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A REGGEISED MULTIPERIPHERAL MODEL FOR INELASTIC PROCESSES
AT HIGH ENERGY

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A B S T R A C T

By supplementing the multi-Regge model with the assumption that the structure of non-resonant low mass clusters is governed only by phase space, we have constructed a phenomenological model for production processes which allows us to calculate single-particle distributions for varying multiplicity and energy. We have applied the model to analyse data from :

$$(a) \quad \pi^{\pm} p \longrightarrow p + (n-1) \pi,$$

and

$$(b) \quad K^{-} p \longrightarrow \Lambda + (n-1) \pi,$$

at multiplicities n ranging from three to nine, and at energies ranging from 5 to 16 GeV/c. Good qualitative agreement with experiment is obtained for the distributions of both baryons and pions in

$$(i) \quad \cos \theta^* \quad (ii) \quad p_L^* \quad (iii) \quad p_T.$$

The model is, at least in the cases examined, particularly successful in describing the interesting ways in which the distributions vary with respect to the multiplicity, the incoming energy, and the quantum numbers of the final particles.

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1. INTRODUCTION

In this paper we propose a phenomenological model for reactions of the type:

$$A + B \rightarrow 1 + 2 + 3 + \dots \quad (1)$$

in collisions of strongly interacting particles at energies $\gtrsim 5$ GeV/c. Our purpose is a general qualitative description of as wide a range of data as possible, and not, at present, an accurate parametrization of any particular reaction.

The model is based on our present knowledge of high energy reactions of the type:

$$A + B \rightarrow 1 + 2. \quad (2)$$

Recent experiments on these have shown several important facts ¹⁾:

- A. the differential cross-sections show forward or backward peaks depending on whether the quantum numbers of the t and u channels admit the exchange of a single particle or resonance;
- B. the peaks are roughly exponential in the four-momentum transfers t or u ;
- C. cross-sections have a power dependence on the energy, $\sim s^{2\alpha-2}$ where α is characteristic of the exchange quantum numbers.

These features are in general agreement with the Regge theory of elementary particles, though there still remain some apparent discrepancies requiring further clarification.

This success of the Regge model in two-body collisions leads naturally to an attempt at its generalization to production processes of type (1). Such a multi-Regge model with more than one Regge pole exchange has been considered theoretically by many authors in the asymptotic region ²⁾-19). In the special case of reactions with three bodies in the final state, some applications to actual data analysis have been made with encouraging results ¹¹⁾, 14), 20)-22). These considerations, however, apply only to events in that region of phase space where every pair of the final particles has a high effective mass. At present experimental energies, even for the most favourable cases, only a small fraction of production events satisfy such a criterion. For our present purpose, therefore, a pure multi-Regge model is insufficient; it has to be supplemented by some account of those regions of phase space where the final particles emerge in clusters with low effective masses.

The particles within a cluster with low effective mass have small momenta relative to one another. They thus stay together for a long time and interact strongly via "final state interactions". If the mass and quantum numbers of the cluster are such as to correspond exactly to a resonant state of its components, the cluster may be treated simply as a quasi-particle; this is familiar already in the analysis of quasi-two-body reactions. In general, however, the interaction within the cluster is much more complicated, producing what is normally described as a "smooth featureless background". Of these interactions we know very little. We shall then simply admit our ignorance and assume that the structure of such a non-resonant cluster be governed only by phase space; i.e., we shall replace by an effective constant that part of the multi-particle amplitude which corresponds to interactions within the cluster.

