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PION MASS DIFFERENCE AND CURRENT ALGEBRA

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A B S T R A C T

A recently derived sum rule for the electromagnetic mass difference of zero mass pions is extended to pions of finite mass. A particular correction, due to the variation of the function involved in the neighbourhood of a branch point, is estimated. It is quite small.

1. INTRODUCTION

Das et al.¹⁾ have presented a sum rule for the electromagnetic mass difference of the pions in the case of hypothetical pions of zero unperturbed mass. We have tried to reformulate this for massive pions. One could not be sure a priori that the corrections would be small. Dimensionally we would expect a factor m_π^2 and a factor α , whence a correction

$$\delta(m_{\pi^+}^2 - m_{\pi^0}^2) \approx \alpha m_\pi^2$$

or

$$\delta(m_{\pi^+} - m_{\pi^0}) \approx \frac{1}{2} \text{ Mev}$$

However, the argument in the case of massive pions involves continuing a function from zero momentum squared (where the usual current algebra manipulations provide information) to the momentum squared of a real pion - which happens to be a branch point of the function. This could enhance the correction, so we feared finally a contribution by no means small compared with the observed 5 MeV. Actually we find that the branch point in itself is not very dangerous, and from it we estimate only a very small effect.

2. ZERO MOMENTUM

We define a quantity $\Delta(s)$ such that $\Delta(m_\pi^2)$ is the required electromagnetic mass squared difference:

$$\Delta(m_\pi^2) = m^2(\pi^+) - m^2(\pi^0) \quad (1)$$

The defining formula is, with $s = -p^2$,

$$\Delta(s) = \frac{ie^2}{2} \int \frac{d^4 q}{i(2\pi)^4} \frac{1}{q^2} (\delta_{\mu\nu} + \lambda \frac{q_\mu q_\nu}{q^2}) T_{\mu\nu}(p, q) \quad (2)$$

$$T_{\mu\nu}(p, q) = (m_\pi^2 + p^2)^2 \langle 0 | \int dx dy dz e^{ip(x-y) + iqz} T^*((\pi^2(x) \pi^2(y) - \vec{\pi}(x) \cdot \vec{\pi}(y)) j_\mu(z) j_\nu(0)) | 0 \rangle \quad (3)$$

where λ is a gauge parameter, j is the isovector part of the electromagnetic current, and $\vec{\pi}$ is a renormalized pion field related to the divergence of the axial current by ²⁾

$$\partial_\mu \vec{A}_\mu = F_\pi m_\pi^2 \vec{\pi} \quad (4)$$

The symbol T^* means that if the simple time-ordered product is not equal to the covariant off-mass-shell Compton amplitude, it is the latter that we mean. In the version of current algebra that we have in mind ³⁾ this only means, in effect, that Schwinger terms are to be ignored in the conventional manipulations at fixed finite momenta p and q .

In the usual way, assuming in particular that " σ commutators" are isoscalar, we obtain the zero-momentum theorem

$$T_{\mu\nu}(0, q) = \frac{2}{F_\pi^2} \langle 0 | \int dz e^{iqz} T^*(V_\mu^3(z) V_\nu^3(0) - A_\mu^3(z) A_\nu^3(0)) | 0 \rangle$$

Let us write

$$T_{\mu\nu}(0, q) = \frac{2}{i F_\pi^2} \left[P(q^2) \delta_{\mu\nu} + Q(q^2) q_\mu q_\nu \right] \quad (6)$$

Then

$$\Delta(0) = \frac{e^2}{F_\pi^2} \int \frac{d^4 q}{i(2\pi)^4} \frac{1}{q^2} \left[3P + (\lambda+1)(P + Qq^2) \right] \quad (7)$$

If we take Landau gauge, $\lambda = -1$, and if we assume an unsubtracted dispersion relation for P

$$P = \int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2 + q^2} \quad (8)$$

we have (indicating Landau gauge by a subscript)

$$\Delta_L(0) = \frac{3e^2}{F_\pi^2} \int \frac{d^4 q}{i(2\pi)^4} \frac{1}{q^2} \int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2 + q^2} \quad (9)$$

If we ignore the difference between $\Delta(0)$ and $\Delta(m_\pi^2)$, this is the result quoted by Das et al.

The expression (9) is finite if Weinberg's second conjectured sum rule ⁴⁾ is assumed:

$$\int dm^2 (\rho_V(m^2) - \rho_A(m^2)) = 0 \quad (10)$$

The first conjectured sum rule of Weinberg

$$\int \frac{dm^2}{m^2} (\rho_V(m^2) - \rho_A(m^2)) = F_\pi^2 \quad (11)$$

plays no direct role in our derivation of (9). However, it is useful in the numerical evaluation. It is also contained to some extent in our assumptions.

4.

