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ANALYSIS OF K_{e4} DECAY, EVIDENCE FOR A VECTOR CURRENT,
AND ITS CONSEQUENCESF.A. Berends ^{*)}, A. Donnachie and G.C. Oades
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K_{e4} decay data are analysed under various assumptions about the behaviour of the s wave $\pi - \pi$ phase shift, and the s wave scattering length and the axial vector form factors are evaluated. An interesting feature of the analysis is the possible existence of a large vector current term. The form factor is determined and $SU(3)$ (with vector octet dominance) used to relate this to η and η' decay. If taken literally, the results obtained are not compatible with the experimental evidence on these decays. However, the errors on the vector form factor are so large that the discrepancy is, at worst, not much more than two standard deviations. There are also several theoretical loopholes which are discussed. The ρ dominance model is used to calculate the vector form factor, and the theoretical value is found to be of the order of two standard deviations below the experimental result.

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1. INTRODUCTION

K_{e4} decay data are analysed under various assumptions about the behaviour of the s wave $\pi - \pi$ phase shift. Apart from possible enhancement factors, the form factors are assumed to be constant.

Allowing suitable enhancement factors, a good fit to the data can be obtained with a Chew-Mandelstam parametrization of the s wave $\pi - \pi$ phase shift with a scattering length $a_0^{(0)} = 0.8$. A simple (two parameter) Breit-Wigner resonance is excluded by the data, but a more complicated resonance structure is allowed.

The axial vector form factors are found not to depend very strongly on the nature of the s wave $\pi - \pi$ phase shift and to be in reasonable agreement with the predictions of current algebra.

There is evidence for a vector current contribution and it is possible that it is much stronger than has hitherto been supposed. The vector form factor is found to be almost completely independent of the nature of the $\pi - \pi$ phase shift. $SU(3)$, with the assumption of vector octet dominance, is used to relate the vector part of K_{e4} decay to the decays $\eta \rightarrow \pi^+ \pi^- \chi$ and $\eta' \rightarrow \pi^+ \pi^- \chi$. Assuming no $\eta - \eta'$ mixing, the η width may be evaluated directly from the vector form factor. Allowing $\eta - \eta'$ mixing, and constraining the η width to lie within the accepted experimental range, the η' width may be evaluated directly from the vector form factor. In both cases, if the results of the analysis of the K_{e4} decay data are taken literally, the results are much larger than the current experimental limits on the η and η' widths. However, the errors on the K_{e4} vector form factor are so large that even in the most extreme case the discrepancy is not much more than two standard deviations. There are also a number of theoretical loop-holes, which are discussed.

2.

Finally, the ρ dominance model, which gives a reasonable value for the width of the decay $\eta \rightarrow \pi^+ \pi^- \gamma$, is used to compute the vector form factor. The theoretical value is much less than the observed experimental value (taken literally) but there is the loop-hole of the large errors on the latter.

2. FORMALISM

We consider the K_{e4} decay

$$K^+(p_K) \longrightarrow \pi^+(p_+) + \pi^-(p_-) + e^+(p_e) + \nu_e(p_\nu) \quad (2.1)$$

The formalism of this decay has been described by several authors ¹⁾⁻⁷⁾. Here we follow the notation of Cabibbo and Maksymowicz ⁵⁾, where full details may be obtained.

Apart from kinematical factors, the decay matrix element is given by

$$\frac{G}{\sqrt{2}} \bar{u}(p_\nu) \gamma^\lambda (1 - i\gamma_5) v(p_e) \langle \pi^+ \pi^- | J_\lambda^V + J_\lambda^A | K^+ \rangle \quad (2.2)$$

where G is the universal Fermi coupling constant, J_λ^V is the hadron weak vector current and J_λ^A is the hadron weak axial vector current. From invariance considerations, the matrix elements of the hadron vector and axial vector currents have the forms

$$\langle \pi^+ \pi^- | J_\lambda^V | K^+ \rangle = \frac{i\hbar}{m_K^3} \varepsilon_{\lambda\mu\nu\sigma} (p_K)^\mu (p_+ + p_-)^\nu (p_+ - p_-)^\sigma \quad (2.3)$$

and

$$\langle \pi^+ \pi^- | J_\lambda^A | K^+ \rangle = \frac{f}{m_K} (p_+ + p_-)_\lambda + \frac{g}{m_K} (p_+ - p_-)_\lambda \quad (2.4)$$

where we have omitted a term in Eq. (2.4) which gives a contribution proportional to m_e^2 in the matrix element.

The decay matrix element is thus described by three form factors f , g and h which are, in general, functions of the invariants $(p_+ + p_-)^2$, $(p_K + p_+)^2$ and $(p_K + p_-)^2$. If, as originally suggested by Van Hieu ²⁾, the dependence on $(p_K + p_+)^2$ and $(p_K + p_-)^2$ can be neglected, then the calculation of the decay amplitude is much simplified.

Let ϕ be the angle between the dipion and dilepton planes in the K^+ rest system and let θ be the angle, in the dipion c.m. system, of the π^+ with regard to the dipion line of flight in the K^+ rest system. Defining x by

$$x^2 = (p_+ + p_-)^2 / m_K^2 \quad (2.5)$$

then the decay angular distribution is given by ⁵⁾

$$\begin{aligned} d\Gamma(x^2, \cos \theta, \phi) = & \frac{G^2 \pi^2 m_K^5}{8 (2\pi)^8} dx^2 d\cos \theta d\phi \left\{ |f|^2 A(x^2) \right. \\ & + |g|^2 (B(x^2) + C(x^2) \cos^2 \theta + D(x^2) \sin^2 \theta \sin^2 \phi) \\ & + |h|^2 E(x^2) \sin^2 \theta (1 + 2 \cos^2 \phi) \\ & + \operatorname{Re}(fg^*) S(x^2) \cos \theta \\ & \left. + \operatorname{Im}(fg^*) T(x^2) \sin \theta \sin \phi + \right. \end{aligned} \quad (2.6)$$

$$\begin{aligned}
& + \operatorname{Re}(fh^*) U(x^2) \sin \theta \cos \phi \\
& + \operatorname{Re}(gh^*) V(x^2) \cos \theta \sin \theta \cos \phi \\
& + \operatorname{Im}(gh^*) Y(x^2) \sin^2 \theta \sin \phi \cos \phi \}
\end{aligned}$$

The functions $A, B, C, D, E, S, T, U, V, Y$ are given by Cabibbo and Maksymowicz ⁵⁾.

Further assumptions are now made about the form factors f, g and h . With the assumption that f, g and h depend only on x^2 , then the f term in Eq. (2.4) is symmetric under interchange of the π^+ and π^- , while the g term in Eq. (2.4) and the h term in Eq. (2.3) are antisymmetric under interchange of the π^+ and π^- . These results, together with the $\Delta I = \frac{1}{2}$ rule, imply that the f term corresponds to the amplitude for emission of the two pions in an $I=0$ state, while the g and h terms give the amplitude for emission of the two pions in an $I=1$ state.

