CM-P00057737

Archives

Ref. TH.819

MESON REGGE EXCHANGE AMPLITUDES AND THE πN
RESONANCE SPECTRUM ^{+))}

V. Barger ^{*)} and R.J.N. Phillips ^{**)}

CERN -- Geneva

ABSTRACT

Regge exchange amplitudes in πN scattering are related to the πN bound state and resonance spectrum, through sum rules based on analyticity. Various ambiguities in dynamical approximations are noted. Characteristic features of ρ exchange can be consistently generated, but the method seems unsuitable for Pomeron amplitudes.

^{+))} Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U.S. Atomic Energy Commission under contract AT(11-1)881.

^{*)} Permanent address : University of Wisconsin,
Madison, Wisconsin, U.S.A.

^{**)} Permanent address : A.E.R.E., Harwell, Berks., U.K.

Sum rules based on analyticity relate asymptotic behaviour to low energy scattering, and provide a powerful tool for determining properties of Regge exchange amplitudes ¹⁾⁻⁴⁾. In this letter we apply such sum rules to the πN system. The crucial role of dynamical assumptions for intermediate energy amplitudes is noted. The information contained in the known πN resonance spectrum is used in evaluating the sum rules.

The sum rules are obtained from the standard relation

$$\int_{-\infty}^{\infty} \nu^m \operatorname{Im} \tilde{a}(\nu + i\epsilon, t) d\nu = 0, \quad (1)$$

where $\tilde{a}$ is an amplitude analytic in the cut ν plane, decreasing faster than $|\nu|^{-m-1}$ as $|\nu| \rightarrow \infty$. To obtain an amplitude $\tilde{a}$ with the desired convergence, it is usually necessary to subtract some part of the asymptotic behaviour from the physical amplitude a : $\tilde{a} = a - a(\text{asym})$. This asymptotic behaviour is usually expressed in terms of Regge exchange amplitudes. High moment sum rules (i.e., with large m) require many leading powers of ν to be correctly subtracted, and many Regge amplitudes may be expected to play a role. The lower moment sum rules are therefore more profitable and reliable.

For πN scattering we use the conventional invariant amplitudes, $T = -A + i \gamma \cdot Q_B$, with $\nu = (s-u)/(4M)$. For convenience we use the isospin even and odd combinations

$$A^{\pm}(\nu, t) = \frac{1}{2} [A(\pi^- p) \pm A(\pi^+ p)], \quad (2)$$

with similar definitions for $B^{\pm}$. Under the transformation $\nu + i\epsilon \rightarrow -\nu + i\epsilon$, $\operatorname{Im} A^+$ and $\operatorname{Im} B^-$ are odd, $\operatorname{Im} A^-$ and $\operatorname{Im} B^+$ are even. For these symmetrized amplitudes, the sum rule Eq. (1) is non-trivial only for $m = \text{odd}$ with A^+ , B^- and for $m = \text{even}$ with A^- , B^+ . For these moments, the range of integration can be converted to (ν_0, ∞) , where ν_0 extends down to the nucleon pole.

To use the sum rules in practice, we assume the amplitude is well approximated at high energies $\nu > \nu_1$ (with $\nu_1 \approx 4-6$ GeV) by a few Regge poles

$$a(\text{asym}) = \sum_j \gamma_j \nu^{\alpha_j - \lambda}, \quad (3)$$

where $\alpha_j(t)$ are the Regge trajectories, $\gamma_j(t)$ are the residue functions; $\lambda = 0$ for A and $\lambda = 1$ for B. Subtracting these terms, the sum rules become

$$\int_{\nu_0}^{\nu_1} \nu^m \text{Im } a \, d\nu = \sum_j \gamma_j \frac{\nu_1^{(\alpha_j - \lambda + m + 1)}}{(\alpha_j - \lambda + m + 1)} \quad (4)$$

Thus a knowledge of the amplitude for $\nu \leq \nu_1$ gives information about the Regge terms evaluated at $\nu = \nu_1$. We shall use the usual P and P' Regge poles for the amplitudes A^+ , B^+ and the ρ pole for A^- , B^- .

