PHOTOPRODUCTION AND ELECTROPRODUCTION OF PIONS

II. - PHOTOPION PRODUCTION BELOW 500 MeV

F.A. Berends^{*)}, A. Donnachie and D.L. Weaver^{**)}

CERN - Geneva

A B S T R A C T

Fixed- t dispersion relations are used to evaluate photopion production below 500 MeV, using conformal mapping techniques to solve the integral equations for the multipole transition amplitudes. Comparison is made with all existing experimental data in this energy region and agreement is excellent except for very low energy π -zero production, where there is a distinct discrepancy, which shows up because of the almost complete cancellation of different multipoles. Some general qualitative statements are made about photoproduction at higher energies, which are in accord with the experimental situation.

^{*)} Instituut-Lorentz, Leiden, The Netherlands.

^{**)} Department of Physics, Tufts University, Medford, Mass., U.S.A.

1. INTRODUCTION

Since the initial application of dispersion relations to photopion production by Chew, Goldberger, Low and Nambu¹⁾,*) the development of the dispersive approach has been steady and its success remarkable. C.G.L.N. evaluated the dispersion relations in what was essentially the static limit (with some minor corrections thereto), projecting out equations for the multipole amplitudes and retaining only those which lead to s- and p-wave pion-nucleon final states. The main line of the multipole approach can be traced firstly through Höhler and Müllensiefen²⁾ and Höhler, Dietz and Müllensiefen³⁾ who evaluated the C.G.L.N. equations in more detail, demonstrated the importance of recoil effects and, by comparison with experiment for photon laboratory energies in the region of the first resonance, showed that the theory was sound. The extension to fully relativistic kinematics was made by McKinley⁴⁾, although his attempt to solve the equations by comparison with the scattering equations via a comparison function was unsuccessful. The most recent calculation is that of Donnachie and Shaw⁵⁾ who obtained relativistic solutions for the $M_{1+}^{(3)}$ and the E_{0+} multipoles, and evaluated the other transitions to p- and d- wave pion-nucleon final states in the $(\text{Born} + M_{1+}^{(3)})$ approximation. Comparison with experiment was good, but there remained some small, but significant discrepancies, due largely to errors in the evaluation of the $E_{1+}^{(3)}$ and $M_{1-}^{(0,1)}$ multipoles, which had posed some difficulty. By parameterizing these multipoles (and in addition the E_{0+} multipoles) and fitting to experiment, Donnachie and Shaw⁶⁾ were able to remove the discrepancies, obtain values for the $E_{1+}^{(3)}$ and $M_{1-}^{(0,1)}$ multipoles and show that their evaluation of the E_{0+} transitions was correct.

An alternative approach was that of Ball⁷⁾, using the dispersion relations for the invariant amplitudes directly. It was assumed that the imaginary part was given entirely by $M_{1+}^{(3)}$, for which the C.G.L.N. result was used, and the real parts were evaluated from the fixed-t dispersion relations. This method has been discussed in detail by Höhler

*) Hereafter referred to as C.G.L.N.

and Schmidt^{8,9)}. The main advantage of this approach is its comparative simplicity, giving a relativistic calculation without the complexities of the relativistic projections. Its disadvantages are that it is based entirely on the static M_+ ⁽³⁾, it neglects the integrals over other imaginary parts (not all of which are negligible). Imaginary parts of the multipoles (for inclusion in cross-section formulae) can be estimated, via Watson's theorem¹⁰⁾ by numerical projection of the real parts of the multipoles from the total amplitudes. However, this destroys the simplicity of the approach and is a less satisfactory way of obtaining the imaginary parts than using the multipole dispersion relations in the first place. This extension of the Ball method has been applied to a study of polarization phenomena by Müllensiefen¹¹⁾.

In a previous paper¹²⁾,*) the dispersion theoretical formalism was discussed in detail and the fully relativistic projections for all multipoles obtained, including analytic expressions for all the important kernels which occur in the multipole dispersion relations. The purpose of this paper is to evaluate the theory in as complete a form as is at present practicable, to present a realistic discussion of the errors which arise in evaluating the dispersion relations, and to compare the results with all existing data up to photon lab. energies of 500 MeV. This energy is the upper limit of prediction for the theory in the elastic form.

The phase shifts used are discussed in Section 2. The method of solution uses conformal mapping techniques (Section 3) which have the twin merits of allowing an easy evaluation of the errors in the solution of the dispersion relations and of the imposition of reasonable physical asymptotic behaviour of the multipole amplitudes (as opposed to the more usual prescription of guessing the asymptotic behaviour of the multipole phases - a very dubious procedure). A discussion of general qualitative features of the more interesting multipoles is also given in this section. The sources of error are discussed, and the errors evaluated

*) Hereafter referred to as I.

in Section 4 and the final results presented in Section 5. A detailed comparison is made with experiment, and a discussion given of those places where the errors in the multipoles have a significant effect.

2. PHASES

The pion-nucleon scattering phase shifts used are those of Donnachie, Kirsopp, Lea and Lovelace¹³⁾. Accounts of this work have been given separately by Donnachie¹⁴⁾ and by Lovelace¹⁵⁾. From the point of view of photopion production, the interesting fact about most of the pion-nucleon scattering amplitudes is that they are essentially elastic for some considerable distance above the nominal inelastic threshold, the main exception being P_{11} and to a lesser extent D_{33} (see Section 12 of I), and so the Watson theorem can be applied without reservation to most multipoles in the energy region with which we are concerned (and in many cases to a much higher energy).

The scattering phase shifts used, and the fits made to them, are shown in Figs. 1(i) - 1(xiv). These fits arise in two ways.

A. If the phase shift is large in the energy range under discussion (i.e. its magnitude is greater than four or five degrees) then the rescattering term cannot be ignored and a specific solution for the multipole is obtained using the conformal mapping techniques of Section 3. This automatically gives a fit to the scattering phase shift.

B. If the phase shift is sufficiently small, then the rescattering term can be ignored. However, we still want to include the imaginary part of the corresponding multipole amplitudes in the cross-section evaluation, so the phase shift is required. Fits to the experimental phase shifts were done using a power-series expansion in q^2 of $q^{2\ell+1} \cot \delta_\ell$, with zeroes of the phase inserted explicitly.

3. METHOD AND SOLUTION

It was shown in I that the general form of the dispersion relation for a multipole amplitude $\mathcal{M}_\ell(W)$ is

$$\begin{aligned} \text{Re } \mathcal{M}_\ell(W) = & B_\ell(W) + \frac{P}{\pi} \int_{(m+\mu)}^{\infty} dW' \frac{\text{Im } \mathcal{M}_\ell(W')}{(W' - W)} \\ & + \frac{1}{\pi} \sum_{\ell'} \int_{(m+\mu)}^{\infty} dW' K_{\ell\ell'}(W, W') \text{Im } \mathcal{M}_{\ell'}(W') \end{aligned} \quad (3.1)$$

where $B_\ell(W)$ denotes the contributions from the Born terms, the principal value integral (the rescattering term) arises from intermediate two-or more particle states in the s-channel and the second integral (the crossed term) arises from intermediate two- or more particle states in both the s- and u-channels. The kernels $K_{\ell\ell'}(W, W')$ are non-singular. In practice the summation over ℓ' is truncated [the missing part being approximated, where appropriate, by the sum-rule pole (see Section 11 of I)] and it was argued in I (see Section 7) that only transitions to s- and p-waves need be included in this sum. Of the transitions to s- and p-waves, the dominant one is the $M_{1+}^{(3)}$ transition to a final P_{33} state. Eqs. (3.1) reduce to a set of coupled inhomogeneous singular integral equations for the multipole transitions to final s- and p-wave pion-nucleon states, and a set of individual (i.e. uncoupled) inhomogeneous singular integral equations for all other multipoles, the Born term and the crossed term (obtainable once the initial coupled set of equations has been solved) together with the sum-rule pole, where applicable, providing the inhomogeneous term.

There is also a separation in isotopic spin, the $\mathcal{M}_\ell^{(0)}$ multipoles being completely separate from the $\mathcal{M}_\ell^{(1)}$ and $\mathcal{M}_\ell^{(3)}$ multipoles, which couple with each other.

In the case of the $\mathcal{M}_\ell^{(0)}$ multipoles, since the large $M_{1+}^{(3)}$ multipole cannot couple in, the main contribution to the crossed cut comes from the only transition to s-waves, $E_{0+}^{(0)}$, the transitions to p-waves being

unimportant and consequently all the $m_\ell^{(0)}$ multipoles are uncoupled. Having solved for $E_{0+}^{(0)}$ (and we leave the discussion of how to solve until later), the inhomogeneous terms for all other $m_\ell^{(0)}$ multipoles can be obtained and the equations for the latter can be solved.

The case of the $m_\ell^{(1)}$ and $m_\ell^{(3)}$ multipoles is more complicated, although simplifications can be made before attempting to obtain a solution. Inspection of the kernels reveals firstly that all transitions are coupled very weakly to the dispersion relation for the $M_{1+}^{(3)}$ multipole, so that this multipole may be treated in isolation, and secondly that the only important coupling in the remaining multipoles is that of the $E_{0+}^{(1)}$ and $E_{0+}^{(3)}$ transitions with each other. Thus having solved for $M_{1+}^{(3)}$, $E_{0+}^{(1)}$ and $E_{0+}^{(3)}$ (and we leave the discussion of how to solve until later) the contributions of these multipoles to all other multipoles can be obtained, and the equations for the latter can be solved.

The solutions can be divided into three categories.

- A. The scattering phase shift is small both in the region under consideration and at higher energies

In this case, since

$$\text{Im } m_\ell = \text{Re } m_\ell \tan \delta_\ell \quad (3.2)$$

the rescattering contribution will be negligible, and can be ignored. The real part of the multipole is then given in the (Born + $E_{0+}^{(0)}$) approximation for the $m_\ell^{(0)}$ multipoles and the (Born + $M_{1+}^{(3)}$ + $E_{0+}^{(1)}$ + $E_{0+}^{(3)}$) approximation for the $m_\ell^{(1)}$ and $m_\ell^{(3)}$ multipoles, and the imaginary part evaluated using Eq. (3.2).