If it is further assumed that the interaction is invariant with regard to time reversal, the Watson-Fermi ⁸⁾ theorem may be applied to fix the phases of f, g and h as

$$\begin{aligned}
f &= \tilde{f} e^{i\delta_0} \\
g &= \tilde{g} e^{i\delta_1} \\
h &= \tilde{h} e^{i\delta_1}
\end{aligned} \tag{2.7}$$

where δ_0 is the $I=0$ s wave $\pi-\pi$ phase shift, δ_1 is the $I=1$ p wave $\pi-\pi$ phase shift and $\tilde{f}$, $\tilde{g}$, $\tilde{h}$ are real.

If the dependence of f , g and h on the variables $(p_K+p_+)^2$ and $(p_K+p_-)^2$ is retained, then since this can be written as a dependence on $(p_K+p_+)^2 \pm (p_K+p_-)^2$, the odd part can give rise to an additional p wave term in f and additional s wave terms in g and h . This dependence can also give rise to higher angular momentum contributions to f , g and h .

3. ANALYSIS OF DATA I

Results of measurements on 208 K_{e4} decays were presented at the 1966 Berkeley Conference ⁹⁾. Since that date more events have been analysed and preliminary results are now available for 310 events ¹⁰⁾.

The data are presented in the form of the dipion mass spectrum, the $\cos\theta$ and ϕ angular distributions and the absolute decay rate, which has the value ¹¹⁾

$$\Gamma = (2.9 \pm 0.6) \times 10^3 \text{ sec}^{-1} \quad (3.1)$$

From Eq. (2.6) and the assumption that the form factors depend only on x^2 it follows that the dipion mass spectrum is given by

$$\begin{aligned} \frac{d\Gamma(x^2)}{dx^2} = & \frac{G^2 m_\pi^5}{16 \cdot (2\pi)^5} \left\{ |f|^2 A(x^2) \right. \\ & \left. + |g|^2 \left(B(x^2) + \frac{1}{3} C(x^2) + \frac{1}{3} D(x^2) \right) + |h|^2 \frac{4}{3} E(x^2) \right\} \end{aligned} \quad (3.2)$$

6.

the $\cos\theta$ angular distribution is given by

$$\begin{aligned} \frac{d\Gamma(\cos\theta)}{d\cos\theta} = & \frac{G^2 \pi^3 m_\kappa^5}{4 (2\pi)^8} \int_{4m_\pi^2/m_\kappa^2}^1 dx^2 \left\{ |f|^2 A(x^2) \right. \\ & + |g|^2 \left(B(x^2) + C(x^2) \cos^2\theta + \frac{1}{2} D(x^2) \sin^2\theta \right) \\ & + 2|h|^2 E(x^2) \sin^2\theta \\ & \left. + \operatorname{Re}(fg^*) S(x^2) \cos\theta \right\} \end{aligned} \quad (3.3)$$

the ϕ angular distribution is given by

$$\begin{aligned} \frac{d\Gamma(\phi)}{d\phi} = & \frac{G^2 \pi^2 m_\kappa^5}{4 (2\pi)^8} \int_{4m_\pi^2/m_\kappa^2}^1 dx^2 \left\{ |f|^2 A(x^2) \right. \\ & + |g|^2 \left(B(x^2) + \frac{1}{3} C(x^2) + \frac{2}{3} D(x^2) \sin^2\phi \right) \\ & + \frac{2}{3} |h|^2 E(x^2) (1 + 2 \cos^2\phi) \\ & + \frac{\pi}{4} \operatorname{Im}(fg^*) T(x^2) \sin\phi + \frac{\pi}{4} \operatorname{Re}(fh^*) U(x^2) \cos\phi \\ & \left. + \frac{2}{3} \operatorname{Im}(gh^*) Y(x^2) \sin\phi \cos\phi \right\} \end{aligned} \quad (3.4)$$

and the total rate by

$$\begin{aligned} \Gamma = & \frac{G^2 m_\kappa^5}{16 (2\pi)^5} \int_{4m_\pi^2/m_\kappa^2}^1 dx^2 \left\{ |f|^2 A(x^2) \right. \\ & + |g|^2 \left(B(x^2) + \frac{1}{3} C(x^2) + \frac{1}{3} D(x^2) \right) \\ & \left. + |h|^2 \frac{4}{3} E(x^2) \right\} \end{aligned} \quad (3.5)$$

Analysing the $\cos\theta$ and ϕ distributions in successive approximations gives the results shown in Tables 1 and 2. The $\cos\theta$ distribution is adequately fitted by

$$A + B \cos\theta ,$$

higher order terms in $\cos\theta$ being unnecessary. The existence of an important term in $\cos\theta$ shows clear evidence of an fg interference term [see Eq. (3.3)].

The ϕ distribution is adequately fitted by

$$A + B \sin \phi + C \cos \phi$$

terms in $\sin^2\phi$ and $\sin \phi \cos \phi$ being unnecessary. The existence of the $\sin \phi$ term is again evidence of fg interference and the existence of the $\cos \phi$ term, which is essential to the fit, is evidence of fh interference, i.e., of the existence of a non-negligible vector current contribution to K_{e4} decay.

It should be noted that, because of the limited statistics, the fact that a term in $\sin^2 \phi$ is not necessary to fit the data cannot be used to conclude that Eq. (2.7) is valid. At best one can say that there is no obvious inconsistency.

4. ANALYSIS OF DATA II

In analysing the data in more detail, we assume that f, g and h are given by Eq. (2.7). As we have seen above, this is consistent with the experimental evidence. The $I=0$ s wave $\pi - \pi$ phase shift and the form factors $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ are now represented by various models and a simultaneous least squares fit made to the ϕ and $\cos\theta$ distributions. The errors are taken to be given by the square root

of the number of events in each bin. Because of possible bias in the experimental analysis¹⁰⁾ we did not fit to the dipion mass spectrum and in the $\cos\theta$ distribution we excluded events with $\cos\theta > 0.6$. This fit gives values for the ratios $\tilde{g}/\tilde{f}$ and $\tilde{h}/\tilde{f}$ from which absolute values for $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ can be obtained from the total rate.

The s wave phase shift δ_0 is parametrized in two ways.

- (i) A Chew-Mandelstam¹²⁾ parametrization with a variable scattering length a_0

$$\cot \delta_0 = \frac{1}{\beta a_0} + \frac{2}{\pi} \ln \left[\frac{m_\pi}{2m_\pi} x(1+\beta) \right] \quad (4.1)$$

where

$$\beta = \left(1 - \frac{4m_\pi^2}{x^2 m_\pi^2} \right)^{1/2} \quad (4.2)$$

- (ii) A Breit-Wigner resonance with a variable width δ and energy squared S_R

$$\cot \delta_0(x^2) = 2(x^2 m_\pi^2 - S_R) / (\delta m_\pi x \beta) \quad (4.3)$$

It should be noted that only the lower half-width is significant for the analysis of K_{e4} decay, so that, although the Breit-Wigner resonance of Eq. (4.3) is a symmetrical one, it does not imply that we are requiring that a σ resonance, if it exists, must be symmetrical.

- (a) Constant form factors

The simplest model is that in which it is assumed that $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ are independent of energy. The p wave phase δ_1 is ignored, which is a reasonable approximation in the dipion energy region available in K_{e4} decay.