Although at low energies, $\nu < 1$ GeV, some information is becoming available through phase shift analyses, there is a broad range of intermediate energies ($1 < \nu < 4$ GeV) where we are obliged ⁵⁾ to make dynamical assumptions about $\text{Im } a(\nu, t)$. Widely differing approximations have been proposed, using direct channel resonances, or Regge exchanges, or both:

- (I) Resonances plus Regge at intermediate energies ["interference model" ^{6), 7)}].
- (II) Resonances plus Regge at intermediate and low energies ⁸⁾. In this extreme, Regge terms completely cancel out of the sum rules Eq. (4).
- (III) Regge alone at intermediate energies ⁴⁾, neglecting known resonances here.
- (IV) Resonances alone at intermediate energies.

According to the approximation chosen, the upper limit on the resonance sum and the energy at which the Regge term is evaluated vary, and are not necessarily equal. A particular approximation can only be justified a posteriori: it seems unlikely that any one prescription will be suitable for all amplitudes.

Our attitude is this : resonances at intermediate energies provide dynamical information which should be used if possible, so that prescriptions (I) and (IV) above are preferable. It is also probable that resonances give an adequate approximation to the low energy amplitude. With this motivation, we evaluate the left-hand side of Eq. (4) as a sum over resonances, up to an upper boundary $\nu_1 = 4.16$ GeV (corresponding to $\sqrt{s} = 3$ GeV), which fairly well exhausts the known resonance spectrum^{9),10)}. We then examine to what extent this agrees with the right-hand side of Eq. (4), as determined from phenomenological Regge amplitudes evaluated at an appropriate cut-off energy.

Extracting physics from the resonances is quantitatively reliable only if :

- i) individual resonance terms converge rapidly (i.e., low moment sum rules), and
- ii) the sums are not sensitive to the exact values of resonance parameters (i.e., no large cancellations occur).

Of the low moment sum rules, the integrals of A^- and νB^- satisfy both criteria: on the other hand, B^+ involves strong cancellations and $A^+\nu$ has very slow convergence. Consequently we can only handle the A^- and νB^- sum rules with confidence : these involve the ρ Regge exchange.

The comparison of resonance sums with Regge terms, through the range $0 \leq -t \leq 1$ (GeV/c)² is shown in Fig. 1. Solid curves denote the resonance sums for A^- and νB^- , taken up to $\nu = 4.16$ GeV. Dashed lines denote the ρ Regge term, from a typical empirical fit¹¹⁾ to high energy scattering (above 5 GeV), evaluated at the same boundary energy $\nu = 4.16$ GeV.

The close agreement of resonance and Regge curves in Fig. 1 suggests that prescription (IV) is appropriate for these amplitudes. Various fits to high energy scattering agree that $\text{Im} B^-$ is large and positive at $t=0$ and changes sign near $t=-0.6 \text{ (GeV/c)}^2$; the amplitude $A'^- = A^- + \nu B^- / (1-t/4M^2)$ is found to be relatively small. Prescription (IV) correctly gives $\text{Im} A^-$ and $\text{Im} B^-$ in sign, magnitude and t dependence (and consequently gives a small $\text{Im} A'^-$). Resonance sums are not quantitatively reliable for $\text{Im} A'^-$, since there are enormous cancellations. Furthermore, independent evidence from fitting cross-section data at $t=0$ suggests that prescription (I) may be more suitable for the A'^- amplitude ⁷⁾.

If instead we truncate the resonance sums at $\nu = 1.65 \text{ GeV}$ ($\sqrt{s} = 2 \text{ GeV}$) and also evaluate the Regge terms at this energy, the latter are too small by a factor two to satisfy Eq. (4). Thus prescription (III) fails for A^- and νB^- , at least in normalization.

The convergence of the A^- and νB^- sums, at $t=-0.2$ and -0.3 (GeV/c)^2 , is shown in Fig. 2; contributions from a given recurrence sequence appear to converge satisfactorily. It is interesting that MacDowell symmetric pairs [e.g., $N_\alpha(1688, \frac{5}{2}^+)$ and $N_\beta(1670, \frac{5}{2}^-)$] contribute with opposite signs to each amplitude in Fig. 2. The possible existence of MacDowell symmetric partners for higher mass resonances introduces a further degree of uncertainty into sum rules.

