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ON THE COHERENT PRODUCTION OF RESONANCES ON DEUTERIUM
AT HIGH ENERGIES

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In a recent paper ¹⁾ it has been discussed how to obtain particle scattering amplitudes from nuclear scattering processes, in particular from coherent production of resonances on ${}^4\text{He}$. The main point of the analysis in Ref. ¹⁾ is that, looking to the differential cross-section of the processes

$$\begin{matrix} \text{(pion)} \\ \text{(proton)} \end{matrix} + {}^4\text{He} \rightarrow \begin{pmatrix} M \\ B \end{pmatrix} + {}^4\text{He} \quad (1)$$

(where M and B are respectively any meson or baryon resonance), one can get information about the total cross-section of the

$$\begin{pmatrix} M \\ B \end{pmatrix} + \text{nucleon} \quad (2)$$

scattering, and about the ratio ρ between the real and imaginary parts of the scattering amplitude of the elastic

$$\begin{pmatrix} M \\ B \end{pmatrix} + \text{nucleon} \rightarrow \begin{pmatrix} M \\ B \end{pmatrix} + \text{nucleon} \quad (3)$$

scattering. This result is obtained from the Glauber ²⁾ expansion of the high-energy scattering amplitude on a complex target (light nucleus), which predicts that the different terms in the multiple scattering expansion of any high-energy scattering matrix do interfere, giving rise both to minima in the angular distribution of a process like (1) and to different behaviours of the differential cross-section in different intervals of the momentum transfer, where one particular term of the multiple scattering expansion is the dominant one.

Such a method has already been fruitfully applied to the analysis of existing elastic p -d ^{3),4)} and p - ${}^4\text{He}$ ⁵⁾ experiments; further experiments in order to test the Glauber theory for π -d and π - ${}^4\text{He}$ scattering are planned at CERN ⁶⁾.

The most striking result is that, after a steep decrease of the differential cross-section at small $|t|$ (where the single scattering term is the dominant one, and which is due to the fast decrease with $|t|$ of the target form factor), there will be a minimum and a subsequent flattening of the differential cross-section.

The value of the differential cross-section at the minimum depends upon the value of g , and the position of the dip upon the value of the total $\left(\frac{M}{B}\right)$ -nucleon cross-section. The subsequent shoulder depends mostly on the double scattering term; unfortunately, at least in the case of the ${}^4\text{He}$ target, at the energy of the existing experiment the theoretical explanation of the second maximum is not unique, and depends probably on details of the two-nucleon correlation at very small distances ⁷⁾. On the contrary, the agreement between theory and experiment for the secondary maximum in p-d scattering is rather good ^{8),4)}.

In the case of ${}^4\text{He}$ target there is also the possibility of triple and quadruple scattering, but the corresponding cross-sections are extremely small, so that no reliable information on particle scattering amplitudes can be extracted from that part of the angular distribution in which those terms do dominate.

The point we would like to stress in this short note is that the measurement of the angular distribution of reaction (1) cannot give a good determination of a third interesting parameter of the process (3) namely of the width c of the forward elastic peak of reaction (3) ^{*)} while c can be determined from the corresponding reaction on the deuteron

*) We use heavily here as much as possible all the symbols and notations of Ref. 1), of which this note can be considered as a complement.

The reason of this different behaviour of reactions (3) and (4) is that the only way c comes into the theoretical description of reaction (3) (when we limit ourselves to the interval of $|t|$ where only single and double scattering are important) is in fixing both the slope and the value of the double scattering term (and so of the cross-section on the secondary maximum), but always through the combination $R^2 + 2c$, where R is the ${}^4\text{He}$ "radius", as defined in (1). Since $R^2 \simeq 50(\text{GeV}/c)^{-2}$ and it is reasonable to expect a c of the order of $10(\text{GeV}/c)^{-2}$, we can understand why c will be always badly determined from reaction (1). On the contrary, the slope of the double scattering term in reaction (4) is quite sensitive to the value of c , and therefore the value of the differential cross-section in the region of the secondary maximum of reaction (4) will be strongly dependent on it. An accurate measurement of the differential cross-section of reaction (1) at high energies in the range of momentum transfer between 0.2 and $1.5(\text{GeV}/c)^2$ should give rather good information about the slope of the diffraction peak of (3).