From (4) and (5)

$$q_\mu q_\nu T_{\mu\nu} = 2i m_\pi^4 (d(q^2) - d(0)) \quad (12)$$

where $d(q^2)$ is the pion propagator ⁵⁾,

$$d(q^2) = i \langle 0 | \int dz e^{iqz} T(\pi^3(z), \pi^3(0)) | 0 \rangle \quad (13)$$

and $d(0)$ is a constant coming from a σ term and fixed in magnitude by requiring a zero at $q=0$. Evaluating the left-hand side of (12) from (6)

$$P + Q q^2 = m_\pi^4 F_\pi^2 (d(0) - d(q^2)) q^{-2} \quad (14)$$

Approximating $d(q^2)$ for small q^2 by the pion pole ⁶⁾

$$d(q^2) \approx (m_\pi^2 + q^2)^{-1} \quad (15)$$

we find

$$P(0) = F_\pi^2 \quad (16)$$

which reduces to (11) in virtue of the dispersion relation (8). So again there is some relation between assumptions about subtractions in dispersion relations and about effective ⁷⁾ commutators.

3. CONTINUATION

It remains to estimate $\Delta_L(m_\pi^2) - \Delta_L(0)$. Of course, we cannot do this completely. But we can estimate in particular the effect of the branch point mentioned in the Introduction. If $\Delta_L(s)$ satisfies a dispersion relation with not more than one subtraction, we have

$$\Delta_L(m_\pi^2) - \Delta_L(0) = \frac{1}{\pi} \int_{m_\pi^2}^{\infty} dm^2 (\text{Im } \Delta_L(m^2 + i\epsilon)) \left(\frac{1}{m^2 - m_\pi^2} - \frac{1}{m^2} \right) \quad (17)$$

For $m_\pi^2 < m^2 < 4m_\pi^2$, the absorptive part is determined by the one-photon one-pion intermediate state. Let the photon pion vertex

(in which only pion p' is on its mass shell) be denoted by

$$a(\gamma, s) (p + p')_\mu + b(\gamma, s) (p' - p)_\mu \quad (18)$$

Then we find for this contribution

$$\text{Im } \Delta_L(s) = \frac{e^2}{16\pi} \frac{s - m_\pi^2}{s} \left\{ 3(s + m_\pi^2) |a(0, s)|^2 + (s - m_\pi^2) \left[\frac{d}{d\gamma} |a(\gamma, s)|^2 \right]_{\gamma=0} \right\} \quad (19)$$

For the form factor $a(\gamma, s)$ we have, by a theorem of Low⁸⁾, $a(0, s) = 1$. We do not know anything about the derivative of the form factor; but if we assume

$$\left[\frac{d}{d\gamma} |a(\gamma, s)|^2 \right]_{\gamma=0} \approx m_\rho^{-2} |a(0, s)|^2$$

where m_ρ is the rho mass, the contribution of the second term in (19) will be relatively small. Neglecting it, we have for the contribution to (17) from the region between the first and second thresholds

$$\begin{aligned} \frac{3e^2}{16\pi^2} \int_{m_\pi^2}^{4m_\pi^2} dm^2 \frac{m^4 - m_\pi^4}{m^2} \frac{m_\pi^2}{(m^2 - m_\pi^2)m^2} \\ = \frac{3e^2}{16\pi^2} m_\pi^2 \left[\ln 4 - \frac{3}{4} \right] \end{aligned}$$

Numerically this is a contribution to the mass difference of about one tenth of an MeV, which is quite negligible on the relevant scale of accuracy. While we cannot do anything about the remaining contributions to $\Delta(m_\pi^2) - \Delta(0)$, at least the beginning of the cut is less serious than we feared.

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R E F E R E N C E S

- 1) T. Das et al., Phys.Rev.Letters 18, 759 (1967).
- 2) Our normalizations are like those of Weinberg.
- 3) For example: J.S. Bell, Nuovo Cimento 50, 129 (1967) and CERN TH.810. Note that our T^* amplitude is supposed to include any necessary contact or "seagull" terms, whether or not these have the particular form mentioned in (1).
- 4) S. Weinberg, Phys.Rev.Letters 18, 507 (1967);
T.D. Lee, S. Weinberg and B. Zumino, Phys.Rev.Letters 18, 1029 (1967).
- 5) This result can also be obtained by calculating straightforwardly $q_\mu q_\nu T_{\mu\nu}(0,q)$ using expression (3).
- 6) Clearly we might use (14) and (15) to estimate the gauge dependent contribution in (7). The result is a logarithmic divergence. The requirement that $\Delta(0)$ converges for arbitrary λ (for which we know no strong argument) would imply

$$(d(q^2) - d(0)) \rightarrow 0 \quad \text{for } q^2 \rightarrow \pm \infty$$
- 7) For the distinction between kinds of commutators, see for example, J.S. Bell, Nuovo Cimento 47, 616 (1967).
- 8) F.E. Low, Phys.Rev. 110, 974 (1958).