Despite the reservations above regarding the usefulness of B^+ and νA^+ sum rules, we note that the B^+ resonance sum is relatively small ⁸⁾ while the νA^+ sum is positive at $t=0$ and changes sign near $t=-0.5 \text{ (GeV/c)}^2$. Unfortunately, phenomenological fitting to high energy scattering has not yet produced unique results for the P and P' Regge amplitudes, with which to compare ¹²⁾. However, a small B^+ Regge amplitude is not excluded, and the fits indicate a positive value for $\text{Im} A^+$ at $t=0$.

We thank Professor J. Prentki and Professor L. Van Hove for the hospitality of the CERN Theoretical Study Division, where this work was carried out.

R E F E R E N C E S

- 1) A.A. Logunov, L.D. Soloviev and A.N. Tavkhelidze, Phys.Letters 24B, 181 (1967).
- 2) K. Igi and S. Matsuda, Phys.Rev.Letters 18, 625 (1967).
- 3) Y. Liu and S. Okubo, Phys.Rev.Letters 19, 190 (1967).
- 4) R. Dolen, D. Horn and C. Schmid, California Institute of Technology preprint CALT-68-133 (1967);
D. Horn and C. Schmid, *ibid*, CALT-68-127 (1967).
- 5) An exception is the forward spin averaged amplitude, for which the imaginary part is directly determined by total cross-sections and the real part by Coulomb interference.
- 6) V. Barger and D. Cline, Phys.Rev. 155, 1792 (1967);
J. Baacke and M. Yvert, in Proceedings of the XIIIth International Conference on High Energy Physics, Berkeley (1966).
- 7) V. Barger and M. Olsson, Phys.Rev. 151, 1123 (1966).
- 8) R. Gatto, Phys.Rev.Letters 18, 803 (1967);
R.J. Rivers, Imperial College preprint ICTP/67/27.
- 9) We use the narrow width approximation with resonance parameters from Ref. 10) and take $g^2/4\pi = 14.4$ for the πN coupling.
- 10) A.H. Rosenfeld et al., Revs.Modern Phys. 39, 1 (1967).
- 11) W. Rarita, R.J. Riddell, Jr., C.B. Chiu and R.J.N. Phillips, University of California Laboratory Report UCRL-17523 (1967).
- 12) The qualitative feature that B^+ is small may be useful in eliminating certain classes of Regge fits. With an upper limit $\sqrt{s} = 4.16$ GeV, the resonance contribution to $\int B^+$ at $t=0$ is 66 GeV^{-1} . The $P+P'$ contribution depends on the Regge boundary energy, which in turn depends on the approximation assumed.

FIGURE CAPTIONS

- Figure 1 Comparison of resonance and Regge contributions to the sum rules of Eq. (4) : $A_0^- = \int \text{Im} A^-(\nu, t) d\nu$ and $B_1^- = \int \nu \text{Im} B^-(\nu, t) d\nu$, in the range $0 \leq -t \leq 1 \text{ (GeV/c)}^2$. Solid curves are resonance sums up to $\nu = 4.16 \text{ GeV}$; dashed curves are Regge terms evaluated at $\nu = 4.16 \text{ GeV}$. Solid and dashed curves should coincide if approximation (IV) of the text is valid.
- Figure 2 Individual resonance contributions to the sum rules A_0^- and B_1^- evaluated at $t = -0.2$ and -0.8 (GeV/c)^2 ; cf., Fig. 1. Small contributions from $N_\beta(1570)$ and $\Delta_\beta(1670)$ are not plotted.

ρ REGGE EXCHANGE $[\nu_p \approx 4.16 \text{ or } \sqrt{s_p} \approx 2.95 \text{ GeV}]$

FIG. 2

RESONANCE CONTRIBUTIONS TO ρ EXCHANGE SUM RULES