- B. The scattering phase shift is small in the region under consideration, but may be large elsewhere

In this case there may well be a non-negligible contribution to the rescattering, but it comes from an energy region outwith our control

and can only be estimated crudely. All transitions to final pion-nucleon states with a high-energy resonance are subject to this malady, but only in those cases where the resonance is close to the energy region in which we are interested will this correction be important. In practice this means the $M_{1-}^{(0,1)}$ transitions to a final P_{11} pion-nucleon state and the $E_{2-}^{(0,1)}$, $M_{2-}^{(0,1)}$ transitions to a final D_{13} pion-nucleon state. To estimate the rescattering correction, we made the Ansatz

$$m_{\ell\pm} = \lambda \mathcal{J}_{\ell\pm} f_{\ell\pm} \quad (3.3)$$

where $\mathcal{J}_{\ell\pm}$ is the inhomogeneous part of the multipole dispersion relation, $f_{\ell\pm}$ is the corresponding scattering amplitude and λ is a constant which was determined by a consistency condition at the resonance position. The best that can be said about this Ansatz is that it is by guesswork out of the static model. In the static model for the $M_{1+}^{(3)}$ multipole, the relation with the scattering amplitude is

$$\frac{m_{1+}^{(3)}}{f_{1+}^{(3)}} = \frac{m_{1+}^{(3)} \text{ (static Born)}}{f_{1+}^{(3)} \text{ (static Born)}} \quad (3.4)$$

(in the case of the $M_{1+}^{(3)}$ multipole, the Born term is to all intents and purposes the full inhomogeneous term). In our case we have replaced the scattering amplitude Born term by an arbitrary constant which is not too unreasonable since in the vicinities of the resonances in question the dominant forces in the pion-nucleon partial wave dispersion relation are approximately constant¹⁶⁾. The scattering amplitudes were fitted with a Layson¹⁷⁾ type resonance formula, suitably modified¹⁸⁾ in the P_{11} case to take account of the zero in the phase^{*}). With this type of

*) The exact nature of the P_{11} resonance is not at all clear, and some solutions cannot be well approximated by a simple resonance formula, even when modified to take account of the zero. Since the situation is ambiguous, we have opted for the simplest solution.

fit, the Ansatz Eq. (3.3) gives a zero in the real part of the multipole, and the constant λ was chosen to keep this zero at the resonance position when the multipole was evaluated from the multipole dispersion relation. The contribution of the rescattering to these multipoles at 500 MeV (where it is a maximum) is of the order of 100% of the inhomogeneous term in the $M_{1-}^{(0,1)}$ cases, and of the order of 40% of the inhomogeneous term in the $E_{2-}^{(0,1)}$ and $M_{2-}^{(0,1)}$ cases.

C. The scattering phase shift is large
in the region under consideration

In this case the rescattering term is important in the region being considered and the main contribution comes from this region. The transitions involved are the $M_{1+}^{(3)}$, $E_{1+}^{(3)}$ transitions to the P_{33} pion-nucleon final state, the $E_{0+}^{(0,1)}$ transitions to the S_{11} pion-nucleon final state, the $E_{0+}^{(3)}$ transition to the S_{31} pion-nucleon final state and the $M_{1-}^{(3)}$ transition to the P_{31} pion-nucleon final state. In these cases the integral equations must be solved explicitly. Given a knowledge of the multipole phases from threshold to infinity then this is in principle possible, since the equations are of the Omnès-Muskhelishvili type. However, this information on the phase is denied us. The pion-nucleon scattering amplitudes are known only to 1311 MeV and long before this energy is reached, the presence of strong inelastic effects means that the scattering phase shift bears no relation to the phase of the corresponding multipole. Attempts to "solve exactly" by guessing the behaviour of the phase are thus open to serious criticism, and they must have recourse to rather dubious procedures in order to obtain a "unique" solution^{19,20}.

Our philosophy is that while it is unreasonable to make statements about the high-energy behaviour of the multipole phases, it is perfectly reasonable to make statements about the high-energy behaviour of the magnitude of the multipoles - the essence of this being that they must decrease with energy at least as fast as some (specified) inverse power of the energy (see Section 11 of I). This was also the philosophy of

Donnachie and Shaw⁵⁾, although it was applied there in a somewhat ad hoc way. Our present approach is considerably more sophisticated.

To evaluate the dispersion relations we use a conformal mapping technique^{14,21)}. It can be shown that there exist two sets of functions $g_k(W)$ and $h_k(W)$ ($k = 1, 2, \dots, \infty$) with the properties

$$i) \quad g_k(W) = \frac{P}{\pi} \int_{(m+u)}^{\infty} dW' \frac{h_k(W')}{(W' - W)} \quad (k = 1, 2, \dots, \infty) \quad (3.5)$$

$$ii) \quad \text{Im } M_\ell(W) = \sum_{k=1}^{\infty} (a_\ell)_k h_k(W) \quad (3.6)$$

and this series converges under very general conditions.

iii) $g_k(W)$ and $h_k(W)$ have the correct threshold behaviour and vanish asymptotically sufficiently rapidly to satisfy our requirements there.

Defining

$$\tilde{Y}_\ell(W) = B_\ell(W) + \frac{1}{\pi} \sum_{\ell \neq \ell'} \int_{(m+u)}^{\infty} dW' K_{\ell\ell'}(W, W') \text{Im } M_{\ell'}(W') \quad (3.7)$$

then the dispersion relation Eq. (3.1) takes the form

$$\text{Re } M_\ell(W) = \tilde{Y}_\ell(W) + \sum_{k=1}^{\infty} (a_\ell)_k \{ g_k(W) + \tilde{g}_k(W) \} \quad (3.8)$$

where

$$\tilde{g}_k(w) = \frac{1}{\pi} \int_{(m+\mu)}^{\infty} dw' K_{\ell\ell}(w, w') h_k(w') \quad (3.9)$$

Truncating the series in Eqs. (3.6) and (3.8) at some value $k = K$ and applying the unitarity condition (Watson's theorem),

$$\text{Im } M_{\ell}(w) = \text{Re } M_{\ell}(w) \tan \delta_{\ell}(w) \quad (3.10)$$

at the points $W = W_i$ ($i = 1, 2, \dots, N$) where the scattering phase shift is known, we obtain the set of linear equations

$$\begin{aligned} \sum_{k=1}^K (a_{\ell})_k \{ h_k(w_i) - [g_k(w_i) + \tilde{g}_k(w_i)] \tan \delta_{\ell}(w_i) \} \\ = \frac{1}{2} \tan \delta_{\ell}(w_i) \tan \delta_{\ell}(w_i) \quad (i=1, 2, \dots, N) \end{aligned} \quad (3.11)$$

and the coefficients $(a_{\ell})_k$ can be found by standard linear fitting techniques (in practice $K \ll N$). Since the equations are linear, given the errors on the scattering phase shifts δ_{ℓ} , the evaluation of the errors on the $(a_{\ell})_k$ is straightforward. If necessary, it is not difficult to impose further constraints on the behaviour of the multipoles at high energies, including the addition of a short-range pole to ensure that the sum-rule is satisfied.

Having solved Eqs. (3.11) for the $(a_{\ell})_k$'s, the real and the imaginary parts of the multipoles are obtained immediately from Eqs. (3.8) and (3.6), they automatically satisfy the unitarity condition in the elastic

region, they automatically have the correct threshold behaviour and sensible asymptotic behaviour and, of course, automatically satisfy the dispersion relation. Finally, they are automatically supplied with errors.

To summarize, the order of events is as follows:-

- i) Eq. (3.11) is used to solve for $E_{0+}^{(0)}$, with $\mathcal{F}_{0+}^{(0)} = B_{0+}^{(0)}$.
- ii) Eq. (3.11) is used to solve for $M_{1+}^{(3)}$, with $\mathcal{F}_{1+}^{(3)} = B_{1+}^{(3)}$.
- iii) Eq. (3.11) is used to solve for $E_{0+}^{(1)}$ and $E_{0+}^{(3)}$, with $\mathcal{F}_{0+}^{(1)}$, $\mathcal{F}_{0+}^{(3)}$ containing the $M_{1+}^{(3)}$ contribution as well as the Born term. The equations are then re-solved with the addition of the $E_{0+}^{(1)}$ contribution to $\mathcal{F}_{0+}^{(3)}$ and the $E_{0+}^{(3)}$ contribution to $\mathcal{F}_{0+}^{(1)}$ and the process repeated until a consistent solution is obtained (and consequently the coupled equations are satisfied).
- iv) Eq. (3.11) is used to solve for $E_{1+}^{(3)}$, $M_{1-}^{(3)}$ with $\mathcal{F}_{1+}^{(3)}$, $\mathcal{F}_{1-}^{(3)}$ containing the $E_{0+}^{(1)}$, $E_{0+}^{(3)}$ and $M_{1+}^{(3)}$ contributions in addition to the Born term.
- v) Eq. (3.1) is used to evaluate the remaining multipoles with the rescattering term neglected (except in the case of the $M_{1-}^{(0)}$, $M_{1-}^{(1)}$, $E_{2-}^{(0)}$, $E_{2-}^{(1)}$, $M_{2-}^{(0)}$, $M_{2-}^{(1)}$ transitions when it was estimated as indicated above.

All multipoles leading to s, p, d and f wave final states are given in Tables 1(i) - 1(xii). All higher multipoles were included in the Born approximation only, since explicit evaluation of the other contributions shows that they are negligible compared to the Born terms in the energy range being considered.

Some specific comment on these multipoles is of interest.

The difference between our solution for $M_{1+}^{(3)}$ and the C.G.L.N. solution (which has been the basis of much of the previous work in photoproduction) is shown in Fig. 2. The real part shows the difference

most clearly away from the resonance region, and the imaginary part shows the difference most clearly in the resonance region. The differences are typically of the order of 10% away from the resonance region, and of the order of 5% in the resonance region (which, of course, produces a difference of 10% or so in the cross-section at resonance, particularly in the case of π^0 production).

The $E_{1+}^{(3)}/M_{1+}^{(3)}$ ratio is shown in Fig. 3. Our result is very similar to that of Finkler¹⁹⁾ and to that obtained phenomenologically by Donnachie and Shaw⁶⁾. It differs considerably from that of Korth et al.²⁰⁾ and from the C.G.L.N. solution. In particular the ratio does not change sign at resonance, although it does become very small there. Schmidt and Wunder²²⁾ have pointed out that the Finkler type solution is certainly to be preferred, on experimental grounds, to that of Korth et al. One feature of our solution, which does not appear to be present in the Finkler solution, is that the magnitude of the ratio increases beyond the resonance. (This is also true, of course, in the solutions which change sign at resonance, but the ratio is of the opposite sign to ours.)

It has long been observed experimentally that transitions to the old "second resonance" (in D_{13}) proceed most strongly via an E_{2-} transition, rather than an M_{2-} transition. The reason for this is clearly seen in our solutions for these multipoles in which E_{2-} is considerably larger than M_{2-} . It should also be noted that there is a partial cancellation between $E_{2-}^{(1)}$ and $E_{2-}^{(0)}$ in photoproduction from protons, but an enhancement in photoproduction from neutrons, so that it would be expected that the resonance would show up most strongly in the latter case.

The same is true of the M_{1-} transitions (the only ones) to the P_{11} final state. There is a strong cancellation between the $M_{1-}^{(0)}$ and $M_{1-}^{(1)}$ in photoproduction from protons and a strong enhancement in photoproduction from neutrons [at 500 MeV, the numbers in question are:

$$\begin{aligned} M_{1-}^{(0)} &= -3.185 \times 10^{-3}, & M_{1-}^{(1)} &= 1.119 \times 10^{-2} \\ M_{1-}^{(\pi^+, n)} &= 7.5 \times 10^{-4}, & M_{1-}^{(\pi^0, p)} &= 5.4 \times 10^{-4}, & M_{1-}^{(\pi^-, p)} &= -9.78 \times 10^{-3} \end{aligned}$$

Consequently, the P_{11} resonance should not show up strongly in photoproduction from protons, which is consistent with the sparse data available in the appropriate energy region. Since the enhancement due to rescattering in photoproduction is proportional to the force term; the ratio of the magnitude of the multipoles for photoproduction of the P_{11} resonance will remain as quoted above, and consequently the cross-section for photoproduction of P_{11} on protons will be only a few per cent of that for photoproduction of P_{11} on neutrons. This is further evidence for the inclusion of P_{11} in an anti-decuplet, as has been suggested by Lovelace²³⁾, as opposed to an octet as required by Dalitz²⁴⁾, since this result is one of the two tests for $\overline{10}$ proposed by Lipkin²⁵⁾ (the application of the strict symmetry excludes completely photoproduction of $\overline{10}$ from protons, but it is allowed on neutrons).

