OBSERVATION OF TIME-DEPENDENT K_S AND K_L
INTERFERENCE IN THE $\pi^+\pi^-$ DECAY CHANNEL
FROM AN INITIAL K_0 STATE

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ABSTRACT

The phase φ_{+-} between the amplitudes $K_L \rightarrow \pi^+\pi^-$ and $K_S \rightarrow \pi^+\pi^-$ has been measured. The result is $46^\circ \pm 15^\circ$ for a $K_L - K_S$ mass difference $\Delta m/\hbar = (0.526 \pm 0.020) \times 10^{10} \text{s}^{-1}$, in good agreement with the "superweak model" of Wolfenstein which predicts $\varphi_{+-}^{\text{SW}} = 42.6^\circ \pm 1.0^\circ$.

Geneva - May 1968

(To be submitted to Physics Letters)

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We are reporting an experiment on time-dependent interference effects in the $\pi^+\pi^-$ decay channel from a K_0 (strangeness = +1) initial state. We have measured the phase φ_{+-} between the $K_L \rightarrow \pi^+\pi^-$ and $K_S \rightarrow \pi^+\pi^-$ decay amplitudes:

$$|\eta_{+-}| e^{i\varphi_{+-}} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} . \quad (1)$$

The present work extends previous investigations^{1,2,14,15)} on interference effects after a thick regenerator, except that no regenerator has been used in this experiment.

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From CPT invariance it follows that³⁾

$$|K_0\rangle = \frac{1}{2p} \left[|K_L\rangle + |K_S\rangle \right] \quad (2)$$

$$\text{and } |\bar{K}_0\rangle = \frac{1}{2q} \left[|K_L\rangle - |K_S\rangle \right]$$

that is $|K_0\rangle$ and $|\bar{K}_0\rangle$ states can be described as superposition of the long- and short-lived states with equal amplitudes and respectively equal and opposite relative phases. The time dependence of the decay rate in the $\pi^+\pi^-$ channel $I(t)$ from a state K_0 or $\bar{K}_0$ at $t = 0$ is given by:

$$I(t) = e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2A|\eta_{+-}| e^{-(\Gamma_L + \Gamma_S)t/2} \cos(\Delta m t - \varphi_{+-}) \quad (3)$$

where

$\Delta m = m_{K_L} - m_{K_S}$ is the $K_L - K_S$ mass difference,

Γ_L and Γ_S are the long-lived and short-lived decay rates, and

A is a parameter equal to +1 for a K_0 state and -1 for a $\bar{K}_0$ state.

If, as for the present experiment, the initial state is an incoherent mixture of K_0 and $\bar{K}_0$ then

$$A(p) = \frac{S(p) - \bar{S}(p)}{S(p) + \bar{S}(p)}, \quad (4)$$

where $S(p)$ and $\bar{S}(p)$ are the K_0 and $\bar{K}_0$ yields at the angle at which the observation is made and at kaon momentum p .

Neutral K mesons were produced from a tungsten target by proton collisions at a laboratory momentum of 19.2 GeV/c. In order to have a large interference term we have chosen the relatively large production angle of 140 mrad and $p > 4.5$ GeV/c, for which one could guess $A \sim 0.8$ from K^+ and K^- production yields⁴⁾ assuming

$$\frac{S(p)}{\bar{S}(p)} \simeq \frac{S_{K^+}(p)}{S_{K^-}(p)}. \quad (5)$$

The interference term is dominant in the proper-time region around $t = 2 \ln (|\eta_{+-}|)/(\Gamma_L - \Gamma_S) \simeq 12/\Gamma_S$. Therefore the experiment was designed to permit observation of $K \rightarrow \pi^+ \pi^-$ decays in the time interval $6 < t\Gamma_S < 16$.

The experimental arrangement is shown in Fig. 1. A small fraction of the slow extracted proton beam from the CERN Proton Synchrotron ($\simeq 1$ to 2×10^{10} p/pulse) was focused on a 10 mm high, 6 mm wide and 70 mm long tungsten target. A thick shield protected the detecting equipment from the strong radiation background produced by the proton beam. A channel, of 2×5 cm² cross-section and 220 cm length, passing through the shield, was provided at 140 mrad average angle to the proton beam line to define the secondary neutral K beam. A vertical magnetic field of about 18 kG all along the channel swept away charged particles. In order to eliminate the γ -ray background, about 10 radiation lengths of lead were placed inside the channel at 60 cm from the target. Regeneration in the lead introduces a phase shift of about 2.5° .