In Table 3(a) we give the results, for various values of the phase δ_0 , of fitting the ϕ distribution in 8 bins ($0^\circ \leq \phi \leq 360^\circ$) and the $\cos\theta$ distribution in 8 bins ($-1 \leq \cos\theta \leq 0.6$) for 208 events. We give values of χ^2 , the ratios $\tilde{g}/\tilde{f}$ and $\tilde{h}/\tilde{f}$ and the corresponding values for $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ obtained from fitting the rate. Table 3(b) gives similar results for 310 events where now the ϕ distribution is in 12 bins.

(b) Energy-dependent s wave form factor

Energy dependence is most likely to occur in f due to the possibility of strong low energy $I=0$ s wave $\pi-\pi$ interaction. This is taken into account by use of the relativistic Watson enhancement factor

$$\tilde{f} = f_0 \frac{S_0 \delta_0(x^2)}{a_0 \beta} \quad (4.3)$$

The results of repeating the analysis, still keeping $\tilde{g}$ and $\tilde{h}$ as constants, are given in Table 4.

(c) Energy dependence in all form factors

Finally a ρ enhancement factor for $\tilde{g}$ and $\tilde{h}$ can be introduced in the form

$$\begin{aligned} \tilde{g} &= g_0 \frac{m_\rho^2 - 4m_\pi^2}{m_\rho^2 - x^2 m_\pi^2} \\ \tilde{h} &= h_0 \frac{m_\rho^2 - 4m_\pi^2}{m_\rho^2 - x^2 m_\pi^2} \end{aligned} \quad (4.4)$$

The results of analysing the data using both Eqs. (4.3) and (4.4) are given in Table 5.

When the form factors are assumed to be independent of energy, both the Chew-Mandelstam and the Breit-Wigner parametrizations give dipion mass spectra which are in good qualitative agreement with the

experimental results. However, once an enhancement factor is introduced into $\tilde{f}$, although the resonance parametrization of Eq. (4.3) gives a fit to the $\cos\theta$ and ϕ distributions equally as good as that obtained with the Chew-Mandelstam parametrization of Eq. (4.1), comparison with the dipion mass spectrum, even on a qualitative basis, shows that it should be excluded. This does not mean, however, that a resonance solution is completely out of the question, only that a simple Breit-Wigner resonance is not allowed. A more complicated resonance structure, such as the three-parameter parametrization

$$\cot \delta_s(x^2) = 2(x^2 m_K^2 - s_R)(x^2 m_K^2 - 4m_\pi^2 + \gamma/a_0) / (\gamma m_K x_B(4 - s_R)) \quad (4.5)$$

which allows a specific scattering length to be included, can be used to fit the $\cos\theta$ and ϕ distributions and give an acceptable dipion mass spectrum. Since the data are not really sufficiently adequate to justify searching over all three parameters, the scattering length was held at 1.0, and different resonance positions and widths were investigated. As long as the resonance is above 500 MeV and sufficiently narrow on its low side, then a reasonable dipion mass spectrum can be obtained. Some typical results for the form factors are given in Table 6.

Typical fits to the $\cos\theta$ and ϕ distributions and some predicted dipion mass spectra are given in Figs. 1-6.

5. RESULTS

We have treated separately the 208 events reported at the Berkeley Conference⁹⁾ and the (preliminary) total of 310 events reported at London¹⁰⁾. The form factors $\tilde{g}$ and $\tilde{h}$ are remarkably independent of the nature of the s wave $\pi - \pi$ phase shift and of the inclusion or omission of the enhancement factors. The results obtained

for $\tilde{f}$ and $\tilde{g}$ do not change significantly in going from the 208 events to the 310 events, but the vector form factor $\tilde{h}$ increases considerably. The consistency of $\tilde{f}$ is not surprising, since it is determined largely by the rate, to which it contributes about 80% of the total. The uncertainty in $\tilde{h}$ as compared with that in $\tilde{g}$ is understandable in that the ratio $\tilde{h}/\tilde{f}$ is determined entirely by the coefficient of $\cos \phi$ in the ϕ distribution (the kinematical factors which occur with terms in $\tilde{h}/\tilde{f}$ elsewhere are extremely small) while the ratio $\tilde{g}/\tilde{f}$ occurs in the other terms of the ϕ distribution and in all the terms of the $\cos \theta$ distribution with significant kinematical factors.

(a) Constant form factors

Assuming constant form factors, a least squares fit varying a_0 , the s wave $\pi - \pi$ scattering length in the Chew-Mandelstam parametrization, and $\tilde{g}/\tilde{f}$ and $\tilde{h}/\tilde{f}$ yield the following values:

	<u>208 events</u>	<u>310 events</u>	
a_0	0.93 ± 0.60	1.04 ± 0.50	
$\tilde{g}/\tilde{f}$	1.08 ± 0.21	1.12 ± 0.19	
$\tilde{h}/\tilde{f}$	-2.59 ± 1.52	-4.06 ± 1.38	(5.1)
$\tilde{f}$	1.21 ± 0.13	1.19 ± 0.13	
$\tilde{g}$	1.31 ± 0.33	1.34 ± 0.30	
$\tilde{h}$	-3.14 ± 1.89	-4.84 ± 1.77	

As can be seen from Table 3, the values of $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ do not change appreciably if a Breit-Wigner type resonance is used instead of the Chew-Mandelstam $\pi - \pi$ phase.

It should be noted that an appreciable part of the error on the absolute values of the form factors arises from the error on the rate, since this is based on an analysis of only 72 events ¹¹⁾.

The values of $\tilde{f}$ and $\tilde{g}$ are in good agreement with the result

$$|\tilde{f}| = |\tilde{g}| = 1.2 \quad (5.2)$$

obtained by Weinberg ¹³⁾, who uses current algebra considerations together with the assumption of constant form factors to relate $K\ell_4$ decays to $K\ell_3$ decays. The scattering length, although somewhat larger than the value of 0.2 obtained by Weinberg ¹⁴⁾ from current algebra, cannot be said to be in disagreement with this result, firstly because of the large errors on the experimentally determined scattering length and secondly because of the possible effects of taking into account the unitary cuts in the $\pi - \pi$ scattering amplitude ¹⁵⁾⁻¹⁷⁾ which can increase appreciably the value of the scattering length obtained from current algebra requirements.

The most interesting result is the large vector current term. With 208 events the ratio $\tilde{h}/\tilde{f}$ is non-zero by 1.7 standard deviations while for 310 events it is non-zero by 2.9 standard deviations. This term is much larger than had hitherto been supposed and we discuss some of the consequences of this in the next section.

(b) Form factors with enhancement factors

Once we allow the presence of an enhancement factor in $\tilde{f}$, then, as we have seen in Section 4, the simple Breit-Wigner resonance gives a dipion mass spectrum which is incompatible with the experimental values. The results using the more complicated resonance parametrization of Eq. (4.5) are in qualitative agreement with the experimental values. It should be noted that, even with this parametrization, a resonance cannot exist much below 500 MeV and the particular type of resonance postulated by Brown and Singer ¹⁸⁾ and Brown and Faier ¹⁹⁾ is definitely excluded within the limits of the present assumptions on K_{e4} decay.