The contribution of the D_{15} resonance is also likely to be small. At 500 MeV the corresponding multipoles are smaller in magnitude even than those leading to the F_{15} resonance,

The result of all this is that the D_{13} resonance shows up very clearly in photoproduction, as compared to pion-nucleon scattering, and it is hardly surprising that the quantum numbers of this resonance were first obtained from the photoproduction data and not the scattering data.

4. ERRORS

There are three principal sources of error in the theoretical calculation.

- i) Errors in the Born terms arising from errors in the coupling constant g (the error on e is negligible from our point of view).
- ii) Errors in the phase shifts produce errors in the multipoles for which explicit solutions have been obtained. (They also produce errors in the imaginary parts of the multipoles in which rescattering has been neglected. In this case, however, the imaginary parts themselves are so small that this error can be neglected.) This is the most important source of error.
- iii) Errors arising from the estimation of the rescattering contributions to $M_{1-}^{(0)}$, $M_{1-}^{(1)}$, $E_{2-}^{(0)}$, $E_{2-}^{(1)}$, $M_{2-}^{(0)}$ and $M_{2-}^{(1)}$.

The first two sources of error provide no difficulty of principle since the sources of error are well defined, although care must be taken in evaluating the errors on the experimental predictions. The errors in the multipoles which arise from the Born term errors are correlated, while those arising from the phase shift errors are uncorrelated. The errors on the experimental predictions from the former were obtained by evaluating the results with the coupling constant g (and consequently the Born term) at its standard value (namely $g^2/4\pi = 14.86 \pm 0.33^{26}$) and then at the upper and lower values given by the quoted error. The errors on the experimental predictions from the uncorrelated errors were obtained for each case in the usual way from the formula

$$(\Delta f)^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 (\Delta x_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 (\Delta x_2)^2 + \dots + \left(\frac{\partial f}{\partial x_N}\right)^2 (\Delta x_N)^2 \quad (4.1)$$

where Δf is the error on f , a function of $x_1, x_2, \dots, x_N$ arising from the errors $\Delta x_1, \Delta x_2, \dots, \Delta x_N$ on $x_1, x_2, \dots, x_N$.

The errors on the estimated rescattering contributions are also uncorrelated errors, but it is not possible to give them a precise value. A rather arbitrary 25% was chosen, although some justification of this choice can be made by comparing the contribution of the Ansatz for $E_{2-}^{(1)}$ (the dominant resonant multipole) to the total cross-section in the region of the second resonance to the observed experimental values. The theoretical values are $42 \mu\text{b}$ for π^+ production and $21 \mu\text{b}$ for π^0 production, and the corresponding experimental values, taking off the background as well as one can, are approximately $50\text{-}60 \mu\text{b}$ and $20\text{-}25 \mu\text{b}$ respectively. In both cases the background contribution is of the same order as the resonant contribution. Clearly the Ansatz produces a number which is realistic, particularly when one bears in mind the fact that it is being used only to make a correction to an already well-defined value, and an error assignment of 25% to this correction is perfectly reasonable.

5. RESULTS

The theoretical predictions are compared with all existing experimental data in Figs. 4, 5, 6, 8, 9, 10, 11, 12, 13 for photon laboratory energies between 160 and 500 MeV.

(i) $\gamma + p \rightarrow \pi^+ + n$

The differential cross-sections are given in Figs. 4(i) - 4(xxxix) for photon laboratory energies in increments of 5 MeV from 160 to 200 MeV and thereafter in increments of 10 MeV to 500 MeV. The experimental data have been binned at the same energies. The errors on the theoretical predictions are typically of the order of (and usually less than) $\pm 10\%$. This is larger than the purely statistical errors on most data, but is somewhat less than the systematic errors on the data, which causes a spread, typically, of $\pm 20\%$ (and sometimes worse). At all energies the agreement of theory with data is very good.

The predictions for the ratio $(\sigma_{\perp} - \sigma_{\parallel})/(\sigma_{\perp} + \sigma_{\parallel})$ for cross-sections using plane polarized photons (where $\sigma_{\perp}$ is the cross-section when the plane of photon polarization and the plane of pion emission are perpendicular and $\sigma_{\parallel}$ is the cross-section when the plane of photon polarization and the plane of pion emission are parallel) are given in Figs. 5(i) - 5(xxii) in increments of 20 MeV from 160 to 500 MeV with the addition of a few extra energies to catch all of the data. Again agreement of theory with data is good. This is seen even more clearly in the excitation curves which are given in Figs. 6 (i) - 6 (vii).

The predictions for the recoil neutron polarization are given in Figs. 7(i) - 7(xvi) at increments of 20 MeV from 200 to 500 MeV. There are no experimental data for comparison. The most notable feature is the amount of polarization predicted, reaching as high as 80%.

(ii) $\gamma + p \rightarrow \pi^0 + p$

The differential cross-sections are given in Figs. 8(i) - 8(xxxix) for photon laboratory energies from 160 to 200 MeV in increments of 5 MeV and thereafter in increments of 10 MeV to 500 MeV. The experimental data

are much less numerous than in the case of π^+ photoproduction, but again one can conclude that the agreement between theory and experiment is very good except at very low energies (less than 200 MeV) where there is a distinct discrepancy. This energy region has always been a problem for theory, the problem being caused by the almost complete cancellation of what should nominally be the dominant terms, namely the E_{0+} transitions to s-waves, leaving the cross-section to be determined by the small transitions to the p-waves. Also, of these, the M_{1-} transition (which is one of the least certain ones) is as important as the $M_{1+}^{(3)}$ transition. This situation is shown for a photon laboratory energy of 160 MeV in Table 2.

The cancellation in the transitions to s-waves is so complete that even the sign is unknown and the error is of the same magnitude as the other important terms. With the typical errors which exist on the E_{0+} multipoles (about 6% or 7%) the resulting value is

$$E_{0+}(\pi^0) = (-0.041 \pm 0.142) \times 10^{-2} \quad (\hbar = \mu = c = 1).$$

Donnachie and Shaw⁶⁾ have shown that small adjustments to the E_{0+} and M_{1-} multipoles (of the order of a few per cent in the E_{0+} multipoles, which has a negligible effect on the π^+ results) are sufficient to produce agreement with experiment. Since the theory works so nicely everywhere else, it is tempting to explain the discrepancy as being due to the omission of ω -exchange, which has the advantage of not affecting the π^+ results at all. (ρ -exchange is known to be negligible; an analysis of low-energy data on the photoproduction of π^+ and π^- by Donnachie and Shaw^{5,6)} showed that the ρ -contribution is consistent with zero, and the most recent experimental upper limit on the width $\Gamma_{\rho\pi\gamma}^{27)$, namely $\Gamma_{\rho\pi\gamma} < 0.6$ MeV, also indicates that the ρ -contribution will be small compared to the ω -contribution). That there should be some ω -exchange present is likely is indicated by the way in which Reggeized ω -exchange can explain very adequately the existing high-energy data on π^0 photoproduction²⁸⁾.

The discrepancy vanishes very quickly due to the rapidly increasing $M_1 + {}^{(3)}$ contribution which (because of the cancellations discussed above) dominates completely π^0 photoproduction in the energy region being considered, although there is some indication of some small discrepancy at the upper end of the energy range (see the data at 450 MeV) but the data are too sparse to allow of a definite conclusion.

The predictions for the ratio $(\sigma_{\perp} - \sigma_{\parallel})/(\sigma_{\perp} + \sigma_{\parallel})$ for cross-sections using plane polarized photons are given in Figs. 9(i) - 9(xvi) in increments of 20 MeV from 200 to 500 MeV, again with a few extra additions to catch all of the five existing data. The good agreement is again seen most easily in the 90° excitation curve, given in Fig. 10. Not unexpectedly, there is again evidence for a discrepancy at low energies.

The predictions for the recoil proton polarization are given in Figs. 11(i) - 11(xvii) at increments of 20 MeV from 200 to 500 MeV. Comparison with the existing data is very good, and is shown clearly in the 90° excitation curve, Fig. 12. A notable feature is the high proton polarization predicted when the pion is produced at backward angles. This is greater than 30% for photon energies between 350 and 450 MeV. This region has never been studied experimentally.

(iii) $\gamma + n \rightarrow \pi^- + p$

The predictions for the differential cross-sections, the $(\sigma_{\perp} - \sigma_{\parallel})/(\sigma_{\perp} + \sigma_{\parallel})$ ratios and the recoil proton polarization are given in Figs. 13(i) - (xviii), 14(i) - (xviii), 15(i) - (xvi) respectively.

This process is notable for the almost complete absence of data in the energy region being considered. Most of the data that do exist consist only of π^-/π^+ ratios in deuterium, a discussion of which is outside the scope of the present paper. Garelick and Cooperstein²⁹ have extracted π^- photoproduction cross-sections from their data and the result, shown in Fig. 13(vii), is some 1 - 8 ($\mu\text{b/sr}$) smaller than the predicted values. However, their extracted value of the π^+ photoproduction cross-section (which is the same magnitude as the π^- cross-section) is also some 3 - 5 ($\mu\text{b/sr}$) smaller than the theoretical values and direct experimental measurement, so this difference should not be taken seriously.

6. CONCLUSIONS

We conclude that the fixed- t dispersion relations give a detailed quantitative picture of photopion production below 500 MeV photon laboratory energy, and allow of a qualitative description of the situation above this energy. The only failure of the theory is in very low energy π^0 production where there is a distinct discrepancy which shows up due to the almost complete cancellation of different multipoles. This pathological situation apart, agreement with experiment is very good everywhere. The inclusion of ω -meson exchange has been suggested as a possible way of removing this discrepancy, and this question will subsequently be discussed in detail.

We hope that this rash display of theoretical prediction (there are no free parameters) will encourage further experimental investigation in this energy region, not only of polarization effects, but also of differential cross-sections, the π^0 differential cross-section data in particular being woefully inadequate. And, of course, anything (almost!) on π^- radiative capture will be acceptable.

At the upper end of our energy range and above (say $400 \text{ MeV} < E_\gamma < 700 \text{ MeV}$) experimental evidence in support (or otherwise) of our conclusions regarding P_{11} are of great interest, and a detailed study of charged pion production in deuterium or, alternatively, a comparison of photoproduction from hydrogen with π^- radiative capture would be very valuable.

ACKNOWLEDGEMENTS

We are indebted to C. Lovelace for many helpful discussions on conformal mapping techniques in solving dispersion relations, and for making available his FORTRAN subroutines for evaluating the conformal functions and for linear fitting.

