $t = \mu^2$, while the pion pole term is absent from the B_2 photoproduction amplitude. The pion pole contributions to F_5 and F_7 are readily obtained and when identified with the Regge expressions, this gives:

$$\beta_5(\mu^2, \nu^2) = -\frac{i}{32m} \pi \frac{d\alpha}{dt} e_g (t^2 - 2t(\mu^2 + \nu^2) + (\mu^2 - \nu^2)^2) \quad (3.15)$$

$$\beta_7(\mu^2, \nu^2) = -\frac{i}{8m} \pi \frac{d\alpha}{dt} e_g (t + 3\mu^2 - 3\nu^2)$$

Combining (3.14) and (3.15) one gets

$$\sqrt{2} \beta_2 = \frac{i\pi}{2m} e_g \quad \text{at } t = \mu^2, \nu^2 = 0 \quad (3.16)$$

a relation which, when put back in the Regge expression written for B_2 gives $sB_2 = e.g.$ The Regge expression then reproduces the results known from matching fixed poles. While the elementary pion pole contributes only to B_3 and B_4 , the pion trajectory also contributes to B_2

the proper amount in order to satisfy the constraint of gauge invariance. One also obtains a Regge expression for the photoproduction amplitudes which is exact at $t = \mu^2$, and should remain a good approximation in the near-forward direction. It reads:

$$B_2 \simeq (t - \mu^2) \frac{\pi e g}{2} \frac{d\alpha}{dt} \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} s^{\alpha(t) - 1} \quad (3.17)$$

This analysis can be extended to any trajectory.¹⁷⁾ The particular interest of the pion trajectory is that the corresponding one-elementary particle contribution is not gauge invariant by itself while this is the case for Regge trajectories associated with natural parity particles. A similar analysis can also be extended to the nucleon trajectory applicable to backward production.

4. CONSPIRACY IN PION PHOTOPRODUCTION

If the pion trajectory leads to consistent results for the photoproduction amplitude it does not lead to any forward production since only B_1 and B_5 contribute to it. The pion trajectory as included so far cannot therefore explain the absence of a forward dip in photoproduction which is now found to be one of the prominent facts of π^+ production up to 16 GeV³). As shown by Drell and Sullivan⁹⁾, any standard exchange process would lead to a forward dip and, as mentioned already, any s channel pole term, which would give by itself a non zero contribution to forward photoproduction, should be disregarded in a Regge pole analysis since it would lead otherwise to double counting.

As already pointed out however⁷⁾, the absence of pole in B_2 at $t = 0$ could be obtained in a different way if a new trajectory is postulated. This trajectory would conspire with the trajectory of the pion that is to say they would have opposite parities and coincide at $t = 0$. The new trajectory would then contribute to F_1 and F_3 and lead therefore to a non zero forward amplitude. In effect only F_3 contributes to the forward asymptotic amplitude.

As already mentioned, and as readily checked on (3.3) the gauge invariance constraints are automatically satisfied by this new trajectory since it corresponds to natural parity. We now define the helicity amplitudes free of kinematical singularities relevant for this analysis, $\bar{F}_2$ and $\bar{F}_3$, as follows

$$\bar{F}_2 = 2\sqrt{2} \frac{p}{E} \bar{F}_2 \quad \bar{F}_3 = \sqrt{2} k p^2 \bar{F}_3 \quad (4.1)$$

The coefficients of both $\bar{F}_2$ and $\bar{F}_3$ in B_2 , B_3 and B_4 are now singular at $t = 0$ and the absence of pole at this point requires that

$$\bar{F}_3 = \frac{8}{\ln(\mu^2 - \gamma^2)} \bar{F}_2 \quad \text{at } t = 0 \quad (4.2)$$

The value of the new amplitude $\bar{F}_2$ at $t = \mu^2$ is obtained as before.