Such an assumption, when coupled with the multi-Regge model, has several attractive features. Consider, for example, what happens when we keep the incoming energy s fixed and increase the multiplicity. We note first the trivial identity:

$$s = \sum_{i>j} s_{ij} - (n-2) \sum_j m_j^2 \quad (3)$$

where $s_{ij} = (p_i + p_j)^2$. When the multiplicity is low, the incoming energy s is shared among only a small number of s_{ij} 's, all of which will then have a good chance simultaneously to attain large values. The particles are all at high energy with respect to one another and the amplitude becomes fully reggeised. We expect then the multiperipheral picture to prevail. As the multiplicity increases, the same s has to be shared between more and more s_{ij} 's, some of which are thus forced to be small. This results in an increasing tendency for cluster formations, with each cluster (or fireball) decaying isotropically in its own centre-of-mass. Finally, when the multiplicity is so high that none of the s_{ij} 's has a chance to be large, all the final particles form a single cluster, yielding a picture close to "statistical equilibrium". Conversely, if the multiplicity is held fixed and s allowed to increase, one passes gradually from "statistical equilibrium" to "fireballs", and from "fireballs" to "multiperipheralism". The model will combine in effect the features of all these pictures, and give an automatic transition from one picture to the other as the energy or multiplicity varies, in a manner indicated by experiment.

In the next Section we shall suggest a simple parametrization of the multiparticle amplitude which smoothly interpolates between the fully reggeised amplitude and the amplitude assumed for cluster formations. Though crude and fairly arbitrary, this parametrization allows us to study in terms of a few parameters the many interesting ways in which the spectra of final particles depend on the incoming energy, the multiplicity and the quantum numbers.

The method is applied to two particular classes of reactions for which good data exist ^{23), 24)}

$$(a) \quad \pi^\pm p \rightarrow p + (n-1)\pi \quad (4)$$

$$(b) \quad K^- p \rightarrow \Lambda + (n-1)\pi \quad (5)$$

So far, only single particle distributions have been studied. The reactions (a) have been analysed at 8 GeV/c for multiplicity n ranging from three to nine, and for multiplicity six at energies ranging from 5-16 GeV/c. It is found that our model gives a qualitatively correct description of the following:

- i. the c.m. angular distributions of protons and pions for varying multiplicity and incoming energy;
- ii. the longitudinal momentum distributions of protons and pions for varying multiplicity;
- iii. the average transverse momenta of protons and pions as functions of multiplicity;
- iv. the average transverse momenta of protons and pions as functions of their longitudinal momenta at a fixed multiplicity.

For reactions (b) the collection of data is much poorer in statistics and is available only for multiplicities varying from three to six at two energies, namely 6²⁴⁾ and 10²⁵⁾ GeV/c. They are, however, particularly interesting for our model for studying the dependence of the final particles' spectra on their quantum numbers. For example, the angular and longitudinal momentum distributions are found to be markedly different - i. between the protons of reactions (a) and the Λ 's of reactions (b), - ii. between the π^+ and π^- of reactions (b). These differences are correctly reproduced by the model.

We remark that although the present analysis is far from testing the full implication of the Regge model for production processes, the three features A.-C. of the model enumerated above, which were found to be so successful in two-body reactions, have entered essentially in our analysis in order to give an adequate description of the data.

Also, it shall be obvious from what follows that the model has been deliberately simplified so as to cover the wide range of data we wished to study. In its present form, therefore, it cannot be expected to give an accurate quantitative description of any particular reaction. It is hoped, however, that there is sufficient room left in the model for elaboration to achieve that end. One example of such, namely the inclusion of the important effects of resonance production, is given as an illustration.

2. PARAMETRIZATION OF THE AMPLITUDE

We wish to parametrize the amplitude for a reaction of type (1) in such a way that:

- i. whenever the effective mass of every pair of the final particles is large, the amplitude becomes fully reggeised;
- ii. whenever any group of the final particles forms a cluster with low effective mass, that part of the amplitude which corresponds to interactions within the cluster is replaced by a constant;
- iii. the amplitude interpolates smoothly between the cases i. and ii.