We are deeply grateful to Miss A. Lutke for her patience and for her considerable effort in preparing the figures.

One of us (F.A.B.) would like to thank the CERN Theoretical Division for their frequent hospitality.

REFERENCES IN TEXT

- 1) G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, Phys.Rev. 106, 1345 (1957).
- 2) G. Höhler and A. Müllensiefen, Z. Physik 157, 30 (1959).
- 3) G. Höhler, K. Dietz and A. Müllensiefen, Z. Physik 159, 77 (1960).
- 4) J. McKinley, Tech.Report No. 38 (1962), University of Illinois, Chicago, Illinois.
- 5) A. Donnachie and G. Shaw, Annals of Physics (N.Y.) 37, 333 (1966).
- 6) A. Donnachie and G. Shaw, Nuclear Physics (to be published).
- 7) J.S. Ball, Phys.Rev. 124, 2014 (1961).
- 8) G. Höhler and W. Schmidt, Annals of Physics (N.Y.) 28, 34 (1964).
- 9) W. Schmidt, Z. Physik 182, 76 (1964).
- 10) K.M. Watson, Phys.Rev. 95, 228 (1954).
- 11) A. Müllensiefen, Z. Physik 188, 199 and 238 (1965).
- 12) F.A. Berends, A. Donnachie and D.L. Weaver, CERN Preprint TH.703 (1966).
- 13) A. Donnachie, R.G. Kirsopp, A.T. Lea and C. Lovelace, " π -p phase shifts up to 1311 MeV" (in preparation).
- 14) A. Donnachie, Lecture Notes, Scottish Universities Summer School 1966, (to be published).
- 15) C. Lovelace, International Conference on High-Energy Physics, Berkeley 1966, and CERN Preprint TH. 705 (1966).
- 16) A. Donnachie, J. Hamilton and A.T. Lea, Phys.Rev. 135, B515 (1964).
- 17) W.M. Layson, Nuovo Cimento 20, 1207 (1961).
- 18) L.D. Roper, R.M. Wright and B.T. Feld, Phys.Rev. 138, B190 (1965).
- 19) P. Finkler, UCRL 7953-T (1964).
- 20) W. Korth, H. Rollnik, D. Schwela and R. Weizel, Bonn Preprint (1965).
- 21) C. Lovelace, Nuovo Cimento 25, 730 (1962) and unpublished notes.

- 22) W. Schmidt and H. Wunder, Physics Letters 20, 541 (1961).
- 23) C. Lovelace, CERN Preprint TH.628, (unpublished).
- 24) R.H. Dalitz, "Resonant States and Strong Interactions", International Conference on Elementary Particles, Oxford, 1965.
- 25) H. Lipkin, Physics Letters 12, 154 (1964).
- 26) V.K. Samaranayake and W.S. Woolcock, Phys.Rev.Letters 15, 936 (1965).
- 27) G. Fidecaro, M. Fidecaro, J.A. Poirier and P. Schiavon, Physics Letters 23, 163 (1966).
- 28) M.P. Locher and H. Rollnik, Physics Letters 22, 696 (1966).
- 29) D. Garelick and G. Cooperstein, Phys.Rev. 136, B201 (1964).

EXPERIMENTAL REFERENCES

$\gamma + p \rightarrow \pi^+ + n$ DIFFERENTIAL CROSS-SECTIONS

- B.T. Feld, D.H. Frisch, I.L. Lebow, L.S. Osborne and J.S. Clark, Phys.Rev. 85, 680 (1952).
- J.S. Steinberger and A.S. Bishop, Phys.Rev. 86, 171 (1952).
- R.S. White, M.J. Jacobson and A.G. Shultz, Phys.Rev. 88, 836 (1952).
- D. Luckey, T.J. Jenkins and R.R. Wilson, Phys.Rev. 91, 468 (1953).
- T.R. Palfrey, D. Luckey and R.R. Wilson, Phys.Rev. 91, 468 (1953).
- N. Jarmie, G.W. Repp and R.S. White, Phys.Rev. 91, 1023 (1953).
- G.S. Janes and W.L. Kraushaar, Phys.Rev. 93, 900 (1954).
- G. Bernardini and E.L. Goldwasser, Phys.Rev. 94, 729 (1954).
- T.L. Jenkins, D. Luckey, T.R. Palfrey and R.R. Wilson, Phys.Rev. 95, 179 (1954).
- G. Bernardini and E.L. Goldwasser, Phys.Rev. 95, 857 (1954).
- R.L. Walker, J.G. Teasdale, V.Z. Peterson and J.L. Vette, Phys.Rev. 99, 210 (1955).
- A.V. Tollestrup, J.C. Keck and R.M. Worlock, Phys.Rev. 99, 220 (1955).
- M. Beneventano, G. Bernardini, D. Carlson-Lee, G. Stoppini and L. Tau, Nuovo Cimento 4, 323 (1956).
- M. Heinberg, W.M. McLelland, F. Turkot, W.M. Woodward, R.R. Wilson and D.M. Zipoy, Phys.Rev. 110, 1211 (1958).
- G.M. Lewis and R.E. Azuma, Proc.Phys.Soc. 73, 873 (1959).
- E.A. Knapp, R.W. Kenney and V. Perez-Mendez, Phys.Rev. 114, 605 (1959).
- J.K. Walker and J.P. Burq, Phys.Rev.Letters 8, 37 (1962).
- M.I. Adamovich, E.G. Gorzhevskaya, V.G. Sanosova, N.M. Panova, S.P. Kharlamov and F.R. Yagudin, Proc. Inter. Conf. on High-Energy Physics at CERN, Geneva, 1962, p. 207.

- G.M. Lewis, R.E. Azuma, E. Gabathuler, D.W.G.S. Leith and W.R. Hogg, Phys.Rev. 125, 378 (1962).
- G.M. Lewis, D.W.G.S. Leith, D.L. Thomas, R. Little and E.M. Lawson, Nuovo Cimento 27, 384 (1963).
- C. Freitag, D. Freytag, K. Lübelmeyer and W. Paul, Z. Physik 175, 1 (1963).
- K. Altoff, H. Fischer and W. Paul, Z. Physik 175, 19 (1963).
- M. Bazin and J. Pine, Phys.Rev. 132, 830 (1963).
- D.W.G.S. Leith, R. Little and E.M. Lawson, Physics Letters 8, 355 (1964).
- M.I. Adamovich, V.G. Larionova, A.I. Lebedev, S.P. Kharlamov and F.R. Yagudina, Dokl.Akad.Nauk SSSR 158, 309 (1964), [Translation: Soviet Physics - Doklady 9, 792 (1965)].
- D. Freytag, W.J. Schuille and R.J. Wedermeyer, Z. Physik 186, 1 (1965).
- M. Aleksandrov, V.F. Grushin, V. Zaperalov and E.M. Leiken, Dokl.Akad. Nauk SSSR 160, 796 (1965).
- J.C. Bizot, J. Perez-y-Jorba and D. Treille, Proc. Inter. Symp. on Electron and Photon Interactions at High Energies, Hamburg, 1965, Vol. II, p.227.

$\gamma + p \rightarrow \pi^+ + n$ ASYMMETRY RATIOS

- R.C. Smith and R.F. Mozley, Phys.Rev. 130, 2429 (1963).
- P. Gorenstein, M. Grilli, P. Spillantini, M. Nigro, E. Schiavuta, F. Soso and V. Valente, Physics Letters 19, 157 (1965).
- P. Gorenstein, M. Grilli, M. Nigro, E. Schiavuta, F. Soso, P. Spillantini and V. Valente, Frascati Report LNF-66/23 (1966).
- M. Grilli, F. Soso, P. Spillantini, M. Nigro, E. Schiavuta and V. Valente. Preliminary results reported at the International Conference on Electromagnetic Interactions, Dubna (1967).

$\gamma + p \rightarrow \pi^0 + p$ DIFFERENTIAL CROSS-SECTIONS

- A. Silverman and M. Stearns, Phys.Rev. 88, 1225 (1952).
- R.L. Walker, D.C. Oakley and A.V. Tollestrup, Phys.Rev. 89, 1301 (1953).
- Y. Goldschmidt-Clermont, L.S. Osborne and M.B. Scott, Phys.Rev. 97, 188 (1955).
- R.L. Walker, D.C. Oakley and A.V. Tollestrup, Phys.Rev. 97, 1279 (1955).

- D.C. Oakley and R.L. Walker, Phys.Rev. 97, 1283 (1955).
- L.J. Koester and F.E. Mills, Phys.Rev. 105, 1900 (1957).
- W.S. McDonald, V.Z. Peterson and D.R. Corson, Phys.Rev. 107, 577 (1957).
- P.C. Stein and K.C. Rogers, Phys.Rev. 110, 1209 (1958).
- J.W. de Wire, H.E. Jackson and R. Littauer, Phys.Rev. 110, 1208 (1958).
- J.I. Vette, Phys.Rev. 111, 622 (1958).
- V.I. Goldansky, B.B. Govorkov and R.G. Vassilkov, Nuclear Physics 12, 327 (1959).
- K. Berkelman and J.A. Waggoner, Phys.Rev. 117, 1364 (1960).
- 3) B.B. Govorkov, S.P. Denisov and E.W. Minarik, Proc.Inter.Conf. on High Energy Physics at CERN, 1962, p.213.
- D.B. Miller and E.H. Bellamy, Proc.Phys.Soc. 81, 343 (1963).
- V.L. Highland and J.W. de Wire, Phys.Rev. 132, 1293 (1963).
- B.B. Govorkov, S.P. Denisov, A.I. Lebedev, E.V. Minarik and S.P. Kharlamov, Zh. Eksperim. i Teor. Fiz. 47 1199 (1964) [Translation: Soviet Physics - JETP 20, 809 (1965)].
- E.P. Dally, (unpublished).
- W. Hitzeroth, Proc.Inter.Symp. on Electron and Photon Interactions at High Energies, Hamburg 1965, Vol. II, p.209.
- C. Bacci, C. Mencuccini, G. Penso, G. Salvini, V. Silvestrini, M. Spinetti and B. Stella, Proc.Inter.Symp. on Electron and Photon Interactions at High Energies, Hamburg 1965, Vol. II, p.241.

$\gamma + p \rightarrow \pi^0 + p$ ASYMMETRY RATIOS

- D.J. Drickey and R.F. Mozley, Phys.Rev. 136, B543 (1964).
- G. Barbiellini, G. Bologna, J. de Wire, G. Diambrini, G.P. Murtas and G. Sette, Proc. 12th Inter.Conf. on High-Energy Physics, Dubna, 1964, p.838.

$\gamma + p \rightarrow \pi^0 + p$ RECCIL PROTON POLARIZATION

J.O. Maloy, G.A. Salandin, A. Manfredini, V.Z. Peterson, J.I. Friedman and H. Kendall, Phys.Rev. 122, 1338 (1961).

J.O. Maloy, V.Z. Peterson, G.A. Salandin, F. Waldner, A. Manfredini, J. Friedman and H. Kendall, Phys.Rev. 139, B733 (1965).

K.H. Althoff, K. Kramp, H. Matthay and H. Piel, Z. Physik 194, 135 and 144 (1966).