One gets

$$\bar{F}_2(\mu^2, 0) = \frac{1 \text{ eg } \mu^2}{4 s m}$$

and

$$\beta_2(\mu^2) = \frac{c \pi \mu^2}{8 m} \text{ eg}$$

which one might expect to be close to the value of β_2 at $t = 0$. If we assume that the pion residue function does not change between $t = \mu^2$ and $t = 0$ one obtains

$$\bar{F}_3(0) = \frac{2 \text{ eg } \mu^2}{\ln^2(\mu^2 - \gamma^2) s}$$

Furthermore, if one assumes that the relations that exist at $t = 0$ between the residue functions and Regge trajectories of the pion and its conspirator are approximately true over a range of t then one obtains

$$B_2 \simeq \frac{\mu^2 \pi e g}{2(\mu^2 - \nu^2)} \frac{\beta_2(t)}{\beta_2(\mu^2)} \alpha(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi \alpha(t)} s^{\alpha(t)-1}$$

A result for B_2 which is identical to the contribution from the non conspiring pion trajectory given in equation (3.17), at $\nu^2 = 0$.

The asymptotic forward π^+ photoproduction is given entirely by A_1 where¹³⁾

$$A_1 = B_1 - m B_6 = i m^2 \bar{F}_3 = - \frac{2 e g}{s} \frac{\beta(0)}{\beta(\mu^2)} s^{-\alpha(0)}$$

This is to be compared to the nucleon pole term contribution which, taken alone, would give

$$A_1 = \frac{e g}{s}$$

The forward amplitude obtained from a conspiratorial solution which includes the constraints of gauge invariance, leads therefore to a result which is similar^{in modulus} to what one would calculate from the nucleon Ban term. The shape of the forward peak is further determined from the combination of the pion and of the conspirator trajectories contributions. If such a picture is correct the forward differential cross section should go down slightly faster than s^{-2} , that is $s^{-(2.04)}$, with the slope usually assumed for the pion trajectory.

TABLE CAPTIONS

<u>Table I</u>	Helicity amplitudes expressed in terms of the invariant amplitudes.
<u>Table II</u>	Invariant amplitudes expressed in terms of the parity conserving helicity amplitudes.
<u>Table III</u>	Partial wave expansions for the parity conserving helicity amplitudes
<u>Table IV</u>	Elimination of the t channel kinematical singularities from parity conserving helicity amplitude free of s channel kinematical singularities.

- Table I -

$$f_{1/2, 1/2; 1} = -\frac{i}{\sqrt{2}} \left\{ k \left(\frac{E}{m} - \frac{pk_0}{mk_2} \right) B_1 - \frac{2Ep}{m} B_2 + pk_0 B_6 \right\} \sin \theta$$

$$f_{1/2, -1/2; 1} = \frac{i}{\sqrt{2}} \left\{ k B_1 + \frac{p^2 k}{m} (1 - \cos \theta) B_6 - \frac{p}{m} B_5 \right\} (1 + \cos \theta)$$

$$f_{-1/2, 1/2; 1} = \frac{i}{\sqrt{2}} \left\{ k B_1 + \frac{p^2 k}{m} (1 + \cos \theta) B_6 + \frac{p}{m} B_5 \right\} (1 - \cos \theta)$$

$$f_{-1/2, -1/2; 1} = -\frac{i}{\sqrt{2}} \left\{ k \left(\frac{E}{m} + \frac{pk_0}{mk_2} \right) B_1 + \frac{2Ep}{m} B_2 - pk_0 B_6 \right\} \sin \theta$$

$$f_{1/2, 1/2; 0} = -i \left\{ \frac{kE + k_0 p \cos \theta}{m} B_1 + \frac{2Ep}{m} \cos \theta B_2 - \frac{2Ek}{m} (B_3 - B_4) + \right. \\ \left. - k_0 p \cos \theta B_6 - k_0 k (B_7 - B_8) \right\}$$

$$f_{1/2, -1/2; 0} = -\frac{i p}{m} \left\{ B_5 + pk \cos \theta B_6 + k^2 (B_7 - B_8) \right\} \sin \theta$$

$$f_{1/2, 1/2; S} = -i \left\{ -\frac{k p \cos \theta + k_0 E}{m} B_1 - \frac{2Eq_0}{m} B_3 - \frac{2Ek_0}{m} B_4 - B_5 + \right. \\ \left. + k_0^2 B_7 + k_0 q_0 B_8 \right\}$$

$$f_{1/2, -1/2; S} = \frac{i p k}{m} \left\{ k_0 B_7 + q_0 B_8 \right\} \sin \theta$$