In the case where only $\tilde{f}$ has an enhancement factor, a simultaneous least squares fit varying a_0 , the s wave $\pi - \pi$ scattering length in the Chew-Mandelstam parametrization, and $\tilde{g}/f_0$ and $\tilde{h}/f_0$ yield the following values:

	<u>208 events</u>	<u>310 events</u>	
a_0	0.80 ± 0.54	0.89 ± 0.44	
$\tilde{g}/f_0$	0.92 ± 0.17	0.94 ± 0.15	
$\tilde{h}/f_0$	-2.21 ± 1.28	-3.38 ± 1.11	(5.2)
f_0	1.44 ± 0.16	1.45 ± 0.16	
$\tilde{g}$	1.32 ± 0.32	1.36 ± 0.30	
$\tilde{h}$	-3.18 ± 1.89	-4.89 ± 1.73	

A similar minimization in the case where all the form factors have enhancement factors yields the values

	<u>208 events</u>	<u>310 events</u>	
a_0	0.76 ± 0.52	0.84 ± 0.43	
g_0/f_0	0.87 ± 0.16	0.88 ± 0.14	
h_0/f_0	-2.11 ± 1.22	-3.22 ± 1.06	(5.3)
f_0	1.42 ± 0.15	1.42 ± 0.15	
g_0	1.23 ± 0.30	1.25 ± 0.28	
h_0	-2.99 ± 1.78	-4.58 ± 1.63	

As can be seen from Table 6, the values of f_0 , g_0 and h_0 do not change appreciably if the extended resonance parametrization of Eq. (4.5) is used instead of the Chew-Mandelstam type $\pi - \pi$ phase.

In all cases the vector form factor remains large. With 208 events the ratio $\tilde{h}/\tilde{f}$ is non-zero by 1.7 standard deviations while for 310 events it is non-zero by 3.0 standard deviations.

6. CONSEQUENCES

The values of h which have been obtained in this analysis are larger than has hitherto been supposed, estimates ^{5),7)} using the Gell-Mann - Sharp-Wagner model ²⁰⁾ giving a value of h smaller than the present results by a factor of four. In the following we shall discuss the implications of the values which have been obtained for h .

(a) The η width, without $\eta - \eta'$ mixing

It is possible to relate h to the coupling constant for the transition $\eta \rightarrow \pi^+ \pi^- \gamma$ from which, via the known branching ratio, the total η width may be obtained.

In the analysis it was assumed that, in the vector transition, the $\pi^+ \pi^-$ pair is in an $I=1$ state. In the $\eta \rightarrow \pi^+ \pi^- \gamma$ decay this is necessarily the case, due to the well established C invariance of this process ²¹⁾. In $SU(3)$ symmetry the general form for a transition $M_1 \rightarrow M_2 M_3 \gamma$, where the final meson state must be antisymmetric and which must be C invariant, is given by

$$a \text{Tr}([M_2, M_3] \{M_1, Q\}) + b \text{Tr}(M_2 M_1 M_3 Q - M_3 M_1 M_2 Q) \quad (6.1)$$

The first term in Eq. (6.1) contains the octet part of the $\pi^+ \pi^-$ state and the second term contains the decuplet part. For the transition $K \rightarrow \pi^+ \pi^- W$ (where we use W symbolically to denote the lepton pair) the Clebsch-Gordon coefficient is zero in the decuplet part of Eq. (6.1) (for the weak current) but for the transition $\eta \rightarrow \pi^+ \pi^- \gamma$ the Clebsch-Gordon coefficient in the decuplet part of Eq. (6.1) is non-zero. Since the octet part of the 2π interaction leads to a ρ meson, it is not unreasonable to consider only the first term of Eq. (6.1) for the decay $\eta \rightarrow \pi^+ \pi^- \gamma$, and first of all we shall examine the consequences of this assumption.

Defining the matrix element $\langle \pi^+ \pi^- | J_\mu^{e.m.} | \eta \rangle$ by

$$M(\eta \rightarrow \pi^+ \pi^- \gamma) = e \langle \pi^+ \pi^- | J_\mu^{e.m.} | \eta \rangle \varepsilon_\mu \quad (6.2)$$

where ε_μ is the photon polarization vector, then, in the framework of the Cabibbo theory²²⁾, it can be related to the matrix element $\langle \pi^+ \pi^- | J_\mu^V | K^+ \rangle$. Since in both cases we are considering only the octet part of the $I=1$ 2π state, the same $SU(3)$ connection may be obtained by using

$$T_r(V\{M, Q\}) \quad (6.3)$$

as the electromagnetic current in place of the first term of Eq. (6.1). If it is assumed further that the decay $\phi \rightarrow \pi \gamma$ is zero and that the $\omega - \phi$ mixing angle is given by $\sin \Theta_{\omega\phi} = \frac{1}{\sqrt{3}}$, then V may be considered as the vector meson nonet, and we shall adopt this standpoint in the computations given here.

In the Cabibbo theory, the weak current is obtained from Eq. (6.3) by the replacement

$$Q \rightarrow \cos \Theta I_- + \sin \Theta K_- \quad (6.4)$$

where I_- lowers the isospin by 1 and K_- lowers the hypercharge by 1 and the isospin by $1/2$.

From this prescription one readily obtains that

$$h_\eta = \sqrt{\frac{2}{3}} \frac{h_{K^+}}{\sin \Theta} \quad (6.5)$$

where h_η has the same meaning for the η decay as h_{K^+} has for the K^+ decay, and h_{K^+} is, of course, exactly the same as h of Eq. (2.3). For the Cabibbo angle we take the value²³⁾

$$\sin \Theta = 0.222 \quad (6.6)$$

In this derivation, η has been considered as a pure octet state.

In terms of h_η , the partial width is given by

$$\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) = e^2 \frac{h_\eta^2}{3 m_K^6} \cdot \frac{(m_\eta^2 - 4 m_\pi^2)^5}{2^9 \pi^3 m_\eta^3} \cdot I \quad (6.7)$$

where I is given by

$$I = \int_0^1 dz \frac{z^3 (1-z)^{3/2}}{\left\{ \frac{m_\eta^2}{m_\eta^2 - 4 m_\pi^2} - z \right\}^{1/2}} \quad (6.8)$$

if no ρ^0 enhancement is considered, and by

$$I = (m_\rho^2 - 4 m_\pi^2)^2 \int_0^1 dz \cdot \frac{1}{\left\{ [m_\rho^2 - m_\eta^2 + (m_\eta^2 - 4 m_\pi^2)z]^2 + \Gamma^2 [m_\eta^2 - (m_\eta^2 - 4 m_\pi^2)z]^2 \right\}^{1/2}} \times \frac{z^3 (1-z)^{3/2}}{\left\{ \frac{m_\eta^2}{m_\eta^2 - 4 m_\pi^2} - z \right\}^{1/2}} \quad (6.9)$$

if ρ^0 enhancement is included. The ρ^0 width was taken to be 140 MeV.

Table 7 gives the η width as a function of h_{K^+} both without ρ^0 enhancement [Eqs. (6.7) and (6.8)] and with ρ^0 enhancement [Eqs. (6.7) and (6.9)]. If the results of the analysis of Section 5 are taken literally, then the values of h_{K^+} obtained lead to an width well outside the experimental range of between 2 and 4 KeV²⁴⁾.