D. Lundquist, D.M. Ritson, J.V. Allaby and R. Anderson, Stanford Preprint HEPL 451 (1966).

TABLE 1(i) - (xii)

Photoproduction multipoles

(The units are $\hbar = \mu = c = 1$ and the phases are in degrees.)

TABLE 1

(i) The $E_{\ell}^{(0)}$ Multipoles

Energy (MeV)	$E_0^{(0)}$		$E_1^{(0)}$		$E_2^{(0)}$		$E_3^{(0)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	-1.253 E-3	3.61	6.031 E-6		-8.099 E-8		1.294 E-9	
170	-1.252 E-3	5.20	8.206 E-6		-1.564 E-7		3.551 E-9	
180	-1.253 E-3	6.10	9.706 E-6		-2.273 E-7		6.354 E-9	
190	-1.255 E-3	6.41	1.086 E-5		-2.946 E-7		9.554 E-9	
200	-1.259 E-3	6.62	1.179 E-5		-3.588 E-7		1.307 E-8	
210	-1.264 E-3	6.79	1.257 E-5	-0.29	-4.203 E-7		1.683 E-8	
220	-1.269 E-3	6.95	1.324 E-5	-0.37	-4.794 E-7		2.082 E-8	
230	-1.275 E-3	7.12	1.383 E-5	-0.46	-5.365 E-7		2.500 E-8	
240	-1.282 E-3	7.31	1.435 E-5	-0.55	-5.919 E-7		2.935 E-8	
250	-1.289 E-3	7.52	1.483 E-5	-0.65	-6.457 E-7		3.386 E-8	
260	-1.296 E-3	7.76	1.526 E-5	-0.76	-6.982 E-7		3.851 E-8	
270	-1.303 E-3	8.03	1.566 E-5	-0.87	-7.493 E-7		4.328 E-8	
280	-1.310 E-3	8.33	1.603 E-5	-0.99	-7.994 E-7		4.818 E-8	
290	-1.318 E-3	8.65	1.637 E-5	-1.12	-8.485 E-7	0.26	5.319 E-8	
300	-1.325 E-3	9.00	1.670 E-5	-1.25	-8.966 E-7	0.29	5.831 E-8	
310	-1.332 E-3	9.36	1.700 E-5	-1.38	-9.438 E-7	0.32	6.352 E-8	
320	-1.339 E-3	9.73	1.729 E-5	-1.52	-9.903 E-7	0.35	6.883 E-8	
330	-1.346 E-3	10.10	1.757 E-5	-1.66	-1.036 E-6	0.38	7.423 E-8	
340	-1.353 E-3	10.47	1.783 E-5	-1.81	-1.081 E-6	0.42	7.971 E-8	
350	-1.360 E-3	10.84	1.809 E-5	-1.96	-1.125 E-6	0.45	8.526 E-8	
360	-1.367 E-3	11.21	1.833 E-5	-2.11	-1.169 E-6	0.50	9.090 E-8	
370	-1.373 E-3	11.58	1.856 E-5	-2.27	-1.212 E-6	0.54	9.660 E-8	
380	-1.379 E-3	11.96	1.878 E-5	-2.42	-1.255 E-6	0.58	1.024 E-7	
390	-1.385 E-3	12.36	1.900 E-5	-2.58	-1.297 E-6	0.63	1.082 E-7	
400	-1.391 E-3	12.79	1.921 E-5	-2.74	-1.339 E-6	0.69	1.141 E-7	
410	-1.397 E-3	13.26	1.941 E-5	-2.90	-1.380 E-6	0.74	1.200 E-7	
420	-1.402 E-3	13.80	1.961 E-5	-3.06	-1.421 E-6	0.80	1.260 E-7	
430	-1.407 E-3	14.40	1.980 E-5	-3.23	-1.461 E-6	0.86	1.321 E-7	
440	-1.413 E-3	15.08	1.998 E-5	-3.39	-1.500 E-6	0.92	1.382 E-7	
450	-1.417 E-3	15.83	2.016 E-5	-3.55	-1.540 E-6	0.99	1.443 E-7	
460	-1.422 E-3	16.63	2.033 E-5	-3.71	-1.578 E-6	1.07	1.505 E-7	
470	-1.427 E-3	17.46	2.050 E-5	-3.86	-1.617 E-6	1.14	1.567 E-7	
480	-1.431 E-3	18.26	2.067 E-5	-4.02	-1.655 E-6	1.23	1.630 E-7	
490	-1.435 E-3	18.96	2.083 E-5	-4.17	-1.692 E-6	1.31	1.693 E-7	
500	-1.439 E-3	19.50	2.098 E-5	-4.32	-1.729 E-6	1.40	1.756 E-7	

TABLE 1 (contd.)

(ii) The $E_{\ell+}^{(1)}$ Multipoles

Energy (MeV)	$E_{0+}^{(1)}$		$E_{1+}^{(1)}$		$E_{2+}^{(1)}$		$E_{3+}^{(1)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	4.284 E-2	3.61	2.155 E-3		1.870 E-4		1.943 E-5	
170	4.064 E-2	5.20	2.836 E-3		3.404 E-4		4.901 E-5	
180	3.871 E-2	6.10	3.247 E-3		4.677 E-4		8.089 E-5	
190	3.700 E-2	6.41	3.519 E-3		5.738 E-4		1.125 E-4	
200	3.550 E-2	6.62	3.704 E-3		6.628 E-4		1.427 E-4	
210	3.416 E-2	6.79	3.832 E-3	-0.29	7.375 E-4		1.709 E-4	
220	3.296 E-2	6.95	3.920 E-3	-0.37	8.006 E-4		1.970 E-4	
230	3.187 E-2	7.12	3.979 E-3	-0.46	8.539 E-4		2.210 E-4	
240	3.086 E-2	7.31	4.015 E-3	-0.55	8.991 E-4		2.429 E-4	
250	2.993 E-2	7.52	4.035 E-3	-0.65	9.373 E-4		2.628 E-4	
260	2.905 E-2	7.76	4.042 E-3	-0.76	9.698 E-4		2.808 E-4	
270	2.822 E-2	8.03	4.040 E-3	-0.87	9.973 E-4		2.972 E-4	
280	2.743 E-2	8.33	4.030 E-3	-0.99	1.021 E-3		3.120 E-4	
290	2.667 E-2	8.65	4.014 E-3	-1.12	1.040 E-3	0.26	3.254 E-4	
300	2.594 E-2	9.00	3.994 E-3	-1.25	1.057 E-3	0.29	3.374 E-4	
310	2.523 E-2	9.36	3.969 E-3	-1.38	1.071 E-3	0.32	3.483 E-4	
320	2.454 E-2	9.73	3.942 E-3	-1.52	1.082 E-3	0.35	3.581 E-4	
330	2.387 E-2	10.10	3.913 E-3	-1.66	1.092 E-3	0.38	3.669 E-4	
340	2.321 E-2	10.47	3.882 E-3	-1.81	1.099 E-3	0.42	3.749 E-4	
350	2.257 E-2	10.84	3.850 E-3	-1.96	1.106 E-3	0.45	3.820 E-4	
360	2.195 E-2	11.21	3.816 E-3	-2.11	1.111 E-3	0.50	3.884 E-4	
370	2.135 E-2	11.58	3.782 E-3	-2.27	1.114 E-3	0.54	3.941 E-4	
380	2.076 E-2	11.96	3.748 E-3	-2.42	1.117 E-3	0.58	3.992 E-4	
390	2.019 E-2	12.36	3.713 E-3	-2.58	1.119 E-3	0.63	4.038 E-4	
400	1.963 E-2	12.79	3.678 E-3	-2.74	1.120 E-3	0.69	4.078 E-4	
410	1.909 E-2	13.26	3.643 E-3	-2.90	1.120 E-3	0.74	4.114 E-4	
420	1.857 E-2	13.80	3.608 E-3	-3.06	1.120 E-3	0.80	4.146 E-4	
430	1.807 E-2	14.40	3.573 E-3	-3.23	1.119 E-3	0.86	4.174 E-4	
440	1.759 E-2	15.08	3.538 E-3	-3.39	1.118 E-3	0.92	4.198 E-4	
450	1.712 E-2	15.83	3.503 E-3	-3.55	1.116 E-3	0.99	4.220 E-4	
460	1.667 E-2	16.63	3.469 E-3	-3.71	1.114 E-3	1.07	4.238 E-4	
470	1.624 E-2	17.46	3.435 E-3	-3.86	1.112 E-3	1.14	4.253 E-4	
480	1.582 E-2	18.26	3.401 E-3	-4.02	1.110 E-3	1.23	4.267 E-4	
490	1.542 E-2	18.96	3.368 E-3	-4.17	1.107 E-3	1.31	4.277 E-4	
500	1.504 E-2	19.50	3.335 E-3	-4.32	1.104 E-3	1.40	4.286 E-4	

TABLE 1 (contd.)