- Table II -

$$B_1 = \frac{im}{\sqrt{2} k E} F_1$$

$$B_2 = \frac{-im}{2\sqrt{2}} \left(\frac{k_0}{k p^2} \left(F_1 + \frac{m}{E} F_3 \right) + \frac{1}{E p} F_2 \right)$$

$$B_3 = \frac{im}{4E^2} \left(\frac{E \cos \theta}{\sqrt{2} p} \left(F_1 + \frac{m}{E} F_3 \right) + \frac{k_0}{\sqrt{2} k} \cos \theta F_2 + \right. \\ \left. + \frac{m}{\sqrt{2} p k^2} (k^2 - k_0^2) F_4 + \frac{k_0}{k} F_5 + \frac{m k_0^2}{p k^2} F_6 + F_7 + \frac{m k_0}{p k} F_8 \right)$$

$$B_4 = \frac{-im}{4E^2} \left(\frac{\sqrt{2} E}{k} F_1 + \frac{E \cos \theta}{\sqrt{2} p} \left(F_1 + \frac{m}{E} F_3 \right) - \frac{q_0}{\sqrt{2} k} \cos \theta F_2 + \right. \\ \left. + \frac{m}{\sqrt{2} p k^2} (k_0 q_0 + k^2) F_4 - \frac{q_0}{k} F_5 - \frac{m k_0 q_0}{p k^2} F_6 + F_7 + \frac{m k_0}{p k} F_8 \right)$$

$$B_5 = \frac{im}{\sqrt{2} p} \left(\cos \theta \left(F_3 + \frac{m}{E} F_1 \right) + F_4 \right)$$

$$B_6 = \frac{-im}{\sqrt{2} p^2 k} \left(F_3 + \frac{m}{E} F_1 \right)$$

$$B_7 = \frac{im q_0}{2E p k^2} \left(-\frac{F_4}{\sqrt{2}} + F_6 - \frac{k}{q_0} F_8 \right)$$

$$B_8 = \frac{-im k_0}{2E p k^2} \left(-\frac{F_4}{\sqrt{2}} + F_6 + \frac{k}{q_0} F_8 \right)$$

- Table III -

$$F_1 = \sum_J (2J+1) F_1^J P_J'(\cos \theta)$$

$$F_2 = \sum_J (2J+1) F_2^J P_J'(\cos \theta)$$

$$F_3 = \sum_J \frac{2J+1}{J(J+1)} \left\{ F_3^J \left(P_J'(\cos \theta) + \cos \theta P_J''(\cos \theta) \right) - F_4^J P_J''(\cos \theta) \right\}$$

$$F_4 = \sum_J \frac{2J+1}{J(J+1)} \left\{ F_4^J \left(P_J'(\cos \theta) + \cos \theta P_J''(\cos \theta) \right) - F_3^J P_J''(\cos \theta) \right\}$$

$$F_5 = \sum_J (2J+1) F_5^J P_J(\cos \theta)$$

$$F_6 = \sum_J (2J+1) F_6^J P_J'(\cos \theta)$$

$$F_7 = \sum_J (2J+1) F_7^J P_J(\cos \theta)$$

$$F_8 = \sum_J (2J+1) F_8^J P_J'(\cos \theta)$$

- Table IV -

$$F_1 = \frac{\sqrt{2} k E}{m} \overline{F}_1$$

$$F_2 = 2\sqrt{2} E p \overline{F}_2 \text{ (evasion)} ; F_2 = 2\sqrt{2} \frac{p \overline{F}_2}{E} \text{ (conspiracy)}$$

$$F_3 = \sqrt{2} k \overline{F}_3 , F_4 = \sqrt{2} p \overline{F}_4 , F_5 = \frac{1}{E k} \overline{F}_5$$

$$F_6 = p \overline{F}_6 , F_7 = \overline{F}_7 , F_8 = \frac{k E}{p} \overline{F}_8$$

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One pion exchange contribution