Both charged particles from K_0 decays occurring between 237 cm and 580 cm from the centre of the production target were detected by a magnetic spectrometer with optical spark chambers. As in our previous work¹⁾, the magnetic field integral in the spectrometer magnet was arranged so as to compensate the transverse momentum of the pion pair from the K_0 decay. Events in which both particles emerged parallel within about $\pm 2.3^\circ$ after the magnet, were selected by an 8-channel counter hodoscope. The normal trigger logic required in addition a veto condition at the anticoincidence counter in front of the decay volume, and a charged particle at each side of the decay volume in front of the first spark chamber before the magnet.

In order to minimize effects of the direct neutral beam, all counters and spark chambers, with the exception of the veto counter, were split vertically into two halves to let the main neutral beam through the central blanked-out region.

Electrons from $K \rightarrow \pi e \nu$ decays were identified by a CO₂ threshold gas Čerenkov counter, 200 cm long, inserted between the first and second layer of the hodoscope counters. The counter was viewed by four 58 AVP

photomultipliers selected for high gain and very-low dark current. Measurements carried out both before and after the experiment in an electron beam gave a detection efficiency uniform over the acceptance of the hodoscope and $\geq 98\%$ for relativistic particles. In these conditions the over-all feedthrough for π^+ having momentum less than 5.0 GeV/c was determined to be less than 10^{-2} .

Muon signatures from $K \rightarrow \pi\mu\nu$ decays were obtained by a threefold scintillation counter coincidence after traversing an iron shield, 140 cm thick, corresponding to a threshold of 1.7 GeV/c. The estimated efficiency was about 90%. Muon signatures and $\checkmark$ Čerenkov pulse heights were directly recorded on each picture.

Accidental counts were monitored continuously in the veto, muon and $\checkmark$ Čerenkov channels by scaling random coincidences one PS radio-frequency cycle apart. The proton beam intensity was often adjusted so as to keep the instantaneous beam-rate to a sufficiently low value.

About 211,000 pictures have been collected during 5.4×10^5 CERN-PS pulses. The pictures have been scanned and measured on the cathode-ray digitizer, Luciole⁵⁾, at the rate of one thousand frames/hour. Out of this sample, 122,000 events were accepted by the pattern recognition programme requiring at least one track in each side of the chambers. The V^0 was spatially reconstructed and momenta of both particles determined. After fiducial volume cuts, rejection of multiple solutions and accidental tracks, 118,000 events were retained.

Out of these, 53,000 events were identified as $K \rightarrow \pi^+ e^+ \nu$ by a count in one side of the $\checkmark$ Čerenkov counter by a particle of less than 5.0 GeV/c. These events were used to determine the sum of the K_0 and $\bar{K}_0$ production spectra. Each event can be fitted with two different K_0 incident momenta $p^{(1)}$ and $p^{(2)}$ corresponding to alternatives in which the neutrino is emitted in different directions in the centre-of-mass system with same momentum and same component transverse to the K_0 direction of flight. Ambiguities of solutions have been handled with a simple iterative procedure, in which to each solution has been attributed a weight proportional to the a priori probability that the true K_0 momentum of the observed event is $p^{(1)}$ or $p^{(2)}$ respectively. The procedure is rapidly

convergent since the trigger logic is highly selective to events in which a small momentum is carried away by the neutrino. The resulting sum of the K_0 and $\bar{K}_0$ spectra is shown in Fig. 2.

Let us now consider $K \rightarrow \pi^+ \pi^-$ events. Only events with visible momentum in a ± 2.5 GeV/c interval around $p = 7.0$ GeV/c have been retained. A total of 16,900 events with neither electron nor muon signature survive the momentum cuts.

For each event the following quantities have been evaluated:

- a) The invariant mass of the charged particle pair,

$$m_{\pi\pi} = \sqrt{(E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2}$$

where $E_{\pm}^2 = m_{\pi}^2 + p_{\pm}^2$ is the total energy of each track, computed under the assumption of a pion,

- b) The cosine of the angle Θ_K between the sum of the vector momenta of the pair

$$\vec{P}_{\pi\pi} = \vec{p}_+ + \vec{p}_-$$

and the presumed direction of flight of the neutral particle, $\vec{u}_K$, oriented as a line passing through the centre of the target and the decay vertex,

$$\cos \Theta_K = \frac{\vec{P}_{\pi\pi} \cdot \vec{u}_K}{|\vec{u}_K| \cdot |\vec{P}_{\pi\pi}|}.$$

The invariant mass distribution for various intervals of the variable Θ_K^2 is shown in Fig. 3. A peak corresponding to the K_0 mass appears for events in a narrow cone around $\Theta_K \simeq 0$, as expected from $K \rightarrow 2\pi$ decays from target.