There are, of course, several loop-holes. Firstly the errors on h_{K^+} are so large that even in the most extreme case the discrepancy is not much more than two standard deviations on h_{K^+} . Secondly the assumption of ρ dominance in the decay $\eta \rightarrow \pi^+ \pi^- \gamma$ may be at fault and there may be a considerable contribution from the decuplet part of Eq. (6.1) which interferes destructively with the octet part to produce a reasonable η width, even when the results of the analysis are taken literally. Thirdly it is possible to get agreement by arguing that the weak current can be renormalized since the non-renormalization theorem of Ademollo-Gatto²⁵⁾ does not apply, although a renormalization factor of more than 2, which is what is required if the results are taken literally, is not a very attractive possibility, since in other cases where renormalization is allowed, for instance in K_{e2} decays and in the axial vector current of the hyperon decays, the change required is only of the order of 20%. Fourthly any combination of the above three possibilities is allowable.

(b) The η width, with $\eta - \eta'$ mixing

Another possible loop-hole is to allow $\eta - \eta'$ mixing²⁶⁾ and use the η' width as an adjustable parameter to obtain agreement between the η width and the value for h_{K^+} . The Cabibbo angle is again taken to be given by Eq. (6.6).

The $\eta - \eta'$ mixing takes the form

$$\begin{aligned}\eta &= -\eta_1 \sin \alpha + \eta_8 \cos \alpha \\ \eta' &= \eta_1 \cos \alpha + \eta_8 \sin \alpha\end{aligned}\tag{6.10}$$

and the electromagnetic current becomes

$$c \eta_1 \text{Tr}(VQ) + d \text{Tr}(V\{M, Q\})\tag{6.11}$$

which gives the following relations for the coupling constants:

$$h_{\eta} = -\frac{c}{\sqrt{2}} \sin \alpha + \frac{d}{\sqrt{3}} \cos \alpha \quad (6.12)$$

$$h_{\eta'} = \frac{c}{\sqrt{2}} \cos \alpha + \frac{d}{\sqrt{3}} \sin \alpha \quad (6.13)$$

The mixing angle α is known from the mass formula to be given by

$$\tan \alpha = 0.19 \quad (6.14)$$

and taking h_{η} from the η width and using

$$\frac{d}{\sqrt{3}} \cos \alpha = \sqrt{\frac{2}{3}} \frac{h_{K^+}}{\sin \theta} \quad (6.15)$$

then $h_{\eta'}$ can be obtained as a function of h_{K^+} and hence, using Eqs. (6.7) and (6.9), the width for the $\eta' \rightarrow \pi^+ \pi^- \gamma$ transition which is mediated through the ρ^0 .

In this case the large $\eta - \eta'$ mass difference poses a problem. In the transitions $K \rightarrow \pi^+ \pi^- W$ and $\eta \rightarrow \pi^+ \pi^- \gamma$ the masses of the decaying particles are compatible, so in both cases one could use the same dimensional coupling constant h_K/m_K^3 or h_{η}/m_{η}^3 .

If this normalization ($h_{\eta'}/m_K^3$) is again used, then we obtain the values of $\Gamma(\eta' \rightarrow \pi^+ \pi^- \gamma)$ shown in Table 8 for η widths of 3 and 4 KeV. In each case two results are given, since the sign of h_{K^+} , and consequently of d in Eq. (6.15), is unknown. If the mass difference is taken into account by replacing the coupling constant by $m_{\eta} h_{\eta'}/m_{\eta} m_K^3$, then the results are reduced by a factor of 4.

If the results of the analysis of Section 5 are again taken literally, then, even in the latter of the above two cases when the results are reduced by a factor of 4, they are still well above the experimental upper limit of 1 MeV²⁷⁾, i.e., η' mixing by itself is not sufficient to resolve the discrepancy (if it exists!). Of course the same loopholes are available as in the previous case of no η' mixing.

(c) The ρ dominance model

A different approach is to make use of some model to evaluate the vector form factor h_{K^+} and to compare this with the results obtained in the analysis. We have already mentioned the Gell-Mann - Sharp - Wagner model and we now consider the ρ dominance model. One may have some belief in this model, since it leads to a reasonable η width, without η' mixing, of about 2.2 KeV²⁸⁾. Also, as was pointed out above, since the Glebsch-Gordon coefficient is zero for the $K \rightarrow \pi^+ \pi^- W$ transition in the decuplet part of Eq. (6.1) for the weak current, there is no approximation made in using only the octet part to calculate h_{K^+} while there is one in the $\eta \rightarrow \pi^+ \pi^- \gamma$ transition, since there the Glebsch-Gordon coefficient in the decuplet part of Eq. (6.1) is non-zero.

Given $\Gamma(\omega \rightarrow \pi \gamma)$, the weak $K^+ \rho$ coupling constant can be calculated directly and hence, from the known ρ width, the complete vector decay. From Eqs. (6.3) and 6.4) one obtains the relations between the coupling constant $f_{\omega \pi \gamma}$ (corresponding to

i.e. $f_{\omega \pi \gamma} / m_K \cdot \epsilon_{\mu \nu \sigma \tau} p_\omega^\mu \epsilon_\omega^\nu p_\gamma^\sigma \epsilon_\gamma^\tau$) and a similar one, $f_{K \rho W}$, as

$$f_{K \rho W} = \sqrt{\frac{3}{2}} \sin \theta \frac{1}{\sqrt{3}} f_{\omega \pi \gamma} \quad (6.16)$$

which leads to a value for h_{K^+} of

$$h_{K^+} = 0.62 \quad (6.17)$$

using a width $\Gamma (\omega \rightarrow \pi \gamma)$ of 1.1 MeV and a width $\Gamma (\rho^0 \rightarrow \pi \pi)$ of 140 MeV. This is again much less than the results obtained in Section 5, although as usual there is the ever-open loop-hole of the large errors in the analysis.

7. CONCLUSIONS

With respect to the s wave $\pi - \pi$ scattering length and the axial vector form factors f_0 and g_0 , our results do not differ significantly from those reported previously 9), 10). The new result is that concerning the vector form factor h_0 . Unfortunately, with the statistics available at present, the errors on this form factor are so large that it is not possible to draw any definitive conclusion. However, if the results are taken literally, then there are three possible consequences.

Firstly there is the possibility of octet-decuplet interference in processes of the type octet meson decaying to two octet mesons and a current. At the moment, this possibility cannot be tested elsewhere, the only three decays of this type available being those considered here, namely $K^+ \rightarrow \pi^+ \pi^- W$, $\eta \rightarrow \pi^+ \pi^- \gamma$ and $\eta' \rightarrow \pi^+ \pi^- \gamma$.

Secondly there is the possibility that, in the Cabibbo theory, much more drastic renormalization is required than has hitherto been thought possible or desirable. In view of the success of the Cabibbo theory elsewhere one would hesitate to modify it to the extent required here.

