Archives

PHOTOPRODUCTION AND ELECTROPRODUCTION OF PIONS
III. ω - AND B EXCHANGE IN LOW ENERGY π^0 PHOTOPRODUCTION

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A B S T R A C T

The inclusion of ω and B exchange in low energy π^0 photoproduction is found to remove a long standing discrepancy between experiment and theoretical prediction. The ωNN coupling constants are found to be in reasonable accord with those obtained elsewhere.

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1. - INTRODUCTION

In a detailed evaluation ¹⁾ of photopion production from fixed- t dispersion relations, Berends, Donnachie and Weaver (BDW) showed that the theoretical results are in good agreement with all experimental data below 500 MeV except for very low energy π^0 production, where there is a distinct discrepancy. As was pointed out by BDW, low energy π^0 photoproduction is, in a sense, a pathological situation in that there are considerable cancellations between multi-poles and any small omission which could not be detected elsewhere shows up readily. BDW suggested that the most likely omission was ω exchange, particularly in view of the success of ω exchange ²⁾, especially if some background B exchange is included ³⁾, in explaining π^0 photoproduction above 2 GeV.

This possibility is now investigated in detail, inserting ω exchange as a simple t -channel pole-term in the fixed- t dispersion relations, leaving the coupling constants free to be determined by the data. The details of the formalism are given in Section 2 and the fit to the data given in Section 3. The possibility of there being some B exchange is considered, but its inclusion improves the fit only marginally. In view of the large mass of the B meson it is not surprising that it is unimportant at low energies. The coupling constants obtained in the fit are discussed in Section 4, and are found to be reasonably in accord with those obtained elsewhere. The possibility of ρ^0 contamination and its effect on the coupling constants is also discussed. The final conclusions are given in Section 5.

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2. - THE FORMALISM

The most direct way in which to study the effect of ω and B exchange on low energy photopion production is to insert them as simple t -channel poles in the fixed- t dispersion relations. To this end, let P_1 and P_2 be the four-momenta of the initial and final nucleons, let Q be the pion four-momenta and K the photon four momenta and $\epsilon^{(\gamma)}$ its polarization vector. Then the $\gamma\pi\omega$ vertex is given by

$$i\lambda\omega\epsilon_{\mu\nu\rho\sigma}\epsilon_{\mu}^{(\gamma)}K_{\nu}Q_{\rho} \quad (2.1)$$

the $NN\omega$ vertex is given by

$$i\bar{u}(P_2)\{g_{\omega}\gamma_{\mu} + g_{\omega_2}\gamma_{\mu\nu}(P_2 - P_1)_{\nu}\}u(P_1) \quad (2.2)$$

and the ω propagator by

$$\frac{\delta\sigma_{\tau} - (Q-K)_{\sigma}(Q-K)_{\tau}/m_{\omega}^2}{(m_{\omega}^2 - t)} \quad (2.3)$$

Only the first term of the propagator contributes to the amplitudes because of the asymmetry of $\epsilon_{\mu\nu\rho\sigma}$, so that the complete ω exchange contribution is given by

$$\frac{\lambda\omega}{t - m_{\omega}^2} \{\epsilon_{\mu\nu\rho\sigma}\epsilon_{\mu}^{(\gamma)}K_{\nu}Q_{\rho}\bar{u}(P_2)[g_{\omega}\gamma_{\sigma} + g_{\omega_2}\gamma_{\sigma\tau}(P_2 - P_1)_{\tau}]u(P_1) \quad (2.4)$$

Expanding the expression (2.4) in terms of the usual invariants ^{*}), the contribution of the ω exchange pole to the dispersion relations for the invariant amplitudes A_1, A_2, A_3, A_4 is

$$A_1 = \frac{\lambda \omega g_{\omega 2} m_\omega^2}{(t - m_\omega^2)} \quad (2.5)$$

$$A_2 = \frac{-\lambda \omega g_{\omega 2}}{(t - m_\omega^2)} \quad (2.6)$$

$$A_3 = 0 \quad (2.7)$$

$$A_4 = \frac{-\lambda \omega g_{\omega 1}}{(t - m_\omega^2)} \quad (2.8)$$

Particular note should be taken of expression (2.5), which is a consequence of considering ω exchange as a pole term in the t channel, i.e., the residue must be taken at the pole value. The usual expression obtained in the literature ^{5), 6), 7)} reads

$$\frac{\lambda \omega g_{\omega 2} t}{(t - m_\omega^2)} \quad (2.9)$$

which arises from the ω exchange Born term. Of course, a similar difference also occurs for the ρ exchange. The latter is a limiting case of the $\pi - \pi$ contribution from the t channel, which is dealt with in Refs. ⁵⁾ and ⁷⁾. The result for A_1 in this approach is a subtraction constant plus a dispersion integral. In a $\delta(t - m_\rho^2)$ function approximation this result reduces to ⁵⁾

$$C_1 + \frac{C_2 t}{(t - m_\rho^2)} \quad (2.10)$$

^{*}) For a discussion of the invariants and the invariant amplitudes, see for example Section 4 of Ref. ⁴⁾.