Two additional small effects can be observed:

- I) a broad background incompatible with $K \rightarrow 2\pi$ in the mass distribution, most likely due to neutron stars, and to leptonic decays which escape detection in the Čerenkov counter or in the muon telescope;

II) a peak around the K_0 mass value in the mass distribution of events with $\Theta_K \neq 0$, due to $K \rightarrow 2\pi$ events in which the kaon has scattered.

In order to correct for background of class (I), only events within $\pm 10 \text{ MeV}/c^2$ from the K_0 mass have been retained, and one half of the sum of events in the intervals $-30 \text{ MeV}/c^2$ to $-10 \text{ MeV}/c^2$ and $+10 \text{ MeV}/c^2$ to $+30 \text{ MeV}/c^2$ from the K_0 mass have been subtracted out. This subtraction is about 3% of the counts in the first bin $\Theta_K^2 \leq 10^{-5}$.

Events of class (II) are understood as 2π decays of a neutral kaon scattered somewhere in the collimator walls and for which the reconstructed direction of flight is not pointing to the production target. Because of measurement errors a fraction of scattered events may fall into the sample of good events. The angular distribution of the scattered events has been evaluated by a Monte Carlo calculation in which scattering and regeneration effects have been included. The error in the determination of the K_0 direction has been evaluated by taking into account uncertainties due to

- a) target size,
- b) Coulomb scattering in the apparatus, mainly in the front coincidence counters, and
- c) errors in the reconstruction of the spark position, the precision in the position determination of each spark has been determined using single high-energy muon tracks traversing the spectrometer with no analysing magnetic field; the result is $\pm (0.3 \pm 0.03) \text{ mm}$ in the horizontal plane and $\pm (0.60 \pm 0.06) \text{ mm}$ in the vertical plane.

After subtraction of the background of class (I) the measured distribution in Θ_K^2 is well described as the sum of two curves: a narrow one at $\Theta_K^2 = 0$, smeared by the predicted resolution function; and another one, interpreted as background events, with intensity and angular distribution in good agreement with the Monte Carlo calculation of effect (II). The extrapolated background subtraction in the sample of events for which $\Theta_K^2 < 3 \times 10^{-5}$ amounts approximately to 4%.

Finally, in order to eliminate K_S coherently regenerated in the collimator material (for which the reconstructed flight path would still point to the target), only events emerging at more than 2.5 mm inside the

walls of the collimator have been retained, and an additional small correction has been applied.

Data have been divided into separate (p, t) bins, in each of which the above indicated background subtractions have been performed. The final sample consists of 3,560 events. A fit was attempted to a distribution of the form

$$N(p, t) = \text{constant} \times \left[S(p) + \bar{S}(p) \right] \epsilon(p, t) I(t) \quad (6)$$

where $I(t)$ has the form of Eq. (3) in which $A(p)$ is defined as in Eq. (4) and $\epsilon(p, t)$ is the geometrical detection efficiency evaluated with a Monte Carlo calculation, in which measurement errors and multiple scattering effects are included. The calculation was verified by comparison with distributions of many quantities in the cases of 17,700 $K \rightarrow \pi e \nu$ decays and of 5,000 $K_S \rightarrow \pi^+ \pi^-$ after a 20 cm thick copper regenerator placed in two different positions along the decay volume. The agreement between the Monte Carlo calculation and the experimental data is excellent.

The K_0 production spectrum, $S(p)$ was parametrized around the average momentum $\langle p \rangle = 6.5$ GeV/c with an expression of the form

$$S(p) = S(\langle p \rangle) \exp [-a(p - \langle p \rangle) - b(p - \langle p \rangle)^2] \quad (7.1)$$

The corresponding expansion for the small $\bar{K}_0$ contamination was limited to the linear term in the exponential,

$$\bar{S}(p) = \bar{S}(\langle p \rangle) \exp [-\bar{a}(p - \langle p \rangle)] . \quad (7.2)$$

It has been checked that the conclusions of the present paper are not sensitive to this parametrization.