As a simple example we suggest the following parametrization for the graph in Fig. 1:

$$|A| \sim \prod_{i=1}^{n-1} \left(\frac{g_i s_i + ca}{s_i + a} \right) \left(\frac{s_i + a}{a} \right)^{\alpha_i} \left(\frac{s_i + b_i}{b_i} \right)^{t_i} \quad (6)$$

In (6), a , b_i , c and g_i are constant parameters whose significance will soon be apparent; α_i is the intercept of the i th Regge pole; the variables s_i and t_i we define by:

$$\begin{aligned} s_i &= (p_i + p_{i+1})^2 - (m_i + m_{i+1})^2 \\ t_i &= (p_A - \sum_{r=1}^i p_r)^2 \end{aligned} \quad (7)$$

- Figure 1 -

In that region of phase space (if such exists at a given incoming energy s) where all s_i 's are $\gg a$ and b_i , the amplitude (6) becomes

$$\prod_i g_i \left(\frac{s_i}{a}\right)^{\alpha_i} \exp [B_i + \log s_i] t_i \quad (8)$$

with $B_i = -\log b_i$. This has the form of the fully reggeised multiparticle amplitude ¹¹⁾ with exponential approximations to the vertices and linear approximations to the trajectories, all trajectories being assumed to have slope 1 GeV^{-2} . g_i here play the role of coupling constants; and B_i that of parameters describing the exponential t dependence of the vertices.

When any of the s_i 's are small, however, say $s_r \rightarrow 0$, then the r th term in the product (6) is replaced by the constant c . Moreover, whenever any group of the final particles forms a cluster with a small effective mass, all the s_i 's within the cluster become small also. That part of the amplitude corresponding to interactions within the cluster is then replaced by a constant, while the rest of the amplitude remains reggeised.

The parametrization (6) thus satisfies our criteria i.-iii. Though crude and quite arbitrary, it is hopefully sufficient for our present purpose. It should be pointed out also that (6) is intended to imitate only the production of non-resonant clusters; the production of sharp resonances is not included.

We turn next to specific parametrizations of the two classes of reactions under study.

Consider the reactions:

$$(a) \quad \pi^\pm p \rightarrow p + (n-1) \pi$$

In order to avoid uninteresting and combinatorial complications, we shall first simplify the problem by ignoring the dependence of pion distributions on the pion charge. Indeed, at incoming energies of $\sim 8 \text{ GeV/c}$, the experimental distributions of pions²³⁾ of different charges are very similar for all reactions with multiplicity $n > 4$. For $n = 3$, and to a lesser extent, for $n = 1$ also, there are interesting differences between pions of different charges. These, however, have a simple qualitative explanation in terms of the present model. In the particularly prominent case of $n = 3$, such effects will be considered together with effects of resonance production in a later section. Ignoring then, in the general discussion, the charge of the pions, we are left to consider only the sequence of graphs shown in Fig. 2, where single lines represent mesons and double lines, baryons.

- Figure 2 -

In computing these graphs, we shall distinguish meson exchanges of opposite G parities. We note that since $G = -$ for pions, meson exchanges in any graph of Fig. 2 should alternate down the chain between $G = +$ and $G = -$, starting with $G = +$ on top. In each case, we shall take α_i as the intercept of the leading trajectory with the correct quantum numbers. Thus,

- i. for meson exchanges with $G = +$, $\alpha_+ = \alpha_P = 1$;
- ii. for meson exchanges with $G = -$, $\alpha_- = \alpha_\omega$ or $\alpha_{A_2} \approx 0.5$;
- iii. for nuclear exchanges, $\alpha_N \approx -0.35$ ²⁶⁾.

Among the constants g_i in (6) which represent the couplings of various Regge poles, we shall distinguish only two types:

- i. g_M for meson exchange;
- ii. g_N for nucleon exchange.

We shall ignore the difference in couplings between meson exchanges of opposite G parities since this is important only when the charges of mesons are considered. Moreover, we are interested at the moment only in distributions of final particles, and not yet in absolute values of cross-sections, so that only the ratio of the couplings g_N/g_M is relevant. The physical meaning of this ratio is represented schematically as the ratio in couplings between the diagrams of Fig. 3.

- Figure 3 -

Of the constants b_i which describe the exponential dependence on t of the Regge couplings, we know very little. For the reggeons which are attached to two external particles, the corresponding constants, say b_E , occur in three-body reactions and can be estimated by an analysis of the type given in Ref.14). For the purely internal reggeons, however, they occur only in reactions of higher multiplicities, of which we know nothing. We shall then simply replace all such constants by an effective average b_I and treat it as a free parameter of the model.