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where however C_1 is not any longer arbitrary but given by

$$C_1 = -C_2 \quad (2.11)$$

giving back Eq. (2.5).

The $\gamma\pi B$ vertex is given by

$$i\lambda_B F_{\mu\nu} Q_\nu \quad (2.12)$$

where

$$F_{\mu\nu} = \varepsilon_{\mu}^{(\gamma)} K_\nu - \varepsilon_\nu^{(\gamma)} K_\mu \quad (2.13)$$

and the NNB vertex is given by

$$ig_B \bar{u}(P_2) \gamma_5 \not{P}_\mu u(P_1), \quad P = \frac{1}{2}(P_1 + P_2) \quad (2.14)$$

being restricted to only one coupling constant by the requirements of C invariance. The propagator is again given by expression (2.3), and again only the first term contributes, so that the complete B exchange contribution is given by

$$\frac{\lambda_B}{m_B^2 - t} \left\{ ig_B \bar{u}(P_2) \gamma_5 \not{P}_\mu u(P_1) : F_{\nu\mu} Q_\nu \right\} \quad (2.15)$$

which contributes only to the invariant amplitude A_2 , the contribution being

$$\frac{\lambda_B g_B}{2(m_B^2 - t)} \quad (2.16)$$

Of the coupling constants λ_ω , $g_{\omega 1}$, $g_{\omega 2}$, λ_B and g_B only λ_ω is known, being determined from the width of the $\pi\gamma$ decay mode of the ω . The coupling constants $g_{\omega 1}$, $g_{\omega 2}$ and the product $(\lambda_B g_B)$ are to be considered as three independent parameters to be determined by fitting to the low energy photoproduction data.

3. - THE FIT

Since the rôle of the ω and B mesons in π^0 photo-production below 500 MeV is to give a correction which is relatively very small, except in the very low energy region, they were inserted in the fixed- t dispersion relations as simple t -channel poles, added on directly to the invariant amplitudes as calculated by BDW.

The fit to the data was straightforward, a least squares fit being done firstly with only ω exchange (i.e., g_{ω_1} and g_{ω_2} as free parameters) and secondly with ω and B exchange [i.e., g_{ω_1} , g_{ω_2} and $(\lambda_B g_B)$ as free parameters]. A simultaneous fit was made to all data below 220 MeV. This cut-off in energy was chosen as the upper limit at which the discrepancy is noticeable and at which the small corrections induced by ω and B exchange are beginning to be swamped by the rapidly increasing $M_{1+}^{(3)}$ amplitude, which dominates completely at higher energies. The data used were those of Koester et al. ⁸⁾, Goldansky et al. ⁹⁾, Govorkov et al. ¹⁰⁾, and Hitzeroth ¹¹⁾. The Hitzeroth data were new values which supersede those quoted by BDW and are, in fact, considerably changed at some energies.

The inclusion of the ω exchange and the B exchange terms make a dramatic improvement to the fit. Without the vector exchange contributions, χ^2 was more than 500 for 68 data points. With ω exchange only, χ^2 was 61.16 for 66 degrees of freedom and with the addition of B exchange there was a marginal change to a χ^2 of 57.17 for 65 degrees of freedom. The statistical evidence for the B is slight. The product of the coupling constants is non-zero by only slightly more than the statutory three standard deviations (3.19 to be precise!). However, in view of the large mass of the B meson it is not at all surprising that its effect at low energies is small.

The results of the fit are shown in Figs. 1-5 at 160, 165, 180, 200 and 210 MeV, the energies at which most data exist. Of these energies, the most serious gap between the fit and the data appears at 180 MeV, where the data appear to require a steeper drop of the differential cross-section in the forward and backward directions (this is particularly true of the Hitzeroth data). However, if the errors on the theoretical multipoles are taken into account (see BDW for a discussion of this point) then this problem is resolved. The extreme forward point at 180 MeV is, experimentally $0.45 \pm 0.1 \mu\text{b/sr}$ and theoretically $0.60 \pm 0.14 \mu\text{b/sr}$, and the extreme backward point at 180 MeV is, experimentally $0.9 \pm 0.08 \mu\text{b/sr}$, and theoretically $1.09 \pm 0.18 \mu\text{b/sr}$.