The chi-squared function used has the form

$$\chi^2 = \chi^2(2\pi) + \chi^2(\pi e \nu) + \chi^2(\eta_{+-}, \Gamma_S, \Delta m), \quad (8)$$

where $\chi^2(2\pi)$ is the chi-squared of the $K \rightarrow 2\pi$ data in all (p, t) bins compared with Eq. (6); $\chi^2(\pi e \nu)$ is the chi-squared of the spectrum as

determined from $K \rightarrow \pi e \nu$ decay events and the sum of spectra (7.1) and (7.2); and $\chi^2(\eta_{+-}, \Gamma_S, \Delta m)$ constrains the parameters of the fit to the values and errors,

$$\eta_{+-} = (1.95 \pm 0.072) \times 10^{-3} \text{ } ^6)$$

$$\Gamma_S = (1.144 \pm 0.014) \times 10^{10} \text{ sec}^{-1} \text{ } ^7)$$

and

$$\Delta m = (0.460 \pm 0.017) \times \Gamma_S \hbar \text{ } ^8).$$

We wish to make the following remarks:

- i) An over-all best fit is found in which all three contributions to the chi-squared are separately acceptable. The results of the fit are summarized in Table 1. The decay time distribution of $K \rightarrow 2\pi$ events after summing over momentum bins is shown in Fig. 4. Finally in Fig. 5 data have been manipulated in order to display the interference term.
- ii) The best fit for the spectra $S(p) + \bar{S}(p)$ are shown in Fig. 2. They are very similar to the corresponding production spectra for charged kaons measured for 23.0 GeV/c protons on beryllium at a production angle of 157 mrad^4). On this basis we discard the alternative $\bar{S}(p) \gg S(p)$. This choice determines the sign of the phase ϕ_{+-} with respect to the sign of Δm .
- iii) The result of the experiment is (in degrees)^{*)}

$$\phi_{+-} = + (46 \pm 15)^\circ \text{ sign } (\Delta m) .$$

The main single contribution to the quoted error is, as expected, due to the indetermination in the mass difference Δm . For different values of Δm the result can be stated as

$$\phi_{+-} = \left[(46 \pm 9)^\circ + 590^\circ \times (\Delta m - 0.46) \right] \text{ sign } (\Delta m)$$

where Δm is expressed in units of $\hbar\Gamma_S$, and the quoted error does not include the error in Δm .

*) Preliminary results have been reported at the Heidelberg International Conference on Elementary Particles. Since then errors in the background subtractions and in the fitting program have been found. Although the quoted result accidentally agrees with the present one, Figs. 5 and 6 of Ref. 6 should be disregarded.

- iv) The present result is in good agreement with the prediction of the "superweak model" of Wolfenstein⁹⁾, $\phi_{+-} = \tan^{-1} (2\Delta m/\Gamma_S) = 42.6^\circ \pm 1.0^\circ$. The somewhat larger value $\phi_{+-} = 84^\circ \pm 18^\circ$ ²⁾ obtained combining the earlier regeneration experiments¹⁾ and K^+ , K^- total cross-section measurements on copper is not confirmed. The regeneration amplitude for copper determined from the 5000 $K_S \rightarrow \pi^+\pi^-$ events comes out $|f(0) - \bar{f}(0)|/k = 2.22 \pm 0.2 \text{ Fermi}^2$ at $\langle p \rangle = 2.7 \text{ GeV/c}$, that is substantially larger than the one $|f(0) - \bar{f}(0)|/k = 1.71 \pm 0.11 \text{ Fermi}^2$ employed in Ref. 2. This new value can increase the regeneration phase by an amount sufficient to remove the discrepancy. However we do not wish to revise the value of Ref. 2 at least until the reason for the discrepancies on the regeneration amplitude is fully understood.
- v) Using the value $\phi_{+-} = 84^\circ \pm 18^\circ$, a satisfactory one constraint fit to the Wu-Yang triangle¹⁰⁾ has been reported by Bennett et al.¹¹⁾ based on the experimental information available at that time^{12, 13)}

$$2\eta_{+-} + \eta_{00} = 3\epsilon \quad (9)$$

where $\epsilon = (p - q)/(p + q)$ with p, q defined in Eq. (2)

$$|\eta_{00}| = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}$$

and the condition $\arg(\epsilon) = \tan^{-1} (2\Delta m/\Gamma_S)$ following from unitarity, CPT and from neglecting CP violating decay amplitudes in all modes other than the two pions. Repeating the same fit for $\phi_{+-} = 46^\circ \pm 15^\circ$, it gives the much larger chi-squared of 10.5, corresponding to a confidence level of 10^{-3} .