Thirdly there is the possibility of disbelieving some of the assumptions about the form factors, which would have the unhappy consequence that the elegant scheme of correlation measurements proposed by Cabibbo and Maksymowicz is invalid. As was mentioned in Section 2 this is open to direct experimental test, since in the Cabibbo and

Maksymowicz theory the coefficient of the $\sin 2\phi$ term in the ϕ distribution is identically zero.

However, as we have stressed frequently, this depends very much on taking the data literally, and with the present statistics the discrepancy is not much more than two standard deviations in the most extreme case. The considerable change in the vector form factor in going from 208 events to 310 events may give some hope that the present result is indeed the consequence of a statistical fluctuation, although it should be noted that with the increased number of events the result unfortunately is going in the wrong direction.

The data at present are at the tantalizing stage of being sufficient to indicate that something interesting and unforeseen may be occurring, but insufficient to confirm it or to make it precise. More data on this decay would be welcome, either to refute or to confirm our present suspicions. If they are confirmed, then K_{e4} decay is a much more interesting and complicated decay than has been supposed.

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T A B L E 1

N_F	χ^2	A	B	C	
6	4.8	1.00 ± 0.07	0.78 ± 0.11	-	208 events
5	4.8	1.00 ± 0.09	0.76 ± 0.23	-0.29 ± 0.31	
6	3.6	1.00 ± 0.05	0.90 ± 0.08	-	310 events
5	2.4	1.00 ± 0.05	0.69 ± 0.13	-0.28 ± 0.17	

Fit to the $\cos\theta$ distribution with successive terms of the series

$$\frac{d\Gamma}{d\cos\theta} = A + B \cos\theta + C \cos^2\theta$$

All fits have been normalized to make the leading term of unit magnitude.

T A B L E 2

N_F	χ^2	A	B	C	D	E	
6	8.3	1.00 ± 0.08	0.13 ± 0.12	-	-	-	208 events
5	2.7	1.00 ± 0.05	0.12 ± 0.07	0.22 ± 0.07	-	-	
4	1.4	1.00 ± 0.08	0.14 ± 0.07	0.23 ± 0.06	0.24 ± 0.13	-	
4	2.4	1.00 ± 0.05	0.11 ± 0.07	0.22 ± 0.07	-	-0.10 ± 0.15	
3	1.1	1.00 ± 0.07	0.12 ± 0.06	0.21 ± 0.06	0.22 ± 0.12	-0.10 ± 0.12	
10	17.1	1.00 ± 0.08	0.15 ± 0.11	-	-	-	310 events
9	2.7	1.00 ± 0.03	0.14 ± 0.04	0.29 ± 0.04	-	-	
8	1.8	1.00 ± 0.05	0.16 ± 0.04	0.31 ± 0.04	0.16 ± 0.08	-	
8	2.7	1.00 ± 0.03	0.14 ± 0.05	0.29 ± 0.05	-	-0.01 ± 0.09	
7	1.7	1.00 ± 0.05	0.16 ± 0.04	0.30 ± 0.04	0.16 ± 0.09	-0.01 ± 0.09	

Fit to the ϕ distribution with successive terms of the series

$$\frac{d\Gamma}{d\phi} = A + B \sin \phi + C \cos \phi + D \sin^2 \phi + E \sin \phi \cos \phi$$

All fits are normalized to make the leading term of unit magnitude.

T A B L E 3(a)

I-0 s wave $\pi - \pi$ phase		χ^2	$\tilde{g}/\tilde{f}$	$\tilde{h}/\tilde{f}$	$\tilde{f}$	$\tilde{g}$	$\tilde{h}$
Chew-Mandelstam type phase	$a_0=0.3$	13.58	0.98	-2.42	1.23	1.21	-2.98
	$a_0=0.5$	12.84	1.00	-2.45	1.23	1.23	-3.01
	$a_0=1.0$	12.30	1.09	-2.62	1.21	1.32	-3.17
	$a_0=1.5$	13.01	1.17	-2.87	1.19	1.39	-3.42
	$a_0=2.0$	14.51	1.22	-3.16	1.18	1.44	-3.72
Breit-Wigner resonance $E_R = 500$ MeV	$\Gamma = 100$ MeV	13.59	0.98	-2.43	1.23	1.21	-2.99
	$= 200$ MeV	12.78	1.02	-2.48	1.22	1.25	-3.03
	$= 300$ MeV	12.47	1.06	-2.54	1.22	1.29	-3.09
Breit-Wigner resonance $E_R = 600$ MeV	$\Gamma = 100$ MeV	14.39	0.96	-2.42	1.24	1.19	-2.99
	$= 200$ MeV	13.72	0.97	-2.42	1.23	1.20	-2.99
	$= 300$ MeV	13.27	0.99	-2.43	1.23	1.22	-2.99
Breit-Wigner resonance $E_R = 700$ MeV	$\Gamma = 100$ MeV	14.75	0.95	-2.42	1.24	1.18	-2.99
	$= 200$ MeV	14.25	0.96	-2.42	1.24	1.19	-2.99
	$= 300$ MeV	13.88	0.97	-2.42	1.23	1.20	-2.99
Breit-Wigner resonance $E_R = 800$ MeV	$\Gamma = 100$ MeV	14.95	0.95	-2.42	1.24	1.18	-2.99
	$= 200$ MeV	14.56	0.96	-2.42	1.24	1.19	-2.99
	$= 300$ MeV	14.26	0.96	-2.42	1.24	1.19	-2.99
Typical percentage errors			20%	59%	11%	26%	61%

208 events fitted with constant form factors.

T A B L E 3(b)

I-0 s wave $\pi - \pi$ phase		χ^2	$\tilde{g}/\tilde{f}$	$\tilde{h}/\tilde{f}$	$\tilde{f}$	$\tilde{g}$	$\tilde{h}$
Chew-Mandelstam type phase	$a_0=0.3$	16.77	0.99	-3.61	1.22	1.21	-4.41
	$a_0=0.5$	15.43	1.02	-3.69	1.22	1.24	-4.49
	$a_0=1.0$	14.14	1.11	-4.02	1.20	1.33	-4.81
	$a_0=1.5$	14.84	1.20	-4.48	1.17	1.41	-5.26
	$a_0=2.0$	16.78	1.27	-5.02	1.15	1.47	-5.79
Breit-Wigner resonance $E_R = 500$ MeV	$\Gamma = 100$ MeV	16.78	0.99	-3.62	1.22	1.21	-4.42
	$= 200$ MeV	15.30	1.04	-3.73	1.21	1.26	-4.52
	$= 300$ MeV	14.64	1.08	-3.86	1.20	1.30	-4.64
Breit-Wigner resonance $E_R = 600$ MeV	$\Gamma = 100$ MeV	18.18	0.97	-3.58	1.23	1.19	-4.39
	$= 200$ MeV	17.01	0.99	-3.61	1.22	1.21	-4.41
	$= 300$ MeV	16.22	1.00	-3.64	1.22	1.22	-4.44
Breit-Wigner resonance $E_R = 700$ MeV	$\Gamma = 100$ MeV	18.80	0.96	-3.58	1.23	1.18	-4.40
	$= 200$ MeV	17.94	0.97	-3.59	1.23	1.19	-4.40
	$= 300$ MeV	17.29	0.98	-3.60	1.22	1.20	-4.41
Breit-Wigner resonance $E_R = 800$ MeV	$\Gamma = 100$ MeV	19.13	0.96	-3.58	1.23	1.18	-4.40
	$= 200$ MeV	18.48	0.97	-3.58	1.23	1.19	-4.39
	$= 300$ MeV	17.95	0.97	-3.58	1.23	1.19	-4.39
Typical percentage errors			17%	34%	11%	23%	37%

310 events fitted with constant form factors.

