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CONSPIRACIES AND FORWARD PRODUCTION OF VECTOR MESONS

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ABSTRACT

It is shown that the result of Sawyer, that a conspiring Regge pole (or equivalently a Toller pole) can contribute fully to the amplitude in the forward direction, can be obtained only if the high-energy limit is performed along the boundary curve of the physical region corresponding to $\vartheta_s = 0$.

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It was recently shown by Sawyer^{1,2)} that the predictions of the $O(4)$ symmetry for the forward ρ production

$$\pi + N \rightarrow \rho + N \quad (1)$$

are completely different from those of a single Regge pole exchange.

In fact, a single Regge pole exchange predicts that in the forward direction all the helicity amplitudes, which are allowed by angular momentum conservation, cannot grow with the leading energy behaviour s^α , showing in this way a dip in the differential cross-section near the forward direction.

On the contrary, using group theoretical arguments, Sawyer was able to show that an exchange of single Toller pole³⁾, either of class II or of class III in the Freedman-Wang terminology⁴⁾, gives the following dominant amplitudes in the forward direction

$$\text{Class II} \quad \langle 0, \frac{1}{2} | T | 0, \frac{1}{2} \rangle \sim s^\alpha \quad (2a)$$

$$\text{Class III} \quad \langle 0, -\frac{1}{2} | T | 1, +\frac{1}{2} \rangle \sim s^\alpha \quad (2b)$$

where α is the angular momentum of the leading Regge pole in the relevant Toller pole.

Our aim is to derive Sawyer's results without the use of $O(3,1)$ or $O(4)$ symmetry, but only satisfying the constraints of $t = 0$ directly by conspiracies. In this way we are able to show in which sense Sawyer's results have to be understood.

As it is well known^{5,6)}, the reaction (1) can be described by six independent t -channel helicity amplitudes, from which one can form the following parity-conserving combinations

$$\bar{f}_1^- = f_{\frac{1}{2}, \frac{1}{2}}^{(+)}; 0, 0$$

$$\bar{f}_2^- = \frac{1}{\sin \theta_t} f_{\frac{1}{2}, -\frac{1}{2}}^{(+)}; 0, 0 \quad (3)$$

$$\bar{f}_3^\pm = \frac{1}{\sin \theta_t} \left[f_{\frac{1}{2}, \frac{1}{2}}^{(+)}; 1, 0 \pm f_{\frac{1}{2}, \frac{1}{2}}^{(+)}; -1, 0 \right]$$

$$\bar{f}_4^\pm = \frac{1}{2 \cos^2 \frac{\theta_t}{2}} f_{\frac{1}{2}, -\frac{1}{2}}^{(+)}; 1, 0 \pm \frac{1}{2 \sin^2 \frac{\theta_t}{2}} f_{\frac{1}{2}, -\frac{1}{2}}^{(+)}; -1, 0$$

where θ_t is the angle in the crossed t -channel.

The amplitudes free from kinematical singularities can also be found in the literature^{5,6}); the essential result is that the following most singular behaviours are possible near $t = 0$

$$\bar{f}_1^-, \bar{f}_2^-, \bar{f}_3^-, \bar{f}_4^- \sim (t)^{-\frac{1}{2}} \quad (4a)$$

$$\bar{f}_3^+, \bar{f}_4^-, \text{ regular.} \quad (4b)$$

In order to satisfy the analyticity of the invariant amplitudes, the following constraints at $t = 0$ have to be satisfied^{5,6})

$$\bar{f}_1^- - i \bar{f}_2^- \sim (t)^{\frac{1}{2}} \quad (5a)$$

$$\bar{f}_3^- - i \bar{f}_4^+ \sim (t)^{\frac{1}{2}} \quad (5b)$$

The t-channel quantum numbers which can contribute to the amplitudes involved in the constraints (5a, 5b), if we consider only $I = 1$ exchanges, are given in Table 1.

Table 1

Amplitude	G-Parity	Isospin	Parity	Signature	Particle
$\bar{f}_1^-$	-1	1	-1	even (unnatural parity)	π
$\bar{f}_2^-$	-1	1	+1	odd (unnatural parity)	A_1
$\bar{f}_3^-$	-1	1	-1	even (unnatural parity)	π
$\bar{f}_4^+$	-1	1	+1	even (unnatural parity)	π_c

We shall call π the particle contributing to $\bar{f}_1^-$ and $\bar{f}_2^-$, A_1 that contributing to $\bar{f}_2^-$, π_c that to $\bar{f}_4^+$.

The usual Reggeization of the helicity amplitudes gives

$$\bar{f}_1^- = \beta_1(t) S^{\alpha_\pi} \quad \bar{f}_2^- = \alpha_{A_1} \beta_2(t) S^{\alpha_{A_1} - 1} \quad (6a)$$

$$\bar{f}_3^- = \beta_3(t) S^{\alpha_\pi - 1} \quad \bar{f}_4^+ = \beta_4(t) S^{\alpha_{\pi_c} - 1} \quad (6b)$$

The exponents $\alpha-1$ are due to the fact that the amplitudes $\bar{f}_2^- - \bar{f}_4^+$ are helicity flip; the factor α_{A_1} in front of $\beta_2(t)$ is the sense-nonsense factor present near $\alpha_{A_1} = 0$ (the amplitude $\bar{f}_2^-$ represents there a sense-nonsense transition).