4. - THE COUPLING CONSTANTS

The coupling constants of interest are the two ω NN coupling constants, $g_{\omega 1}$ and $g_{\omega 2}$. In evaluating these coupling constants we have taken $T_{\omega\pi\gamma}$ as 1.15 ± 0.15 MeV¹²⁾.

In the fit with ω exchange only we obtain *)

$$\frac{g_{\omega 1}^2}{4\pi} = 42 \pm 13, \quad g_{\omega 2}/g_{\omega 1} = .048 \pm .007 \quad (4.1)$$

and in the fit with ω and B exchange we obtain

$$\frac{g_{\omega 1}^2}{4\pi} = 36 \pm 10, \quad g_{\omega 2}/g_{\omega 1} = .049 \pm .007 \quad (4.2)$$

and

$$(\lambda_B g_B)/(\lambda_\omega g_{\omega 1}) = -1.81 \pm 0.64 \quad (4.3)$$

The values of $g_{\omega 1}^2$ are in reasonable accord with those obtained elsewhere. From their ω Regge pole fit to high energy π^0 photoproduction, Locher and Rollnik²⁾, taking $T_{\omega\pi\gamma}$ as 1.4 MeV, obtained $g_{\omega 1}^2/4\pi = 36$. With $T_{\omega\pi\gamma}$ as 1.15, this increases to 44. From a study of nucleon-nucleon scattering, Bryan and Scott¹³⁾ obtain $g_{\omega 1}^2/4\pi = 21.5$ and Scotti and Wong¹⁴⁾ obtained $g_{\omega 1}^2/4\pi = 16.7$ in their initial fit, but quoted a somewhat lower value later¹⁵⁾.

It is a straightforward matter to extract the ratio $g_{\omega 2}/g_{\omega 1}$ from the results of Ader et al.³⁾ in their Regge pole fit to high energy π^0 photoproduction.

*) The coupling constants are expressed in terms of the inverse pion mass.

This yields

$$g_{\omega 2}/g_{\omega 1} = 0.7 \quad (4.4)$$

although again there are no errors estimated.

Another estimate of the ratio $g_{\omega 2}/g_{\omega 1}$ may be obtained from the nucleon isoscalar form factors if it is assumed that the latter are dominated by ω and ϕ exchange.

The usual electric and magnetic isoscalar form factors $F_1^S(t)$ and $F_2^S(t)$ are given by

$$F_1^S(t) = e \Lambda_\omega \frac{g_{\omega 1}}{m_\omega^2 - t} + \frac{e \Lambda_\phi g_{\phi 1}}{m_\phi^2 - t} \quad (4.5)$$

and

$$F_2^S(t) = e \Lambda_\omega \frac{g_{\omega 2}}{m_\omega^2 - t} + \frac{e \Lambda_\phi g_{\phi 2}}{m_\phi^2 - t} \quad (4.6)$$

where $g_{\omega 1}$, $g_{\omega 2}$ are the coupling constants defined in (2.2), $g_{\phi 1}$, $g_{\phi 2}$ are the corresponding ones for the ϕ meson and $e \Lambda_\omega$, $e \Lambda_\phi$ are respectively the coupling constants for the transitions $\omega \rightarrow \gamma$ and $\phi \rightarrow \gamma$.

A similar expression with ρ exchange would then account for the isovector form factors. Unfortunately a good fit to the experimental data using the exchange of the known vector mesons is not possible¹⁶⁾. In practice one tries to remedy this, e.g., by inclusion of an unknown vector meson ($I=1$) with higher mass¹⁶⁾ or by assuming an extra t -dependence of the vector meson-nucleon vertex¹⁷⁾.

Although these fits do not have the simplest physical picture of exchanges of vector mesons we use them to get some idea of the ratio $g_{\omega_2}/g_{\omega_1}$. Chan et al. ¹⁶⁾ obtain a fit to the form factors ¹⁸⁾

$$G_E^S(t) = \frac{1}{e} \left[F_1^S(t) + \frac{t}{2m} F_2^S(t) \right] \quad (4.7)$$

and

$$G_M^S(t) = \frac{1}{e} \left[F_1^S(t) + 2m F_2^S(t) \right] \quad (4.8)$$

where F_1^S, F_2^S are normalized to the charge and anomalous magnetic moments

$$F_1^S(0) = e, \quad F_2^S(0) = \frac{e}{2m} (\mu_p + \mu_n) \quad (4.9)$$

From their parametrization

$$G_E^S = \frac{\alpha_\omega}{1 - t/m_\omega^2} + \frac{\alpha_\phi}{1 - t/m_\phi^2} \quad (4.10)$$

$$G_M^S = \frac{\beta_\omega}{1 - t/m_\omega^2} + \frac{\beta_\phi}{1 - t/m_\phi^2} \quad (4.11)$$

we find

$$\frac{g_{\omega_2}}{g_{\omega_1}} = \frac{1 - \frac{\alpha_\omega}{\beta_\omega}}{2m \left[\frac{\alpha_\omega}{\beta_\omega} - \frac{m_\omega^2}{4m^2} \right]} \quad (4.12)$$