In order to improve the fit, either a smaller value for $|\eta_{00}|$ or a different value for δ are required. If the experimental information of $|\eta_{00}|$ is ignored, we obtain the prediction $|\eta_{00}| = (0.7^{+1.0}_{-0.7}) \times 10^{-3}$. Alternatively starting from $|\eta_{00}|$ and ϕ_{+-} we get $\delta = (4.0 \pm 0.2) \times 10^{-3}$ or $\delta = (-0.1 \pm 0.2) \times 10^{-3}$.

Acknowledgements

We would like to thank Professor J. Steinberger who participated in the design and in the earlier part of this experiment, and Professors W. Paul, P. Preiswerk and H. Faissner for support and encouragement. We acknowledge the assistance of Mr. J. Daub and Dr. P. Zanella, who made the measurement of the events on Luciole possible. Dr. L. Caneschi has helped in the running of the experiment. The detection apparatus was built with the help of Messrs. F. Blythe, K. Bussmann, J.M. Fillot and G. Miratori. Finally, we would like to thank Dr. G. Petrucci, the CPS staff and especially Dr. L. Hoffmann for the setting up and operation of the slow ejected proton beam.

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Table 1

Results of least square fit. $\chi^2 = \chi^2(2\pi) + \chi^2(\pi\pi) + \chi^2(\eta_{+-}, \Gamma_s, \Delta m)$

Quantity	Result	Remarks
φ_{+-} (degrees)	46 ± 15	
$ \eta_{+-} $	0.00189 ± 0.00010	Constrained by $\chi^2(\eta_{+-}, \Gamma_s, \Delta m)$ to $ \eta_{+-} = (1.95 \pm 0.072) \times 10^{-3}$
Δm (h Γ_s)	0.460 ± 0.017	Constrained by $\chi^2(\eta_{+-}, \Gamma_s, \Delta m)$ to $\Delta m = (0.460 \pm 0.017) \text{ h}\Gamma_s$
Γ_s (10^{10} s^{-1})	1.144 ± 0.019	Constrained by $\chi^2(\eta_{+-}, \Gamma_s, \Delta m)$ to $\Gamma_s = (1.144 \pm 0.014) 10^{10} \text{ s}^{-1}$
$\bar{S}(\langle p \rangle) / S(\langle p \rangle)$	$0.15^{+0.21}_{-0.15}$	Ratio of $\bar{K}_0$ and K_0 spectra at $\langle p \rangle = 6.5 \text{ GeV}/c$
a (GeV/c^{-1})	0.82 ± 0.12	} $S(p) = S(\langle p \rangle) \exp(-a[p - \langle p \rangle] - b[p - \langle p \rangle]^2)$
b (GeV/c^{-2})	0.059 ± 0.034	
$\bar{a}$ (GeV/c^{-1})	1.09 ± 0.19	
$\chi^2(2\pi)$	136.5	$\bar{S}(p) = \bar{S}(\langle p \rangle) \exp(-\bar{a}[p - \langle p \rangle])$ expected value 136
$\chi^2(\pi\pi)$	20.5	expected value 18
$\chi^2(\eta_{+-}, \Gamma_s, \Delta m)$	0.7	

Figure captions

- Fig. 1 : Experimental set-up.
- Fig. 2 : Production spectrum at 140 mrad of neutral kaons from proton-tungsten collisions deduced from the $K \rightarrow \pi e \nu$ data. Also shown (solid lines) are $S(p) + \bar{S}(p)$ and $\bar{S}(p)$, where $S(p)$ and $\bar{S}(p)$ are the fitted K_0 and $\bar{K}_0$ spectra respectively. They are compared with the K^+ and K^- spectra of Ref. 4 (dashed lines), for 23 GeV/c protons on beryllium at 157 mrad production angle. An over-all normalization between the (K^+, K^-) spectra and the $(K_0, \bar{K}_0)$ spectra has been introduced.
- Fig. 3 : Invariant mass distribution of the charged pion pair for various intervals of the variable Θ_K^2 .
- Fig. 4 : Experimental data and best fit for proper decay-time distributions of $K \rightarrow 2\pi$ events. The time dependence of the detection efficiency calculated by a Monte Carlo method is also shown (dashed line).
- Fig. 5 : Experimental data and best fit time distributions manipulated to display the interference term $\cos(\Delta mt - \varphi_{+-})$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5