We notice that, at large t , all the amplitudes have their leading behaviour s^α , due to the factors $\sin \vartheta_t$ and $\cos \vartheta_t$, which are proportional to s at large t . At $\vartheta_s = 0$, $\sin \vartheta_t = 0$, $\cos \vartheta_t = 1$, so that

$$f_{\frac{1}{2}, \frac{1}{2}; 0, 0}^{(+)} = f_{\frac{1}{2}, \frac{1}{2}; 1, 0}^{(+)} = f_{\frac{1}{2}, \frac{1}{2}; -1, 0}^{(+)} = f_{\frac{1}{2}, -\frac{1}{2}; -1, 0}^{(+)} = 0 \quad (7)$$

but

$$f_{\frac{1}{2}, \frac{1}{2}; 0, 0}^{(+)} \quad \text{and} \quad f_{\frac{1}{2}, -\frac{1}{2}; 1, 0}^{(+)}$$

can survive.

The constraints (5a, 5b) can be satisfied by evasions, namely allowing only one independent Regge pole. This forces one to have all the residues vanishing separately at $t = 0$

$$\beta_1(t), \beta_2(t), \beta_3^-(t), \beta_4^+(t) \sim (t)^{\frac{1}{2}} \quad (8)$$

and therefore all the amplitudes are depressed near $t = 0$ by this factor; moreover, none of them has the leading s^α behaviour.

We can notice that also the amplitudes $\bar{f}_3^+$ and $\bar{f}_4^-$, which are not involved in the constraints, behave as $s^{\alpha-1}$, and that their residues are regular at $t = 0$.

Suppose now that we satisfy the constraint (5a) by a conspiracy; it obviously requires $\alpha_\pi(0) = \alpha_{A_1}(0) - 1$ and this is just a class II conspiracy⁴⁾, which connects the parent and the first daughter trajectories, which have the same conjugation, but opposite parity and signature.

This conspiracy allows now the singular behaviour for β_1 and β_2 in agreement with (4a), $\beta_1, \beta_2 \sim (t)^{-1/2}$, so that

$$\bar{f}_2^- \sim -\frac{\alpha_{A_1}}{\sqrt{t}} s^{\alpha_{A_1}(0)-1}, \quad \bar{f}_1^- \sim \frac{\alpha_{A_1}}{\sqrt{t}} s^{\alpha_\pi(0)} = \frac{\alpha_{A_1}}{\sqrt{t}} s^{\alpha_{A_1}(0)-1} \quad (9)$$

Since at $\vartheta_s = 0$ the crossing matrix is diagonal, we have finally

$$f_{0, \frac{1}{2}; 0, \frac{1}{2}}^{(s)} = \bar{f}_{0, 0; \frac{1}{2}, \frac{1}{2}} \sim \frac{\alpha_{A_1}}{\sqrt{t}} S^{\alpha_{A_1}(0) - 1} \quad (10)$$

Now, how can we get the Sawyer result, (2a)?

$$f_{0, \frac{1}{2}; 0, \frac{1}{2}}^{(s)} \sim \alpha_{A_1} S^{\alpha_{A_1}}$$

The answer is that one has to perform the limit $s \rightarrow \infty$ along the curve $\vartheta_s = 0$, namely in the forward direction of the s-channel. Therefore, in all the expressions regular at $t = 0$, as $\alpha(t)$, we can directly write $\alpha(0)$, but we have to be careful with expressions singular at $t = 0$, as the coefficient $(t)^{-1/2}$. At large s , the relation connecting s and t for $\vartheta_s = 0$ is, in the unequal-equal mass case,

$$s^2 t + m^2 (M^2 - \mu^2)^2 = 0 \quad (11)$$

(m, M, μ are the nucleon, ρ and π masses) and so

$$s \sim \frac{1}{\sqrt{t}} \quad (12)$$

Therefore, along the curve $\vartheta_s = 0$, the factor $1/\sqrt{t}$ adds a power of s , and we have finally the Sawyer result

$$f_{0, \frac{1}{2}; 0, \frac{1}{2}}^{(s)} \sim \alpha_{A_1} S^{\alpha_{A_1}}$$

In a similar way, the constraint (5b) can only be satisfied by a parity doublet (it links amplitudes with opposite natural parities); the conditions are [remember again (4a)]

$$\alpha_{\pi}(0) = \alpha_{\pi_c}(0) \quad ; \quad \beta_3^-(t) \sim \frac{\zeta_3^-}{\sqrt{t}}, \quad \beta_4^+(t) \sim -i \frac{\zeta_3^-}{\sqrt{t}} \quad (13)$$

and again, taking into account the crossing properties and the conditions $\cos \vartheta_t/2 = 1$, $\sin \vartheta_t/2 = 0$ for $\vartheta_s = 0$, we find that the amplitude

$$f_{0, -\frac{1}{2}; 1, \frac{1}{2}}^{(s)} = f_{\frac{1}{2}, -\frac{1}{2}; 1, 0}^{(t)} \sim s^{\alpha_{\pi}(0)} \quad (14)$$

at $\vartheta_s = 0$.

This is now the leading behaviour since in the parity doublet α_{π} is the maximum value of α (in class II it was $\alpha_{A_1} = \alpha_{\pi} + 1$).

The same argument obviously applies to the forward photoproduction, when only class III can be present⁷⁾.

The moral of the story is that the leading behaviour is obtained by a conspiracy only in the forward direction, namely along the curve $\vartheta_s = 0$, and not at fixed t ; at fixed t the amplitude shows always the non-leading behaviour $s^{\alpha-1}$. This result has to be taken into account when one speaks of the presence or the absence of dips in the forward direction.

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