(iii) The $E_{\ell+}^{(3)}$ Multipoles

Energy (MeV)	$E_{0+}^{(3)}$		$E_{1+}^{(3)}$		$E_{2+}^{(3)}$		$E_{3+}^{(3)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	-2.015 E-2	- 1.77	-1.045 E-3	0.45	-9.356 E-5		-9.712 E-6	
170	-1.931 E-2	- 2.74	-1.512 E-3	1.31	-1.703 E-4		-2.450 E-5	
180	-1.854 E-2	- 3.65	-1.886 E-3	2.48	-2.340 E-4		-4.044 E-5	
190	-1.781 E-2	- 4.53	-2.191 E-3	3.98	-2.871 E-4		-5.624 E-5	
200	-1.713 E-2	- 5.40	-2.431 E-3	5.83	-3.316 E-4		-7.132 E-5	
210	-1.649 E-2	- 6.25	-2.590 E-3	8.07	-3.689 E-4		-8.544 E-5	
220	-1.588 E-2	- 7.07	-2.702 E-3	10.72	-4.004 E-4		-9.850 E-5	
230	-1.530 E-2	- 7.85	-2.757 E-3	13.84	-4.271 E-4		-1.105 E-4	
240	-1.476 E-2	- 8.59	-2.650 E-3	17.45	-4.496 E-4		-1.214 E-4	
250	-1.425 E-2	- 9.28	-2.641 E-3	21.61	-4.686 E-4		-1.314 E-4	
260	-1.377 E-2	- 9.93	-2.475 E-3	26.37	-4.848 E-4		-1.404 E-4	
270	-1.333 E-2	-10.55	-2.262 E-3	31.78	-4.985 E-4		-1.486 E-4	
280	-1.291 E-2	-11.15	-1.996 E-3	37.91	-5.100 E-4		-1.560 E-4	
290	-1.253 E-2	-11.73	-1.649 E-3	44.80	-5.197 E-4	-0.25	-1.626 E-4	
300	-1.218 E-2	-12.29	-1.211 E-3	53.03	-5.279 E-4	-0.29	-1.687 E-4	
310	-1.186 E-2	-12.86	-8.798 E-4	60.56	-5.346 E-4	-0.32	-1.741 E-4	
320	-1.157 E-2	-13.42	-5.560 E-4	68.43	-5.402 E-4	-0.36	-1.790 E-4	
330	-1.130 E-2	-13.99	-3.004 E-4	74.31	-5.448 E-4	-0.40	-1.834 E-4	
340	-1.106 E-2	-14.57	-1.386 E-4	83.63	-5.485 E-4	-0.44	-1.874 E-4	
350	-1.085 E-2	-15.16	1.194 E-5	90.58	-5.515 E-4	-0.49	-1.910 E-4	
360	-1.066 E-2	-15.76	1.402 E-4	96.90	-5.537 E-4	-0.53	-1.942 E-4	
370	-1.049 E-2	-16.36	2.599 E-4	102.89	-5.554 E-4	-0.58	-1.971 E-4	
380	-1.035 E-2	-16.97	3.707 E-4	108.48	-5.565 E-4	-0.62	-1.996 E-4	0.25
390	-1.022 E-2	-17.58	4.793 E-4	114.31	-5.572 E-4	-0.67	-2.019 E-4	0.27
400	-1.011 E-2	-18.19	6.275 E-4	120.32	-5.574 E-4	-0.72	-2.040 E-4	0.29
410	-1.002 E-2	-18.80	7.278 E-4	124.92	-5.574 E-4	-0.76	-2.058 E-4	0.32
420	-9.941 E-3	-19.40	8.189 E-4	127.66	-5.570 E-4	-0.81	-2.074 E-4	0.35
430	-9.878 E-3	-19.99	8.960 E-4	130.02	-5.563 E-4	-0.85	-2.088 E-4	0.39
440	-9.827 E-3	-20.57	9.606 E-4	132.09	-5.554 E-4	-0.90	-2.100 E-4	0.43
450	-9.786 E-3	-21.13	1.014 E-3	133.93	-5.542 E-4	-0.94	-2.111 E-4	0.47
460	-9.756 E-3	-21.69	1.057 E-3	135.61	-5.529 E-4	-0.98	-2.120 E-4	0.51
470	-9.734 E-3	-22.23	1.093 E-3	137.16	-5.514 E-4	-1.02	-2.128 E-4	0.56
480	-9.721 E-3	-22.76	1.121 E-3	138.62	-5.498 E-4	-1.06	-2.135 E-4	0.61
490	-9.713 E-3	-23.29	1.142 E-3	140.02	-5.480 E-4	-1.10	-2.141 E-4	0.66
500	-9.711 E-3	-23.80	1.158 E-3	141.38	-5.461 E-4	-1.13	-2.145 E-4	0.72

TABLE 1 (contd.)

(iv) The $E_{\ell-}^{(0)}$ Multipoles

Energy (MeV)	$E_{2-}^{(0)}$		$E_{3-}^{(0)}$	
	Real Part	Phase	Real Part	Phase
160	-6.360 E-5		6.121 E-7	
170	-1.224 E-4		1.671 E-6	
180	-1.775 E-4		2.975 E-6	
190	-2.295 E-4		4.448 E-6	
200	-2.788 E-4		6.048 E-6	
210	-3.260 E-4		7.746 E-6	
220	-3.712 E-4		9.522 E-6	
230	-4.145 E-4		1.136 E-5	
240	-4.573 E-4		1.325 E-5	
250	-4.986 E-4		1.519 E-5	
260	-5.391 E-4	0.25	1.715 E-5	
270	-5.789 E-4	0.31	1.915 E-5	
280	-6.183 E-4	0.38	2.117 E-5	
290	-6.573 E-4	0.46	2.321 E-5	
300	-6.963 E-4	0.56	2.526 E-5	
310	-7.352 E-4	0.66	2.733 E-5	
320	-7.743 E-4	0.77	2.941 E-5	
330	-8.138 E-4	0.90	3.149 E-5	
340	-8.537 E-4	1.03	3.358 E-5	
350	-8.942 E-4	1.13	3.567 E-5	
360	-9.354 E-4	1.34	3.776 E-5	
370	-9.776 E-4	1.51	3.985 E-5	
380	-1.021 E-3	1.70	4.193 E-5	0.25
390	-1.065 E-3	1.90	4.402 E-5	0.29
400	-1.111 E-3	2.11	4.609 E-5	0.33
410	-1.159 E-3	2.33	4.816 E-5	0.38
420	-1.208 E-3	2.58	5.023 E-5	0.43
430	-1.259 E-3	2.83	5.228 E-5	0.49
440	-1.313 E-3	3.11	5.433 E-5	0.55
450	-1.369 E-3	3.41	5.637 E-5	0.61
460	-1.427 E-3	3.73	5.839 E-5	0.68
470	-1.489 E-3	4.07	6.041 E-5	0.75
480	-1.554 E-3	4.44	6.241 E-5	0.83
490	-1.622 E-3	4.83	6.441 E-5	0.91
500	-1.695 E-3	5.26	6.638 E-5	0.98

TABLE 1 (contd.)

(v) The $E_{\ell--}^{(1)}$ Multipoles

Energy (MeV)	$E_{2--}^{(1)}$		$E_{3--}^{(1)}$	
	Real Part	Phase	Real Part	Phase
160	1.156 E-3		6.263 E-5	
170	2.131 E-3		1.587 E-4	
180	2.962 E-3		2.629 E-4	
190	3.676 E-3		3.670 E-4	
200	4.294 E-3		4.672 E-4	
210	4.832 E-3		5.617 E-4	
220	5.305 E-3		6.497 E-4	
230	5.722 E-3		7.310 E-4	
240	6.093 E-3		8.059 E-4	
250	6.425 E-3		8.745 E-4	
260	6.724 E-3	0.25	9.373 E-4	
270	6.996 E-3	0.31	9.946 E-4	
280	7.244 E-3	0.38	1.047 E-3	
290	7.473 E-3	0.46	1.095 E-3	
300	7.687 E-3	0.56	1.138 E-3	
310	7.887 E-3	0.66	1.177 E-3	
320	8.076 E-3	0.77	1.213 E-3	
330	8.257 E-3	0.90	1.246 E-3	
340	8.432 E-3	1.03	1.276 E-3	
350	8.602 E-3	1.18	1.303 E-3	
360	8.769 E-3	1.34	1.327 E-3	
370	8.935 E-3	1.51	1.349 E-3	
380	9.101 E-3	1.70	1.369 E-3	0.25
390	9.269 E-3	1.90	1.387 E-3	0.29
400	9.439 E-3	2.11	1.404 E-3	0.33
410	9.613 E-3	2.33	1.419 E-3	0.38
420	9.793 E-3	2.58	1.432 E-3	0.43
430	9.980 E-3	2.83	1.444 E-3	0.49
440	1.017 E-2	3.11	1.454 E-3	0.55
450	1.038 E-2	3.41	1.464 E-3	0.61
460	1.059 E-2	3.73	1.473 E-3	0.68
470	1.082 E-2	4.07	1.480 E-3	0.75
480	1.106 E-2	4.44	1.487 E-3	0.83
490	1.132 E-2	4.83	1.493 E-3	0.91
500	1.159 E-2	5.26	1.498 E-3	0.98

TABLE 1 (contd.)

(vi) The $E_{\ell-}^{(3)}$ Multipoles

Energy (MeV)	$E_{2-}^{(3)}$		$E_{3-}^{(3)}$	
	Real Part	Phase	Real Part	Phase
160	-6.526 E-4		-3.048 E-5	
170	-1.205 E-3		-7.706 E-5	
180	-1.678 E-3		-1.274 E-4	
190	-2.085 E-3		-1.775 E-4	
200	-2.439 E-3		-2.255 E-4	
210	-2.747 E-3		-2.705 E-4	
220	-3.016 E-3		-3.122 E-4	
230	-3.253 E-3		-3.505 E-4	
240	-3.462 E-3		-3.855 E-4	
250	-3.646 E-3		-4.173 E-4	
260	-3.809 E-3		-4.462 E-4	
270	-3.954 E-3		-4.723 E-4	
280	-4.084 E-3		-4.960 E-4	
290	-4.199 E-3		-5.172 E-4	
300	-4.301 E-3		-5.364 E-4	
310	-4.393 E-3		-5.536 E-4	
320	-4.474 E-3		-5.690 E-4	
330	-4.547 E-3		-5.829 E-4	
340	-4.613 E-3	0.25	-5.951 E-4	
350	-4.671 E-3	0.27	-6.061 E-4	
360	-4.722 E-3	0.28	-6.158 E-4	
370	-4.768 E-3	0.30	-6.244 E-4	
380	-4.809 E-3	0.31	-6.320 E-4	
390	-4.845 E-3	0.32	-6.386 E-4	
400	-4.877 E-3	0.33	-6.444 E-4	
410	-4.905 E-3	0.33	-6.494 E-4	
420	-4.929 E-3	0.34	-6.536 E-4	
430	-4.950 E-3	0.35	-6.573 E-4	
440	-4.968 E-3	0.36	-6.603 E-4	
450	-4.984 E-3	0.37	-6.627 E-4	
460	-4.996 E-3	0.38	-6.647 E-4	
470	-5.007 E-3	0.40	-6.662 E-4	
480	-5.015 E-3	0.43	-6.673 E-4	
490	-5.021 E-3	0.45	-6.680 E-4	
500	-5.026 E-3	0.49	-6.683 E-4	

TABLE 1 (contd.)

(vii) The $M_{\ell+}^{(0)}$ Multipoles

Energy (MeV)	$M_{1+}^{(0)}$		$M_{2+}^{(0)}$		$M_{3+}^{(0)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	1.537 E-4		-1.555 E-6		2.232 E-8	
170	2.172 E-4		-3.135 E-6		6.416 E-8	
180	2.658 E-4		-4.741 E-6		1.198 E-7	
190	3.067 E-4		-6.371 E-6		1.875 E-7	
200	3.426 E-4		-8.021 E-6		2.659 E-7	
210	3.748 E-4	-0.29	-9.689 E-6		3.545 E-7	
220	4.042 E-4	-0.37	-1.137 E-5		4.527 E-7	
230	4.314 E-4	-0.46	-1.307 E-5		5.598 E-7	
240	4.567 E-4	-0.55	-1.477 E-5		6.757 E-7	
250	4.803 E-4	-0.65	-1.649 E-5		7.999 E-7	
260	5.026 E-4	-0.76	-1.821 E-5		9.320 E-7	
270	5.236 E-4	-0.87	-1.993 E-5		1.072 E-6	
280	5.435 E-4	-0.99	-2.166 E-5		1.219 E-6	
290	5.624 E-4	-1.12	-2.339 E-5	0.26	1.373 E-6	
300	5.805 E-4	-1.25	-2.513 E-5	0.29	1.534 E-6	
310	5.976 E-4	-1.38	-2.686 E-5	0.32	1.702 E-6	
320	6.140 E-4	-1.52	-2.858 E-5	0.35	1.876 E-6	
330	6.296 E-4	-1.66	-3.031 E-5	0.38	2.056 E-6	
340	6.447 E-4	-1.81	-3.203 E-5	0.42	2.241 E-6	
350	6.591 E-4	-1.96	-3.374 E-5	0.45	2.433 E-6	
360	6.729 E-4	-2.11	-3.545 E-5	0.50	2.630 E-6	
370	6.861 E-4	-2.27	-3.715 E-5	0.54	2.831 E-6	
380	6.988 E-4	-2.42	-3.885 E-5	0.58	3.038 E-6	
390	7.111 E-4	-2.58	-4.054 E-5	0.63	3.250 E-6	
400	7.228 E-4	-2.74	-4.221 E-5	0.69	3.446 E-6	
410	7.342 E-4	-2.90	-4.388 E-5	0.74	3.686 E-6	
420	7.451 E-4	-3.06	-4.554 E-5	0.80	3.911 E-6	
430	7.555 E-4	-3.23	-4.718 E-5	0.86	4.139 E-6	
440	7.656 E-4	-3.39	-4.882 E-5	0.92	4.371 E-6	
450	7.754 E-4	-3.55	-5.045 E-5	0.99	4.607 E-6	
460	7.848 E-4	-3.71	-5.206 E-5	1.07	4.846 E-6	
470	7.938 E-4	-3.86	-5.366 E-5	1.14	5.088 E-6	
480	8.025 E-4	-4.02	-5.525 E-5	1.23	5.334 E-6	
490	8.109 E-4	-4.17	-5.682 E-5	1.31	5.582 E-6	
500	8.190 E-4	-4.32	-5.838 E-5	1.40	5.834 E-6	

TABLE 1 (contd.)

