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ON SINGH'S LEMMA FOR LOW ENERGY COMPTON SCATTERING

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A new derivation will be given of a lemma of Singh ¹⁾ which has proved useful in establishing low energy theorems for Compton scattering ^{1),2)}. The original treatment depends on the validity of a certain infinite expansion over intermediate states. This can be avoided. The result follows essentially from gauge invariance and the notion that the amplitude, after extraction of the pole terms, is a smooth function.

Let

$$k \equiv (\vec{k}, i\omega) \quad k' \equiv (\vec{k}', i\omega') \quad p_I \equiv (\vec{p}_I, i\sqrt{M^2 + \vec{p}_I^2}) \quad p_F \equiv (\vec{p}_F, i\sqrt{M^2 + \vec{p}_F^2})$$

be initial and final photon and target four-momenta; the photons are not restricted to the mass shell. Let $T_{\mu\nu}$ be the Compton amplitude for some specified target spin states, where μ and ν are photon polarization indices. A particular contribution is that of unexcited intermediate states :

$$U_{\mu\nu} = \sum_N \frac{\langle F | j_\mu | N \rangle \langle N | j_\nu | I \rangle}{\omega + E_I - E_N} \delta(\vec{k} + \vec{p}_I - \vec{p}_N) + (k \rightleftharpoons -k', \mu \rightleftharpoons \nu) \quad (1)$$

where the currents j refer to the origin of co-ordinates, the last bracket indicates the crossed term, and the summation is over unexcited target states N ³⁾. The residual "excited" contribution $E_{\mu\nu}$ is defined by

$$T_{\mu\nu} = U_{\mu\nu} + E_{\mu\nu} \quad (2)$$

Singh's lemma is that for fixed $\vec{p}_I$

$$E_{00} = k'_m k_n \Lambda_{mn}(k', k) \quad (3)$$

2.

where Λ_{mn} is free from singularities for small values of the variables (just "F-S" hereafter), and has the crossing property

$$\Lambda_{mn}(k', k) = \Lambda_{nm}(-k, -k') \quad (4)$$

Latin indices take values 1, 2, 3. In obtaining this result, Singh represents E_{00} by an infinite expansion over intermediate states. However it will be seen a certain property of the finite summation defining V_{00} is sufficient.

Note first that as independent variables we can take $\vec{k}$, $\vec{k}'$, and $\omega + \omega'$; $\omega - \omega'$ is determined ⁴⁾ by the mass shell condition

$$\Delta = (P_{10} + \omega - \omega')^2 - (\vec{P}_1 + \vec{k} - \vec{k}')^2 - M^2 = 0 \quad (5)$$

The relevant property of U_{00} is that it vanishes in general (i.e., avoiding singularities) when either $\vec{k}$ or $\vec{k}'$ approaches zero, the other variables being held fixed. This follows from the construction (1) and

$$\begin{aligned} \sum_N \delta(\vec{P}_F - \vec{P}_N) \langle F | J_0 | N \rangle \langle N | &= Z \langle F | \\ \sum_N \delta(\vec{P}_I - \vec{P}_N) | N \rangle \langle N | J_0 | I \rangle &= Z | I \rangle \end{aligned} \quad (6)$$

which are consequences of the commutation of total charge with spin, Z being the target charge. That T_{00} has the same property follows from

$$\omega' \omega T_{00} = k'_m k_n T_{mn} \quad (7)$$

which is a consequence ⁵⁾ of the gauge invariance conditions $k'_\mu T_{\mu\nu} = T_{\mu\nu} k_\nu = 0$. Then $E_{00} (= T_{00} - U_{00})$ must also vanish in general for $\vec{k}=0$ or $\vec{k}'=0$; since it is an F-S function (U_{00} supposedly accounting entirely for the singularities of T_{00} for small values of the variables)

$$E_{00} = k'_m W_{mn} k_n \quad (8)$$

where W_{mn} is F-S.

In the first instance (8) is subject to the mass shell restriction (5). However we can continue E_{00} and W_{mn} , for example by specifying that they do not depend on $\omega' - \omega$, in such a way that (8) remains true. Then identifying W_{mn} with Λ_{mn} we have the result (3). The crossing property (4) follows from the crossing properties of $T_{\mu\nu}$ and $U_{\mu\nu}$.

The infinite expansion employed by Singh implies the validity of certain unsubtracted dispersion relations in ω . With the present approach, on the contrary, no implication about high energy behaviour is made. Proof of the low energy theorems then depends only on low energy assumptions.

I am much indebted to Professor A. Pais in connection with this question.

R E F E R E N C E S

- 1) V. Singh, Rockefeller preprint (1967).
- 2) A. Pais, Brookhaven preprint, and CERN preprint TH.816 (1967).
- 3) The states N are real, on-the-mass-shell, states.
- 4) It is assumed that $P_{FO} (= P_{IO} + \omega - \omega')$ is positive.
- 5) F. Low, Phys.Rev. 96, 1428 (1954).