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DIVERGENCES OF WEAK NON-LEPTONIC AMPLITUDES

AND THE BREAKING OF $SU(3) \times SU(3)$

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It was generally believed that the strong interactions could provide a natural cut-off in hadronic weak amplitudes. This question has been reconsidered recently, using current algebra techniques ¹⁾⁻⁴⁾. The result was that these amplitudes remain divergent even if the strong interactions are taken into account in all orders. Therefore by looking at specific processes one can derive an "intrinsic cut-off" of weak interactions. An order of magnitude estimation of this, rather ill-defined, parameter gave surprisingly low values. For example the K_1-K_2 mass difference leads to a cut-off of the order of 4 GeV ^{3),4)}. In most of the above quoted investigations, it was argued that in the high energy limit, the chiral symmetry becomes exact, and therefore, the weak hadronic current can be assumed to be conserved. In the present paper we shall examine the effects of the $SU(3) \times SU(3)$ breaking on the divergences of the weak non-leptonic processes. We shall show that, in the general case, the symmetry breaking introduces higher order divergences. However, these terms can be eliminated provided one makes specific assumptions concerning the transformation properties under $SU(3) \times SU(3)$ of the symmetry breaking Lagrangian. We shall work in the framework of the intermediate boson theory.

Let us start by considering the non-leptonic weak decays in second order of the semi-weak coupling constant $f \cdot (f^2/m_W^2 = G/\sqrt{2})$. To this order, the S matrix element for the process $\alpha \rightarrow \beta$ can be written in the form :

$$S_{\alpha \rightarrow \beta}^{(2)} = -\frac{1}{2} \int d^4x_1 d^4x_2 \langle \beta | T(\mathcal{L}_w(x_1), \mathcal{L}_w(x_2)) | \alpha \rangle \quad (1)$$

where the Lagrangian $\mathcal{L}_w(x)$ is given by :

$$\mathcal{L}_w(x) = f J_\mu(x) W^\mu(x) + h.c. \quad (2)$$

$J_\mu(x)$ is the weak Cabibbo current ^{*)} and $W_\mu(x)$ is the intermediate boson field. Substituting (2) into (1) and performing the contraction of W field operators we find :

*) Our results can also be obtained in a theory with neutral currents for example the model proposed by d'Espagnat ⁵⁾.

2.

$$S_{\alpha \rightarrow \beta}^{(2)} = -\frac{f^2}{2} \int d^4x_1 d^4x_2 \Delta^{\mu\nu}(x_1 - x_2) \left\{ \langle \beta | T(J_\mu^+(x_1), J_\nu(x_2)) | \alpha \rangle \right. \\ \left. + \langle \beta | T(J_\mu(x_1), J_\nu^+(x_2)) | \alpha \rangle \right\} \quad (3)$$

where $\Delta^{\mu\nu}(x_1 - x_2)$ is the W propagator

$$\Delta^{\mu\nu}(x) = \frac{1}{2} \left(g^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{m_W^2} \right) \Delta_F(x) \quad (4)$$

The most divergent part of $S_{\alpha \rightarrow \beta}$ is obtained by keeping only the $\partial^\mu \partial^\nu$ part of the propagator. Partial integration yields :

$$S_{\alpha \rightarrow \beta}^{(2)} = \frac{1}{4} \int d^4x_1 d^4x_2 \Delta_F(x_1 - x_2) \left\{ \langle \beta | T(\partial^\mu J_\mu^+(x_1), \partial^\nu J_\nu(x_2)) | \alpha \rangle \right. \\ + \langle \beta | T(\partial^\mu J_\mu(x_1), \partial^\nu J_\nu^+(x_2)) | \alpha \rangle \\ + \frac{2}{\partial x_{1\mu}} \left(\delta^4(x_1 - x_2) \langle \beta | J_\mu'(x_1) | \alpha \rangle \right) \\ \left. + i \delta^4(x_1 - x_2) \langle \beta | (\Sigma(x_1) + \Sigma^+(x_2)) | \alpha \rangle \right\} \quad (5)$$

where, neglecting gradient terms,

$$\delta(x_0 - y_0) [J_0(x), J_\mu^+(y)] = J_\mu'(x) \delta^4(x - y)$$

$$\delta(x_0 - y_0) [J_0^+(x), \partial^\mu J_\mu(y)] = i \Sigma(x) \delta^4(x - y) \quad (6)$$