T A B L E 4(a)

I=0 s wave $\pi - \pi$ phase		χ^2	$\tilde{g}/f_0$	$\tilde{h}/f_0$	f_0	$\tilde{g}$	$\tilde{h}$
Chew-Mandelstam type phase	$a_0=0.3$	13.66	0.93	-2.30	1.31	1.22	-3.01
	$a_0=0.5$	13.02	0.92	-2.25	1.36	1.25	-3.06
	$a_0=1.0$	12.81	0.92	-2.21	1.50	1.38	-3.31
	$a_0=1.5$	14.08	0.89	-2.24	1.65	1.47	-3.69
	$a_0=2.0$	16.36	0.84	-2.29	1.81	1.52	-4.15
Breit-Wigner resonance $E_R = 500$ MeV	$\Gamma = 100$ MeV	13.08	1.42	-3.64	0.82	1.16	-2.98
	$= 200$ MeV	12.28	1.43	-3.56	0.84	1.20	-3.00
	$= 300$ MeV	12.03	1.44	-3.53	0.86	1.24	-3.05
Breit-Wigner resonance $E_R = 600$ MeV	$\Gamma = 100$ MeV	14.05	1.22	-3.16	0.92	1.13	-2.92
	$= 200$ MeV	13.34	1.24	-3.14	0.93	1.15	-2.92
	$= 300$ MeV	12.88	1.25	-3.13	0.93	1.17	-2.93
Breit-Wigner resonance $E_R = 700$ MeV	$\Gamma = 100$ MeV	14.48	1.16	-3.01	0.97	1.13	-2.92
	$= 200$ MeV	13.95	1.17	-3.00	0.97	1.14	-2.92
	$= 300$ MeV	13.56	1.18	-2.99	0.97	1.15	-2.91
Breit-Wigner resonance $E_R = 800$ MeV	$\Gamma = 100$ MeV	14.71	1.13	-2.94	1.00	1.13	-2.93
	$= 200$ MeV	14.30	1.14	-2.93	1.00	1.14	-2.92
	$= 300$ MeV	13.99	1.14	-2.92	1.00	1.14	-2.91
Typical percentage errors			19%	58%	12%	25%	60%

208 events fitted with enhanced s wave form factor and constant p wave and vector form factors.

T A B L E 4(b)

I=0 s wave $\pi - \pi$ phase		χ^2	$\tilde{g}/f_0$	$\tilde{h}/f_0$	f_0	$\tilde{g}$	$\tilde{h}$
Chew-Mandelstam type phase	$a_0 = 0.3$	16.90	0.94	-3.42	1.41	1.33	-4.84
	$a_0 = 0.5$	15.71	0.94	-3.38	1.48	1.39	-5.00
	$a_0 = 1.0$	14.93	0.94	-3.40	1.48	1.39	-5.02
	$a_0 = 1.5$	16.49	0.92	-3.51	1.62	1.49	-5.68
	$a_0 = 2.0$	19.61	0.87	-3.64	1.77	1.54	-6.44
Breit-Wigner resonance $E_R = 500$ MeV	$\Gamma = 100$ MeV	15.92	1.44	-5.43	0.81	1.17	-4.40
	$= 200$ MeV	14.42	1.46	-5.38	0.83	1.22	-4.48
	$= 300$ MeV	13.85	1.47	-5.38	0.85	1.26	-4.60
Breit-Wigner resonance $E_R = 600$ MeV	$\Gamma = 100$ MeV	17.64	1.24	-4.68	0.92	1.14	-4.29
	$= 200$ MeV	16.39	1.25	-4.68	0.92	1.15	-4.31
	$= 300$ MeV	15.57	1.26	-4.68	0.93	1.17	-4.34
Breit-Wigner resonance $E_R = 700$ MeV	$\Gamma = 100$ MeV	18.37	1.18	-4.45	0.96	1.14	-4.28
	$= 200$ MeV	17.46	1.18	-4.44	0.97	1.14	-4.28
	$= 300$ MeV	16.78	1.19	-4.44	0.97	1.15	-4.29
Breit-Wigner resonance $E_R = 800$ MeV	$\Gamma = 100$ MeV	18.75	1.15	-4.33	0.99	1.14	-4.28
	$= 200$ MeV	18.06	1.15	-4.33	0.99	1.14	-4.28
	$= 300$ MeV	17.52	1.16	-4.33	0.99	1.15	-4.28
Typical percentage errors			16%	33%	11%	22%	36%

310 events fitted with an enhanced s wave form factor and constant p wave and vector form factors.

T A B L E 5(a)

I = 0 s wave $\pi - \pi$ phase		χ^2	g_0/f_0	h_0/f_0	f_0	g_0	h_0
Chew-Mandelstam type phase	$a_0 = 0.3$	13.76	0.87	-2.17	1.30	1.13	-2.83
	$a_0 = 0.5$	13.14	0.87	-2.13	1.35	1.18	-2.88
	$a_0 = 1.0$	13.06	0.86	-2.11	1.49	1.28	-3.14
	$a_0 = 1.5$	14.55	0.83	-2.14	1.64	1.36	-3.51
	$a_0 = 2.0$	17.07	0.78	-2.20	1.80	1.41	-3.97
Breit-Wigner resonance $E_R = 500$ MeV	$T = 100$ MeV	13.01	1.30	-3.35	0.82	1.06	-2.74
	$= 200$ MeV	12.23	1.32	-3.30	0.84	1.11	-2.77
	$= 300$ MeV	12.03	1.33	-3.28	0.86	1.15	-2.83
Breit-Wigner resonance $E_R = 600$ MeV	$T = 100$ MeV	14.04	1.13	-2.93	0.92	1.04	-2.70
	$= 200$ MeV	13.32	1.14	-2.91	0.93	1.06	-2.70
	$= 300$ MeV	12.86	1.15	-2.90	0.93	1.07	-2.71
Breit-Wigner resonance $E_R = 700$ MeV	$T = 100$ MeV	14.49	1.07	-2.80	0.97	1.04	-2.71
	$= 200$ MeV	13.95	1.08	-2.78	0.97	1.05	-2.70
	$= 300$ MeV	13.55	1.09	-2.78	0.97	1.06	-2.70
Breit-Wigner resonance $E_R = 800$ MeV	$T = 100$ MeV	14.73	1.05	-2.73	0.99	1.04	-2.71
	$= 200$ MeV	14.32	1.05	-2.72	0.99	1.04	-2.71
	$= 300$ MeV	13.99	1.06	-2.72	1.00	1.05	-2.71
Typical percentage errors			19%	58%	11%	25%	60%

208 events fitted with enhanced form factors.