Again, as for the ${}^4\text{He}$ target, the position of the minimum and the cross-section at the minimum in the angular distribution of (4) will give information on the total cross-section (2) and on the ratio. Therefore, a combined measurement of reactions (1) and (4) is of extreme importance. A measurement of the elastic π^-d angular distribution in the same range of $|t|$ will also help in fixing the deuteron parameters.

The result we have pre-announced above can be easily obtained by applying the Glauber expansion to reaction (4). We use the same notation as in Ref. ¹⁾, namely

$$f = \frac{\sigma \cdot p}{4\pi} (i + \alpha) e^{-\alpha \delta^2/2}$$

is the $\begin{smallmatrix} \text{(pion)} \\ \text{(nucleon)} \end{smallmatrix}$ -nucleon scattering amplitude, σ the corresponding total cross-section, p the incident momentum, δ^2 the momentum transfer.

(5.a)

$$h = \frac{\sigma_s \cdot p}{4\pi} (i + \gamma) e^{-c \delta^2/2}$$

is the $(\frac{M}{B})$ -nucleon scattering amplitude,
 σ_s the corresponding total cross-section.

(5.b)

$$g = \frac{G \cdot p}{4\pi} (i + \gamma) e^{-b \delta^2/2}$$

is the amplitude for the process
 (pion) + nucleon $\rightarrow$ $(\frac{M}{B})$ + nucleon

(5.c)

All the discussions of Ref. 1) concerning these parametrizations or the approximations connected with the Glauber expansion are also relevant here, so we shall not repeat them. We only say again that we assume the proton and the neutron to be identicals for the purpose of calculating the scattering.

In principle, the values of σ , α , a , G , γ and b can be obtained from the experiments on free nucleons [as in (1), only the combination $G^2(1+\gamma^2)$ is relevant], or, as it is the case when α varies strongly with δ^2 , from the elastic scattering on d or ^4He .

A straightforward calculation, corresponding to taking into account the three diagrams in Fig. 1, gives the result for the amplitude $t(\delta^2)$ of reaction (4) *)

*) Strictly speaking, the Glauber formula for the double scattering amplitude cannot directly be applied to the diagram of Fig. 1c, since in the intermediate state one has the propagation of an unstable particle, which can have the possibility of decaying before suffering the second scattering. The correction due to the finite width of the resonance and the mass difference between the resonance and the incident particle can be easily handled, for instance using the Feynman diagram technique ⁹⁾, and keeping the contribution of the (complex) pole of the propagator of the intermediate resonance. The result is that one has to replace the mean value

$$\langle \frac{1}{k^2} \rangle = \frac{1}{2\pi} \int d_2 q G(q^2)$$

by the expression

$$\left\langle \frac{e^{i \vec{q}_2 \cdot \vec{x}}}{k^2} \right\rangle = \frac{1}{2\pi} \int d q_x d q_y G(q_x, q_y, q_z^0); \quad q_z^0 = \frac{M_0^2 - m_x^2 - i \gamma_0 M_0}{2 p}$$

where m_x is the mass of the incident particle, M_0 and γ_0 the position and half width of the resonance. One can check that this correction is negligible already when the incident energy is of the order of few GeV.

$$t(\delta^2) = 2 g(\delta^2) G(\frac{\delta^2}{4}) + \frac{i}{2\pi p} \int d_2 \vec{q} G(q^2) g((\vec{q} - \frac{\vec{\delta}}{2})^2) \left(f((\vec{q} + \frac{\vec{\delta}}{2})^2) + h((\vec{q} - \frac{\vec{\delta}}{2})^2) \right) \quad (6)$$

where $G(\delta^2/4) = \int d_3 x i^{\frac{1}{2}} \vec{\delta} \cdot \vec{x} / 2 \psi_d^2(x)$ is the deuteron form factor, and therefore for the differential cross-section

$$\frac{d\sigma}{d\delta^2} \left((\pi)_p + d \rightarrow (M)_B + d \right) = \frac{d\sigma_n}{d\delta^2} \cdot \left| 2 G(\frac{\delta^2}{4}) + \frac{i e^{b\delta^2/2}}{\delta \pi^2} \int d_2 \vec{q} G(q^2) e^{-\frac{b}{2}(\vec{q} - \frac{\vec{\delta}}{2})^2} \left(\sigma(i+d) e^{-\frac{a}{2}(\vec{q} + \frac{\vec{\delta}}{2})^2} + \sigma(i+g) e^{-\frac{c}{2}(\vec{q} - \frac{\vec{\delta}}{2})^2} \right) \right|^2 \quad (7)$$