If the matrix element of the T product of the two divergences appearing in the first term of Eq. (5) satisfies an unsubtracted dispersion relation according to Bjorken's hypothesis ⁶⁾, this term diverges at most logarithmically. The second term vanishes identically. The third term, integrated over $d^4x_1 d^4x_2$ reads :

$$\frac{i}{4}(2\pi)^4 \delta^4(p_\alpha - p_\beta) \Delta_F(0) \langle \beta | (\Sigma(0) + \Sigma^+(0)) | \alpha \rangle \quad (7)$$

The factor $\Delta_F(0)$ diverges quadratically ⁷⁾. Therefore, we conclude that the non-conservation of the weak current introduces a quadratic divergence in the lowest order non-leptonic amplitudes, while in the case of a conserved current the divergence is only logarithmic provided that the matrix element of the T product of two currents satisfies Bjorken's hypothesis.

In the rest of this Section, we shall try to calculate the coefficient of the quadratically divergent term under definite assumptions concerning the symmetry breaking interaction.

$$1) - \underline{|\Delta S| = 1}$$

We shall sketch the proof of the following statement : if the chiral symmetry is broken by a local Lagrangian which transforms according to the $(3, \bar{3})$ representation of the algebra, the $|\Delta S| = 1$ part of the operator $\Sigma(x) + \Sigma^+(x)$ is proportional to a four-divergence.

The most general form of the symmetry breaking Hamiltonian belonging to a $(\bar{3}, 3)$ representation can be written symbolically as :

$$H_1 = \sum_{i=1}^3 \epsilon_i L^i R_i + h.c. \quad (8)$$

In fact we need only to know the matrix element, Eq. (7), at zero momentum transfer. Therefore, it is only necessary to calculate the operator $\Sigma(x)$ integrated over all three-space. Equation (6) gives :

$$\begin{aligned}
\int \vec{\Sigma}(\vec{x}, t) d^3x &= -i [W^\dagger(t), \dot{W}(t)] \\
&= [W^\dagger(t), [H_1(t), W(t)]]
\end{aligned}
\tag{9}$$

where $W(t)$ represents the space integral of the zero component of the weak current. We assume that the formal expression of the currents in terms of the fundamental fields remains unchanged by the presence of H_1 and, therefore, at equal times, the commutation relations are unaffected. Under this assumption the computation of the double commutator (9) is straightforward and can be rewritten into the following form :

$$\begin{aligned}
\int \left(\vec{\Sigma}(\vec{x}, t) + \vec{\Sigma}^\dagger(\vec{x}, t) \right) d^3x &= \\
&= -4 \cos \theta \sin \theta \left[\lambda \int d^3x \partial_\mu V^\mu{}^7 + \frac{1}{\lambda} \int d^3x \partial_\mu A^\mu{}^7 \right]
\end{aligned}
\tag{10}$$

where

$$\lambda = \frac{\epsilon_2 + \epsilon_3}{\epsilon_2 - \epsilon_3}$$

and V_μ^7 and A_μ^7 are the seventh components of the vector and axial currents respectively. In the limit of exact $SU(3)$ $\epsilon_1 = \epsilon_2 = \epsilon_3$ $\lambda \rightarrow \infty$ and $\partial^\mu V_\mu^7 \rightarrow 0$. Therefore, Eq. (10) does not give any information about the parity conserving part of $\vec{\Sigma}$. However, in this limit the parity conserving non-leptonic weak amplitudes are not well defined anymore.

Equation (10) shows that the coefficient of the quadratically divergent term vanishes for all physical matrix elements between states of the same energy momentum.

In current algebra calculations, one finds relations between amplitudes for processes $\alpha \rightarrow \beta + \pi$ with a soft final pion and $\alpha \rightarrow \beta$ with $p_\alpha \neq p_\beta$. These matrix elements are therefore quadratically divergent.