T A B L E 5(b)

I=0 s wave $\pi - \pi$ phase		χ^2	g_o/f_o	h_o/f_o	f_o	g_o	h_o
Chew-Mandelstam type phase	$a_o=0.3$	17.04	0.88	-3.23	1.29	1.14	-4.17
	$a_o=0.5$	15.89	0.88	-3.21	1.34	1.18	-4.30
	$a_o=1.0$	15.31	0.88	-3.25	1.47	1.29	-4.77
	$a_o=1.5$	17.20	0.86	-3.36	1.61	1.38	-5.40
	$a_o=2.0$	20.66	0.80	-3.50	1.76	1.41	-6.17
Breit-Wigner resonance $E_R = 500$ MeV	$\Gamma = 100$ MeV	15.81	1.32	-5.01	0.81	1.07	-4.06
	$= 200$ MeV	14.33	1.34	-4.98	0.83	1.12	-4.15
	$= 300$ MeV	13.82	1.36	-5.00	0.85	1.16	-4.26
Breit-Wigner resonance $E_R = 600$ MeV	$\Gamma = 100$ MeV	17.62	1.14	-4.34	0.92	1.04	-3.98
	$= 200$ MeV	16.35	1.15	-4.34	0.92	1.06	-4.00
	$= 300$ MeV	15.53	1.17	-4.35	0.93	1.08	-4.03
Breit-Wigner resonance $E_R = 700$ MeV	$\Gamma = 100$ MeV	18.38	1.09	-4.13	0.96	1.05	-3.97
	$= 200$ MeV	17.45	1.09	-4.13	0.96	1.05	-3.98
	$= 300$ MeV	16.77	1.10	-4.13	0.97	1.06	-3.99
Breit-Wigner resonance $E_R = 800$ MeV	$\Gamma = 100$ MeV	18.78	1.06	-4.03	0.99	1.05	-3.98
	$= 200$ MeV	18.08	1.06	-4.03	0.99	1.05	-3.98
	$= 300$ MeV	17.53	1.07	-4.03	0.99	1.06	-3.98
Typical percentage errors			16%	33%	11%	23%	36%

310 events fitted with enhanced form factors.

T A B L E 6(a) 208 events

Resonance width	χ^2	g_o/f_o	h_o/f_o	f_o	g_o	h_o
100 MeV	13.07	0.76	-1.87	1.66	1.26	-3.11
150 MeV	12.88	0.84	-2.07	1.50	1.26	-3.11
200 MeV	12.90	0.90	-2.20	1.42	1.28	-3.12
Typical percentage errors		19%	58%	11%	25%	60%

(b) 310 events

Resonance width	χ^2	g_o/f_o	h_o/f_o	f_o	g_o	h_o
100 MeV	15.43	0.78	-2.87	1.64	1.28	-4.71
150 MeV	14.99	0.87	-3.19	1.48	1.29	-4.72
200 MeV	14.91	0.92	-3.40	1.40	1.29	-4.76
Typical percentage errors		16%	33%	11%	23%	36%

Fits with all form factors enhanced using an $I=0$ s wave $\pi - \tau$ phase having a scattering length of 1.0 and a resonance at 500 MeV.

T A B L E 7

h_{K^+}	η width (KeV)	
	Without ρ^0 enhancement	With ρ^0 enhancement
0.5	1.08	1.52
1.0	4.32	6.07
1.5	9.72	13.65
2.0	17.27	24.27
2.5	26.99	37.92
3.0	38.86	54.60
3.5	52.90	74.32
4.0	69.09	97.07
4.5	87.44	123.00
5.0	108.00	152.00

The η width in KeV as a function of the vector form factor h_{K^+} without ρ^0 enhancement [see Eq. (6.8)] and with ρ^0 enhancement [see Eq. (6.9)], using a Cabibbo angle of $\sin\theta = 0.222$.

T A B L E 8

h_{K^+}	Width of decay $\eta' \rightarrow \pi^+ \pi^- \gamma$ (MeV)			
	η width = 3 KeV		η width = 4 KeV	
0.5	0.24	10.45	0.61	12.40
1.0	0.78	21.21	0.35	23.94
1.5	5.08	35.72	3.86	39.24
2.0	13.14	53.99	11.13	58.31
2.5	24.96	76.03	22.17	81.14
3.0	40.55	101.8	36.96	108.0
3.5	59.90	131.0	55.52	138.0
4.0	83.01	165.0	77.84	172.0
4.5	110.0	202.0	104.0	210.0
5.0	141.0	243.0	134.0	252.0

The width of the decay $\eta' \rightarrow \pi^+ \pi^- \gamma$ in MeV as a function of the vector form factor h_{K^+} for η widths of 3 and 4 KeV. Two solutions exist in each case because of the ambiguity in the overall sign of h_{K^+} .

To take into account the $\eta - \eta'$ mass difference, the above results should be divided by a factor of 4 (see Section 6).

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FIGURE CAPTIONS

- Figure 1 : Experimental and fitted $\cos\theta$ distributions for 208 events, the parameters having the values of Eq. (5.3). The curves corresponding to constant form factors and to the case where only $\tilde{f}$ is enhanced are very similar. The shaded bins were not included in our fit.
- Figure 2 : Experimental and fitted ϕ distributions for 208 events, the parameters having the values of Eq. (5.3). The curves corresponding to constant form factors and to the case where only $\tilde{f}$ is enhanced are very similar.
- Figure 3 : Experimental and predicted dipion mass spectra for 208 events. The curves correspond to:
1. constant form factors, the parameters having the values of Eq. (5.1);
 2. $\tilde{f}$ enhanced, the parameters having the values of Eq. (5.2);
 3. all form factors enhanced, the parameters having the values of Eq. (5.3);
 4. all form factors enhanced, the $I=0$ s wave $\pi - \pi$ phase being given by a Breit-Wigner resonance at 500 MeV with a width of 100 MeV;
 5. all form factors enhanced, the $I=0$ s wave $\pi - \pi$ phase having a scattering length of 1.0 and a resonance at 500 MeV with a width of 100 MeV.

Figure 4 : Experimental and fitted $\cos\theta$ distributions for 310 events, the parameters having the values of Eq. (5.3). The curves corresponding to constant form factors and to the case where only $\tilde{f}$ is enhanced are very similar. The shaded bins were not included in our fit.

Figure 5 : Experimental and fitted ϕ distributions for 310 events, the parameters having the values of Eq. (5.3). The curves corresponding to constant form factors and to the case where only $\tilde{f}$ is enhanced are very similar.

Figure 6 : Experimental and predicted dipion mass spectra for 310 events. The curves correspond to:

1. constant form factors, the parameters having the values of Eq. (5.1);
2. $\tilde{f}$ enhanced, the parameters having the values of Eq. (5.2);
3. all form factors enhanced, the parameters having the values of Eq. (5.3);
4. all form factors enhanced, the $I=0$ s wave $\pi-\pi$ phase being given by a Breit-Wigner resonance at 500 MeV with a width of 100 MeV.
5. all form factors enhanced, the $I=0$ s wave $\pi-\pi$ phase having a scattering length of 1.0 and a resonance at 500 MeV with a width of 100 MeV.

FIG.1

FIG.2

FIG. 3

FIG.4

FIG.5

FIG.6