(viii) The $M_{\ell+}^{(1)}$ Multipoles

Energy (MeV)	$M_1^{(1)}$		$M_2^{(1)}$		$M_3^{(1)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	-2.869 E-3		-9.000 E-5		-6.927 E-6	
170	-3.946 E-3		-1.704 E-4		-1.832 E-5	
180	-4.708 E-3		-2.427 E-4		-3.162 E-5	
190	-5.300 E-3		-3.077 E-4		-4.583 E-5	
200	-5.780 E-3		-3.663 E-4		-6.043 E-5	
210	-6.180 E-3	-0.29	-4.192 E-4		-7.510 E-5	
220	-6.520 E-3	-0.37	-4.669 E-4		-8.963 E-5	
230	-6.810 E-3	-0.46	-5.100 E-4		-1.039 E-4	
240	-7.062 E-3	-0.55	-5.490 E-4		-1.178 E-4	
250	-7.281 E-3	-0.65	-5.843 E-4		-1.312 E-4	
260	-7.472 E-3	-0.76	-6.161 E-4		-1.443 E-4	
270	-7.639 E-3	-0.87	-6.450 E-4		-1.568 E-4	
280	-7.786 E-3	-0.99	-6.710 E-4		-1.689 E-4	
290	-7.915 E-3	-1.12	-6.945 E-4	0.26	-1.806 E-4	
300	-8.028 E-3	-1.25	-7.158 E-4	0.29	-1.917 E-4	
310	-8.127 E-3	-1.38	-7.349 E-4	0.32	-2.025 E-4	
320	-8.214 E-3	-1.52	-7.522 E-4	0.35	-2.128 E-4	
330	-8.290 E-3	-1.66	-7.676 E-4	0.38	-2.226 E-4	
340	-8.356 E-3	-1.81	-7.815 E-4	0.42	-2.321 E-4	
350	-8.414 E-3	-1.96	-7.940 E-4	0.45	-2.412 E-4	
360	-8.463 E-3	-2.11	-8.050 E-4	0.50	-2.499 E-4	
370	-8.504 E-3	-2.27	-8.149 E-4	0.54	-2.583 E-4	
380	-8.539 E-3	-2.42	-8.236 E-4	0.58	-2.663 E-4	
390	-8.568 E-3	-2.58	-8.313 E-4	0.63	-2.741 E-4	
400	-8.592 E-3	-2.74	-8.380 E-4	0.69	-2.815 E-4	
410	-8.610 E-3	-2.90	-8.438 E-4	0.74	-2.885 E-4	
420	-8.624 E-3	-3.06	-8.488 E-4	0.80	-2.954 E-4	
430	-8.633 E-3	-3.23	-8.531 E-4	0.86	-3.020 E-4	
440	-8.638 E-3	-3.39	-8.566 E-4	0.92	-3.083 E-4	
450	-8.640 E-3	-3.55	-8.595 E-4	0.99	-3.144 E-4	
460	-8.638 E-3	-3.71	-8.618 E-4	1.07	-3.203 E-4	
470	-8.633 E-3	-3.86	-8.636 E-4	1.14	-3.259 E-4	
480	-8.626 E-3	-4.02	-8.649 E-4	1.23	-3.314 E-4	
490	-8.615 E-3	-4.17	-8.656 E-4	1.31	-3.366 E-4	
500	-8.602 E-3	-4.32	-8.660 E-4	1.40	-3.417 E-4	

TABLE 1 (contd.)

(ix) The $M_{\ell+}^{(3)}$ Multipoles

Energy (MeV)	$M_1^{(3)}$		$M_2^{(3)}$		$M_3^{(3)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	4.267 E-3	0.45	3.195 E-5		3.646 E-6	
170	6.518 E-3	1.31	5.882 E-5		9.688 E-6	
180	8.652 E-3	2.48	8.131 E-5		1.679 E-5	
190	1.069 E-2	3.98	9.989 E-5		2.446 E-5	
200	1.266 E-2	5.83	1.150 E-4		3.241 E-5	
210	1.447 E-2	8.07	1.270 E-4		4.048 E-5	
220	1.648 E-2	10.72	1.362 E-4		4.856 E-5	
230	1.838 E-2	13.84	1.430 E-4		5.658 E-5	
240	2.023 E-2	17.45	1.475 E-4		6.450 E-5	
250	2.201 E-2	21.61	1.499 E-4		7.228 E-5	
260	2.358 E-2	26.37	1.506 E-4		7.990 E-5	
270	2.459 E-2	31.78	1.496 E-4		8.737 E-5	
280	2.495 E-2	37.91	1.471 E-4		9.466 E-5	
290	2.425 E-2	44.80	1.432 E-4	-0.25	1.018 E-4	
300	2.163 E-2	53.03	1.381 E-4	-0.29	1.087 E-4	
310	1.833 E-2	60.56	1.319 E-4	-0.32	1.155 E-4	
320	1.390 E-2	68.43	1.247 E-4	-0.36	1.222 E-4	
330	1.029 E-2	74.31	1.166 E-4	-0.40	1.287 E-4	
340	4.077 E-3	83.63	1.077 E-4	-0.44	1.350 E-4	
350	-3.512 E-4	90.58	9.798 E-5	-0.49	1.412 E-4	
360	-4.005 E-3	96.90	8.760 E-5	-0.53	1.473 E-4	
370	-7.025 E-3	102.89	7.659 E-5	-0.58	1.532 E-4	
380	-8.826 E-3	108.48	6.502 E-5	-0.62	1.590 E-4	0.25
390	-1.042 E-2	114.31	5.293 E-5	-0.67	1.647 E-4	0.27
400	-1.184 E-2	120.32	4.039 E-5	-0.72	1.703 E-4	0.29
410	-1.201 E-2	124.92	2.742 E-5	-0.76	1.758 E-4	0.32
420	-1.232 E-2	127.66	1.408 E-5	-0.81	1.812 E-4	0.35
430	-1.237 E-2	130.02	3.962 E-7	-0.85	1.865 E-4	0.39
440	-1.207 E-2	132.09	-1.359 E-5	-0.90	1.917 E-4	0.43
450	-1.185 E-2	133.93	-2.785 E-5	-0.94	1.969 E-4	0.47
460	-1.158 E-2	135.61	-4.236 E-5	-0.98	2.019 E-4	0.51
470	-1.122 E-2	137.16	-5.708 E-5	-1.02	2.069 E-4	0.56
480	-1.083 E-2	138.62	-7.199 E-5	-1.06	2.118 E-4	0.61
490	-1.026 E-2	140.02	-8.707 E-5	-1.10	2.167 E-4	0.66
500	-9.771 E-3	141.38	-1.023 E-4	-1.13	2.215 E-4	0.72

TABLE 1 (contd.)

(x) The $M_{l-}^{(0)}$ Multipoles

Energy (MeV)	$M_1 -^{(0)}$		$M_2 -^{(0)}$		$M_3 -^{(0)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	-3.303 E-4		3.462 E-7		-3.252 E-9	
170	-4.646 E-4	-0.38	6.937 E-7		-9.279 E-9	
180	-5.677 E-4	-0.63	1.043 E-6		-1.721 E-8	
190	-6.555 E-4	-0.84	1.393 E-6		-2.674 E-8	
200	-7.342 E-4	-1.04	1.744 E-6		-3.767 E-8	
210	-8.070 E-4	-1.21	2.094 E-6		-4.988 E-8	
220	-8.760 E-4	-1.35	2.444 E-6		-6.326 E-8	
230	-9.425 E-4	-1.46	2.792 E-6		-7.773 E-8	
240	-1.008 E-3	-1.54	3.139 E-6		-9.320 E-8	
250	-1.073 E-3	-1.58	3.484 E-6		-1.096 E-7	
260	-1.138 E-3	-1.57	3.827 E-6	0.25	-1.269 E-7	
270	-1.204 E-3	-1.51	4.166 E-6	0.31	-1.450 E-7	
280	-1.272 E-3	-1.40	4.503 E-6	0.38	-1.639 E-7	
290	-1.342 E-3	-1.23	4.837 E-6	0.46	-1.834 E-7	
300	-1.415 E-3	-0.99	5.167 E-6	0.56	-2.037 E-7	
310	-1.491 E-3	-0.66	5.494 E-6	0.66	-2.245 E-7	
320	-1.571 E-3	-0.24	5.817 E-6	0.77	-2.460 E-7	
330	-1.654 E-3	0.28	6.136 E-6	0.90	-2.680 E-7	
340	-1.742 E-3	0.91	6.451 E-6	1.03	-2.904 E-7	
350	-1.833 E-3	1.67	6.762 E-6	1.18	-3.133 E-7	
360	-1.930 E-3	2.56	7.069 E-6	1.34	-3.366 E-7	
370	-2.030 E-3	3.61	7.371 E-6	1.51	-3.604 E-7	
380	-2.135 E-3	4.80	7.669 E-6	1.70	-3.844 E-7	0.25
390	-2.243 E-3	6.16	7.963 E-6	1.90	-4.088 E-7	0.29
400	-2.354 E-3	7.66	8.251 E-6	2.11	-4.335 E-7	0.33
410	-2.467 E-3	9.31	8.536 E-6	2.33	-4.584 E-7	0.38
420	-2.581 E-3	11.09	8.816 E-6	2.58	-4.836 E-7	0.43
430	-2.693 E-3	12.95	9.091 E-6	2.83	-5.090 E-7	0.49
440	-2.802 E-3	14.88	9.361 E-6	3.11	-5.345 E-7	0.55
450	-2.904 E-3	16.81	9.627 E-6	3.41	-5.602 E-7	0.61
460	-2.996 E-3	18.71	9.888 E-6	3.73	-5.861 E-7	0.68
470	-3.074 E-3	20.53	1.015 E-5	4.07	-6.120 E-7	0.75
480	-3.134 E-3	22.22	1.040 E-5	4.44	-6.381 E-7	0.83
490	-3.172 E-3	23.75	1.064 E-5	4.83	-6.642 E-7	0.91
500	-3.185 E-3	25.09	1.089 E-5	5.26	-6.904 E-7	0.98

TABLE 1 (contd.)