where $d\sigma_n/d\delta^2$ is the differential cross-section of the reaction $(\pi)_p + p \rightarrow (M)_B + p$. One can easily convince oneself that, unless a , b and c are extremely different the one from the others, the slope of the cross-section due to only the double scattering term (which, we repeat, is the dominant one on the secondary maximum) depends essentially only on $a+b$ and $b+c$ (remember that in the elastic case, where $a=b=c$, the double scattering differential cross-section behaves as $e^{-a\delta^2/2}$ *).

*) One could ask the question why, also in the elastic scattering, the slope of the double scattering term on the deuteron is independent of the deuteron radius, while on ^4He it depends strongly on the α particle radius. The answer is that, in the Glauber expansion, for the ^4He once the momentum values of the two nucleons participating into the double scattering are specified, still the momenta of the remaining nucleons can be varied, and the probability of a given momentum distribution of the two first nucleons depends therefore on the motion of the two extra nucleons. On the contrary, in the deuteron problem, the specification of the momenta of both the nucleons completely specifies the kinematics. All what remains is the sum over all the specified configurations, which just defines the form factor.

As a matter of fact, also in the ^4He problem the quadruple scattering term (in which the momenta of all the four nucleons are specified in the sense discussed just below) has a slope independent of the ^4He radius (cf. e.g. formula II.5 of Ref. 1)).

As an example, we can take a Gaussian form factor,
 $G(\delta^2/4) = e^{-b_1 \delta^2/4}$ with $b_1 \approx 33(\text{GeV}/c)^{-2}$, then the double scattering cross-section behaves as

$$\left(\sigma \exp\left(-\frac{b_1(a+b)+2ab}{4(2b_1+a+b)}\right) + \tilde{\sigma}_g \exp\left(-\frac{b_1(c+b)+2cb}{4(2b_1+c+b)}\right) \right)^2$$

In Fig. 2, we report the calculation of the process $\pi + d \rightarrow g + d$, using the Gartenhaus ¹⁰⁾ wave function for the deuteron and the parameters

$$\sigma = 30 \text{ mb} \quad a = b = 8 (\text{GeV}/c)^{-2}$$

$$\frac{|G(1-i\delta)|^2}{16\pi} = 1 \text{ mb} / (\text{GeV}/c)^2$$

$\tilde{\sigma}_g = 31 \text{ mb}$ (as obtained in Ref. ¹¹⁾ from the g^0 photoproduction on nuclei) and taking for simplicity $\alpha = g = -0.2$. We show there the differential cross-section for three values of c , namely $c = a/2$, $c = a$, $c = 2a$, showing the sensitivity of the cross-section on the secondary maximum on the value of c . We can notice that the value of δ^2 at which the minimum in the cross-section shows up is considerably lower than the corresponding value in the elastic process $\pi + d \rightarrow \pi + d$. The reason is that, if $\sigma = \tilde{\sigma}_g$, the relative contribution of the double scattering amplitude with respect to the single scattering amplitude is here twice as big as for the elastic scattering, since one has to take into account the two diagrams 1.b and 1.c in Fig. 1, in which the pion and the g are the intermediate particles. In the elastic case one has only the diagram with the intermediate pion, that with the intermediate g being neglected since it is proportional to the second power of the small quantity G . The same phenomenon is quite general and is present also in Ref. ¹⁾ for the production on ^4He .

Finally, we would like to remark that similar information can be also obtained from an accurate measurement of the momentum transfer distribution of the reaction

$$\begin{pmatrix} \pi \\ p \end{pmatrix} + d \rightarrow \begin{pmatrix} M \\ B \end{pmatrix} + p + n$$

if the kinematical variables specifying the final proton and neutron are all measured (as proposed in a recent paper ¹²⁾ by one of the authors, for the process in which $M = \pi^-$ or $B = p$, namely the coherent break-up of the deuteron).

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a)

b)

c)

FIG. 1

FIG. 2