Using the same technique as above one can verify that the coefficients of the divergent terms in both sides of these equations are the same. Therefore, it is always possible to define an extrapolation for soft pions where the divergent terms never appear and a meaningful approximation can be obtained for the physical amplitude.

$$2) - \underline{|\Delta S| = 0}$$

Using the Hamiltonian (8), one can derive by the same method the following results : the parity violating strangeness conserving part of the operator $\sum$ vanishes identically. Therefore there are no quadratically divergent terms in these amplitudes. On the other hand, the parity conserving part of $\sum(x)$ is different from zero and is not related to a four-divergence. Hence, these amplitudes will contain, in general, quadratically divergent terms.

$$3) - \underline{|\Delta S| = 2}$$

In this Section, we shall examine the fourth order transitions with $|\Delta S| = 2$. The only interesting example is the K_1-K_2 mass difference. The most divergent part of the amplitude is :

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \Delta_F(x_1-x_2) \Delta_F(x_3-x_4) \partial_1^\mu \partial_2^\nu \partial_3^\rho \partial_4^\sigma$$

$$\langle \alpha | T (J_\mu(x_1), J_\nu^\dagger(x_2), J_\rho(x_3) J_\sigma^\dagger(x_4)) | \beta \rangle \quad (11)$$

The case of the conserved current has been examined by Low³⁾ and Mohapatra et al.⁴⁾. They have shown that the most divergent part of the matrix element is of the Λ^2 type. If the current is not conserved a great number of new quadratically divergent terms are present and, in general, there is a possibility of Λ^4 terms, coming from the factor $[\Delta_F(0)]^2$. The coefficients of these terms have one of the following forms :

$$\begin{aligned}
(i) \quad & \langle \alpha | [F_L^a, \dot{F}_L^b] | \beta \rangle \\
& \langle \alpha | [F_L^a, [F_L^b, \dot{F}_L^c]] | \beta \rangle \\
& \langle \alpha | [F_L^a, [F_L^b, [F_L^c, \dot{F}_L^d]]] | \beta \rangle
\end{aligned}$$

where

$$F_L^a = \frac{1}{2} \int (V_0^a(\vec{x}, 0) - A_0^a(\vec{x}, 0)) d^3x \quad \alpha = 1, \dots, 8$$

and

$$(ii) \quad \int \langle \alpha | T(\Sigma(x) + \Sigma^\dagger(x), \Sigma(0) + \Sigma^\dagger(0)) | \beta \rangle d^4x$$

The terms of type (i) can be eliminated if one assumes that the chiral symmetry breaking Hamiltonian belongs to the representation $(\bar{3}, 3)$, since, in this case, all multiple commutators remain inside the $(\bar{3}, 3)$ representation and, therefore, contain no $|\Delta S| = 2$ part. Under the same assumption, the term (ii), using Eq. (10) and performing the partial integrations, can be written as a sum of terms of type (i) :

$$\begin{aligned}
\sim \langle \alpha | \left\{ \lambda^2 [F^7, \dot{F}^7] + \frac{1}{\lambda^2} [F_5^7, \dot{F}_5^7] + \right. \\
\left. + [F^7, \dot{F}_5^7] + [F_5^7, \dot{F}^7] \right\} | \beta \rangle
\end{aligned} \quad (12)$$

Therefore we conclude that if the symmetry breaking Hamiltonian transforms according to the $(\bar{3}, 3)$ representation of the chiral group the fourth order $|\Delta S| = 2$ amplitude does not contain any Λ^4 divergence. Notice that this is not true for the $|\Delta S| \leq 1$ fourth order transitions.

All the above results depend crucially on the assumption that the symmetry breaking Hamiltonian belongs to the $(\bar{3}, 3)$ representation. For example, in the $(1, 8) + (8, 1)$ case the results on the $\Delta S = 0$ parity

violating amplitudes do not hold any more. For higher representations, like $(\bar{6},6)$, none of the results derived in this paper holds true. Notice that the elimination of the divergences obtained here does not occur if one allows symmetry breaking Hamiltonians having components in different representations. For example no linear combination of $(\bar{3},3)$ and $(1,8)+(8,1)$ can be used except in the special case where the couplings of the singlet and octal parts of $(\bar{3},3)$ have the ratio $c_8/c_0 = -\sqrt{2}$.

Note added in proof

After completion of the above work we received the letter by V.S. Mathur and P. Olesen [Phys.Rev.Letters 20, 1527 (1968)] where the same divergences are discussed. However, their point of view is that the divergent $\sum$ terms remain and actually give the dominant contribution being responsible for the octet enhancement in weak decays. With our method, we have shown that, at least in our model, this cannot be the case.

R E F E R E N C E S

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