There remain then in (6) only the constants a and c . The parameter a represents the energy scale which divides the "low" and "high" energy regions. In all our analysis so far, we have kept a fixed at 1 GeV^2 , which is the currently accepted value for Regge phenomenology. The other constant c , which describes the low effective mass region, is to be determined by fitting with the data.

To summarize, for description of the sequence of graphs in Fig. 2, we have introduced in all the following constant parameters: α_+ , α_- , α_N , g_N/g_M , b_{EA} , b_{EB} , b_I , a , c . Among these, α_+ , α_- and α_N are well known and fixed from two-body reactions. All the others except b_I occur in three-body reactions, and can, in principle, be determined using the data there. From the way these parameters are introduced, and from the physical meaning assigned to them, we expect them to be independent of the multiplicity and the incoming energy. Thus once the parameters are fixed by studying three-body reactions, we have only one more parameter b_I to describe (hopefully!) all the reactions (a) for all multiplicities and all energies. As an example for our parametrization, we write down explicitly the amplitude for the graph in Fig. 4.

$$W_2^{(5)} = \left(\frac{g_M s_1 + ca}{s_1 + a} \right) \left(\frac{s_1 + a}{a} \right)^{\alpha_+} \left(\frac{s_1 + b_{EA}}{b_{EA}} \right)^{t_1} \left(\frac{g_M s_2 + ca}{s_2 + a} \right) \left(\frac{s_2 + a}{a} \right)^{\alpha_-} \left(\frac{s_2 + b_I}{b_I} \right)^{t_2} \left(\frac{g_M s_3 + ca}{s_3 + a} \right) \left(\frac{s_3 + a}{a} \right)^{\alpha_+} \left(\frac{s_3 + b_I}{b_I} \right)^{t_3} \left(\frac{g_N s_4 + ca}{s_4 + a} \right) \left(\frac{s_4 + a}{a} \right)^{\alpha_N} \left(\frac{s_4 + b_{EB}}{b_{EB}} \right)^{t_4} \quad (9)$$

- Figure 4 -

Note that Eq. (6) gives only the absolute value of an amplitude; we know little of the relative phases between the various graphs in Fig. 2. In all our calculations therefore, these have been added incoherently, thus:

$$|W^{(n)}|^2 = |W_1^{(n)}|^2 + |W_2^{(n)}|^2 + \dots + |W_n^{(n)}|^2$$

Such an approximation may not actually be too poor for the following reason. Two graphs in Fig. 2 differ by a permutation of the final particles. If the permutation involves particles from different clusters, the final configurations of the two graphs are very different and the wave functions have little overlap¹¹⁾. If, on the other hand, the permutation involves only particles from the same clusters, it has no effect on the structure of the amplitude. In any case, it is obvious that one cannot handle interference effects at this stage.

The parametrization of the reactions (a) is now complete.

For the class of reactions :

(b)

the parametrization is similar, except that here, because of the quantum numbers involved, it turns out that one can distinguish the charge of the pions without doing extra work.

We note that since strangeness has to be transmitted from the incoming K^- to the final Λ , all meson exchanges must also carry strangeness. Now the best known strange meson trajectories are those corresponding to $K^*(890)$ and $K^{**}(1410)$ with comparable intercepts, both having $I = \frac{1}{2}$. All other reasonably well established strange mesons so far seem to have $I = \frac{1}{2}$ also. If any strange meson trajectories exist with higher isospins, they are presumably lower on the Chew-Frautschi plot, and may for our purpose be neglected.

Among baryon exchanges, both the $N(940)$ and $N^*(1326)$ trajectories may contribute to the reactions (b). However, the N^* trajectory is known in backward elastic $\pi^\pm p$ scattering to be very weakly coupled to the πN system, and have negligible effects compared to N exchanges. Accepting this, we have here then only $I = \frac{1}{2}$ also.