Chan et al. give the results of two fits to the data. In one of them,

$$\alpha_\omega = 1.214, \quad \beta_\omega = 1.093$$

yielding

$$g_{\omega_2}/g_{\omega_1} = -0.0088 \quad (4.13)$$

and in the other

$$\alpha_\omega = 1.184, \quad \beta_\omega = 1.180$$

yielding

$$g_{\omega 2} / g_{\omega 1} = -0.0003 \quad (4.14)$$

Unfortunately no estimate of errors is given, although the difference between the two results (4.13) and (4.14) is a reasonable guide at least to the order of magnitude.

Massam and Zichichi made a one parameter fit to the data, by using the exchange of ρ , ω and ϕ mesons, SU(3) relations between them, but allowing for a vector meson nucleon vertex of the form

$$\frac{1}{1 - t/\Lambda^2} \quad (4.15)$$

where Λ is the free parameter. From their fit one easily obtains

$$g_{\omega 2} / g_{\omega 1} = 0.274.$$

Some comment on the possibility of ρ^0 contamination in our present results is called for. Since the ρ and ω masses are almost the same, and since the ρ^0 will contribute to the invariant amplitudes $A_1, \dots, A_4$ in exactly the same way as the ω , what is really being determined in the fit is not $(\lambda_\omega g_{\omega 1})$ and $(\lambda_\omega g_{\omega 2})$ but the combinations $(\lambda_\omega g_{\omega 1} + \lambda_\rho g_{\rho 1})$ and $(\lambda_\omega g_{\omega 2} + \lambda_\rho g_{\rho 2})$. Now, $g_{\rho 1} < g_{\omega 1}$ and almost certainly $\lambda_\rho \ll \lambda_\omega$, so that $(\lambda_\rho g_{\rho 1}) \ll (\lambda_\omega g_{\omega 1})$ and no appreciable effect on the results for $g_{\omega 1}$ would therefore be expected. However, since $g_{\omega 2} \ll g_{\omega 1}$ and $g_{\rho 2} > g_{\rho 1}$, there may well be an effect on the results for $g_{\omega 2}$. To make this quantitative, we made use of the results of Donnachie and Shaw²⁰⁾ who have investigated the effect of ρ meson exchange on charged pion production near threshold. Taking their value [which corresponds to an upper limit on the $\rho \rightarrow \pi \gamma$ width of about $0.2^{+0.6}_{-0.2}$ MeV, compared to the lowest upper limit by direct experimental measurement¹⁹⁾ of about 0.7 MeV] one does indeed find that the effect on $g_{\omega 1}$ is negligible, but the effect on $g_{\omega 2}$ is large. Indeed

$$|\lambda_w g_{w2}| \simeq |\lambda_p g_{p2}| \quad (4.16)$$

Since the relative sign of the coupling constants is unknown, this produces two results for g_{w2}/g_{w1} namely

$$g_{w2}/g_{w1} \simeq 0 \pm 0.15 \quad \text{or} \quad g_{w2}/g_{w1} \simeq 0.15 \pm 0.15 \quad (4.17)$$

Clearly in view of the uncertainties in obtaining this ratio, and its small value, one cannot say that there is any obvious inconsistency among the results given in Eqs. (4.7), (4.13), (4.14), (4.16) and (4.17).

5. - CONCLUSIONS

We conclude that ω and B exchange is a very reasonable mechanism to explain the discrepancy between experimental and theoretical calculation in low energy π^0 photoproduction. All the low energy data are fitted collectively extremely well, and the coupling constants obtained are in reasonable agreement with those obtained elsewhere. At higher energies, the modification to the results of BDW will be only slight and, of course, the effect on the results of BDW for charged pion production will be zero, so that no complications are introduced.

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FIGURE CAPTIONS

Figures 1 to 5

The π^0 differential cross-sections at 160, 165, 180, 200 and 210 MeV. The experimental data points are :

Koester et al. 8)	$\triangle$
Goldansky et al. 9)	$\square$
Govorkov et al. 10)	$\bullet$
Hitzeroth 11)	$\odot$

The theoretical fits are :

ω exchange only	————
ω and B exchange	-----

200 MeV

$\gamma + p \rightarrow \pi^0 + p$

210 MeV

$\gamma + p \rightarrow \pi^0 + p$