(xi) The $M_{\ell-}^{(1)}$ Multipoles

Energy (MeV)	$M_1 -^{(1)}$		$M_2 -^{(1)}$		$M_3 -^{(1)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	2.060 E-3		1.457 E-4		9.077 E-6	
170	2.804 E-3	-0.38	2.770 E-4		2.398 E-5	
180	3.309 E-3	-0.63	3.962 E-4		4.130 E-5	
190	3.685 E-3	-0.84	5.044 E-4		5.978 E-5	
200	3.978 E-3	-1.04	6.031 E-4		7.869 E-5	
210	4.215 E-3	-1.21	6.930 E-4		9.762 E-5	
220	4.410 E-3	-1.35	7.751 E-4		1.163 E-4	
230	4.575 E-3	-1.46	8.502 E-4		1.345 E-4	
240	4.719 E-3	-1.54	9.190 E-4		1.523 E-4	
250	4.847 E-3	-1.58	9.820 E-4		1.694 E-4	
260	4.966 E-3	-1.57	1.040 E-3	0.25	1.858 E-4	
270	5.080 E-3	-1.51	1.093 E-3	0.31	2.016 E-4	
280	5.193 E-3	-1.40	1.142 E-3	0.38	2.167 E-4	
290	5.308 E-3	-1.23	1.186 E-3	0.46	2.312 E-4	
300	5.430 E-3	-0.99	1.228 E-3	0.56	2.450 E-4	
310	5.561 E-3	-0.66	1.265 E-3	0.66	2.581 E-4	
320	5.705 E-3	-0.24	1.300 E-3	0.77	2.707 E-4	
330	5.863 E-3	0.28	1.332 E-3	0.90	2.827 E-4	
340	6.040 E-3	0.91	1.361 E-3	1.03	2.941 E-4	
350	6.237 E-3	1.67	1.388 E-3	1.18	3.050 E-4	
360	6.457 E-3	2.56	1.413 E-3	1.34	3.153 E-4	
370	6.702 E-3	3.61	1.435 E-3	1.51	3.252 E-4	
380	6.972 E-3	4.80	1.456 E-3	1.70	3.346 E-4	0.25
390	7.269 E-3	6.16	1.475 E-3	1.90	3.435 E-4	0.29
400	7.593 E-3	7.66	1.492 E-3	2.11	3.521 E-4	0.33
410	7.943 E-3	9.31	1.507 E-3	2.33	3.603 E-4	0.38
420	8.316 E-3	11.09	1.521 E-3	2.58	3.680 E-4	0.43
430	8.708 E-3	12.95	1.534 E-3	2.83	3.754 E-4	0.49
440	9.114 E-3	14.88	1.546 E-3	3.11	3.825 E-4	0.55
450	9.524 E-3	16.81	1.556 E-3	3.41	3.892 E-4	0.61
460	9.928 E-3	18.71	1.565 E-3	3.73	3.956 E-4	0.68
470	1.031 E-2	20.53	1.573 E-3	4.07	4.018 E-4	0.75
480	1.066 E-2	22.22	1.580 E-3	4.44	4.076 E-4	0.83
490	1.096 E-2	23.75	1.587 E-3	4.83	4.132 E-4	0.91
500	1.119 E-2	25.09	1.592 E-3	5.26	4.186 E-4	0.98

TABLE 1 (contd.)

(xii) The $M_{\ell-}^{(3)}$ Multipoles

Energy (MeV)	$M_{1-}^{(3)}$		$M_{2-}^{(3)}$		$M_{3-}^{(3)}$	
	Real Part	Phase	Real Part	Phase	Real Part	Phase
160	-2.822 E-3		-6.900 E-5		-4.581 E-6	
170	-3.876 E-3		-1.308 E-4		-1.211 E-5	
180	-4.615 E-3	-0.45	-1.864 E-4		-2.088 E-5	
190	-5.185 E-3	-0.77	-2.366 E-4		-3.025 E-5	
200	-5.644 E-3	-1.13	-2.820 E-4		-3.986 E-5	
210	-6.022 E-3	-1.50	-3.230 E-4		-4.950 E-5	
220	-6.339 E-3	-1.84	-3.601 E-4		-5.903 E-5	
230	-6.606 E-3	-2.16	-3.937 E-4		-6.836 E-5	
240	-6.834 E-3	-2.47	-4.242 E-4		-7.743 E-5	
250	-7.028 E-3	-2.77	-4.519 E-4		-8.622 E-5	
260	-7.194 E-3	-3.08	-4.769 E-4		-9.470 E-5	
270	-7.336 E-3	-3.40	-4.997 E-4		-1.029 E-4	
280	-7.456 E-3	-3.74	-5.203 E-4		-1.107 E-4	
290	-7.558 E-3	-4.09	-5.391 E-4		-1.182 E-4	
300	-7.644 E-3	-4.46	-5.560 E-4		-1.254 E-4	
310	-7.715 E-3	-4.83	-5.714 E-4		-1.323 E-4	
320	-7.773 E-3	-5.20	-5.853 E-4		-1.389 E-4	
330	-7.820 E-3	-5.56	-5.978 E-4		-1.452 E-4	
340	-7.856 E-3	-5.91	-6.091 E-4	0.25	-1.512 E-4	
350	-7.882 E-3	-6.23	-6.193 E-4	0.27	-1.570 E-4	
360	-7.901 E-3	-6.52	-6.284 E-4	0.28	-1.625 E-4	
370	-7.911 E-3	-6.79	-6.366 E-4	0.30	-1.677 E-4	
380	-7.914 E-3	-7.02	-6.439 E-4	0.31	-1.728 E-4	
390	-7.910 E-3	-7.22	-6.504 E-4	0.32	-1.776 E-4	
400	-7.901 E-3	-7.41	-6.561 E-4	0.33	-1.822 E-4	
410	-7.885 E-3	-7.59	-6.611 E-4	0.33	-1.866 E-4	
420	-7.865 E-3	-7.76	-6.655 E-4	0.34	-1.908 E-4	
430	-7.840 E-3	-7.93	-6.693 E-4	0.35	-1.948 E-4	
440	-7.811 E-3	-8.12	-6.725 E-4	0.36	-1.987 E-4	
450	-7.777 E-3	-8.31	-6.752 E-4	0.37	-2.024 E-4	
460	-7.740 E-3	-8.53	-6.775 E-4	0.38	-2.059 E-4	
470	-7.699 E-3	-8.77	-6.793 E-4	0.40	-2.093 E-4	
480	-7.655 E-3	-9.03	-6.806 E-4	0.43	-2.125 E-4	
490	-7.608 E-3	-9.32	-6.817 E-4	0.45	-2.156 E-4	
500	-7.558 E-3	-9.64	-6.823 E-4	0.49	-2.186 E-4	

TABLE 2

The leading multipoles at 160 MeV photon lab. energy.

(The units are $\hbar = \mu = c = 1.$)

	E_0+	E_1+	E_2-	M_1-	M_1+
'0' Multipole	-0.129×10^{-2}	0	-0.007×10^{-2}	-0.034×10^{-2}	0.016×10^{-2}
'1' Multipole	4.413×10^{-2}	0.222×10^{-2}	0.119×10^{-2}	0.212×10^{-2}	-0.295×10^{-2}
'3' Multipole	-2.075×10^{-2}	-0.106×10^{-2}	-0.067×10^{-2}	-0.290×10^{-2}	0.439×10^{-2}
π^+ Multipole	2.877×10^{-2}	0.155×10^{-2}	0.078×10^{-2}	0.188×10^{-2}	-0.323×10^{-2}
π^0 Multipole	-0.041×10^{-2}	0.003×10^{-2}	-0.012×10^{-2}	-0.157×10^{-2}	0.210×10^{-2}

FIGURE 1 (i) -- (xiv)

PION-NUCLEON SCATTERING PHASE SHIFTS

FIG. 1(i)

FIG. 1(ii)

FIG. 1(iii)

FIG. 1(iv)

FIG. 1(v)

FIG. 1 (vi)

FIG. 1 (vii)

FIG. 1 (viii)

FIG. 1(ix)

FIG. 1(x)

FIG. 1(xi)

FIG. 1(xii)

FIG. 1(xiii)

FIG. 1(xiv)

FIGURE 2

Comparison of the present solution and the C.G.L.N. type solution
for $M_+^{(3)}$ (— present solution, - - - - C.G.L.N. solution)

FIG. 2

FIGURE 3

The ratio $-E_{1+}^{(3)}/M_{1+}^{(3)}$ for different solutions (— present solution,
- - - Finkler's solution, -.-.-. Korth et al. solution)

FIG. 3

FIGURE 4 (i) - (xxxix)

$\gamma + p \rightarrow \pi^+ + n$: DIFFERENTIAL CROSS-SECTIONS

FIG. 4(vi)

FIG. 4(vii)

FIG. 4(viii)

FIG. 4(ix)

FIG. 4(x)

FIG. 4(x)

FIG. 4 (xix)

FIG. 4 (xviii)

FIGURE 5 (i) - (xxii)

$\gamma + p \rightarrow \pi^+ + n$: DIFFERENTIAL ASYMMETRY RATIO

FIGURE 6 (i) - (vii)

$\gamma + p \rightarrow \pi^+ + n$: EXCITATION CURVES OF ASYMMETRY RATIO

FIG.6 (i)

FIG.6 (ii)

FIG. 6 (iii)

FIG. 6 (iv)

FIG.6(v)

FIG.6 (vi)

FIG.6(vii)

FIGURE 7 (i) - (xvi)

$\gamma + p \rightarrow \pi^+ + n$: DIFFERENTIAL RECOIL NEUTRON POLARIZATION

FIGURE 8 (i) - (xxxix)

$\gamma + p \rightarrow \pi^0 + p$: DIFFERENTIAL CROSS-SECTIONS

FIGURE 9 (i) - (xvi)

$\gamma + p \rightarrow \pi^0 + p$: DIFFERENTIAL ASYMMETRY RATIOS

FIGURE 10

$\gamma + p \rightarrow \pi^0 + p$: 90° EXCITATION CURVE OF ASYMMETRY RATIO

FIG. 10

FIGURE 11 (i) - (xvii)

$\gamma + p \rightarrow \pi^0 + p$: DIFFERENTIAL RECOIL PROTON POLARIZATION

FIGURE 12

$\gamma + p \rightarrow \pi^0 + p$: 90° EXCITATION CURVE OF
RECOIL PROTON POLARIZATION

FIG. 12

FIGURE 13 (i) - (xviii)

$\gamma + n \rightarrow \pi^- + p$: DIFFERENTIAL CROSS-SECTIONS

FIG.13(viii)

FIG.13(vii)

FIGURE 14 (i) - (xviii)

$\gamma + n \rightarrow \pi^- + p$: DIFFERENTIAL ASYMMETRY RATIOS

FIGURE 15 (i) - (xvi)

$\gamma + n \rightarrow \pi^- + p$: DIFFERENTIAL RECOIL PROTON POLARIZATION