The fact that only $I = \frac{1}{2}$ exchanges occur allows us greatly to reduce the number of graphs for consideration. For example, for the reaction:

the only graphs we need consider are those shown in Fig. 5. In general, for any reaction of class (b), the rule is that we admit only graphs

- Figure 5 -

in which, irrespective of the position of the Λ and the π^0 , the charged mesons alternate down the chain, beginning with a π^- . Thus, in Fig. 6, for example, the graph (a) is allowed but the graph (b) is not.

- Figure 6 -

With this selection rule in mind, we proceed further to specify our parametrization. We have again to compute for every reaction in (b) a sequence of graphs which are similar to those of Fig. 2, except that the incoming π has to be replaced by a K^- , and the outgoing p by a Λ . There is now only one type of meson exchange, namely the strange meson with $\alpha_M = \alpha_{K^*}$ or $\alpha_{K^{**}} \approx 0.3$. The nucleon exchange parameter is the same as before, i.e., $\alpha_N \approx -0.35$. For all other parameters of the model, the previous discussion for reactions (a) holds for the present case also. The sequence of graphs is again added incoherently.

However, because of the selection rule just discussed, it is here easy to distinguish the pion charge. For cases where only charged pions are emitted, e.g., the reaction of Fig. 5, the identification of the pions is obvious. In cases where a π^0 is also emitted, the situation is somewhat more complicated. We shall illustrate the method by the following example:

$$K^- p \rightarrow \Lambda \pi^+ \pi^- \pi^0 \quad (11)$$

Consider the graph a. shown in Fig. 7. It is supposed to represent all the three graphs b., c., and d., which are the only graphs of this type allowed by our selection rule. These last graphs are to be added incoherently to give the actual pion distributions. In our parametrization, however, these graphs are identical except for the labelling of the pions.

- Figure 7 -

It is clear then that we need calculate graph a. only once. The distributions for π^+ , π^- and π^0 can be obtained simply as appropriately weighted averages of the distributions of π_1 , π_2 and π_3 in graph a., thus

$$D(\pi^+) = D(\pi_2) + 2 D(\pi_3)$$

$$D(\pi^-) = 2 D(\pi_1) + D(\pi_2)$$

$$D(\pi^0) = D(\pi_1) + D(\pi_2) + D(\pi_3)$$

(12)

We have now given in terms of several parameters full prescriptions for calculating any single particle distributions for any reaction of classes (a) and (b), and at any incoming energy. The actual numerical computation, to be discussed in the next Section, is left to the computer.

3. COMPARISON WITH EXPERIMENTAL DATA

All our calculations have been carried out using the Monte Carlo phase space program FOWL²⁷⁾ with the function $|w^{(n)}|^2$ as WEIGHT. The distributions presented have been computed with the following values of the parameters in units of 1 GeV²:

$$\begin{aligned}\alpha_+ &= 1., \quad \alpha_- = .5, \quad \alpha_M = .3, \quad \alpha_N = -.35 \\ b_{EA} &= 1., \quad b_{EB} = .5, \quad b_I = 1.2 \\ g_N/g_M &= 1.3, \quad c/g_M = 1.4, \quad a = 1.\end{aligned}\tag{13}$$

The values of b_{EA} and b_{EB} quoted represent crude averages of the values for these parameters obtained in Ref. 14). The value of a , as explained before, was fixed arbitrarily. The others, b_I , g_N/g_M and c were regarded as free parameters of the model which were determined by visual fits to the data. These last are to be accepted with caution, since our primitive calculational technique does not at present allow a proper optimization of them. We note also that, apart from the Regge intercepts, we have deliberately put all other parameters the same for both the reactions (a) and (b), so as to emphasize the difference imposed on them by the Regge model. This need not, however, be the case in general.

All the data used in our study come from bubble chamber experiments, the most complete being a set from a π^+p experiment at 8 GeV/c²³⁾ and another from a K^-p experiment at 6 GeV/c²⁴⁾.

In all the distributions shown, except otherwise stated, events from decays of resonances have been included, since there does not exist a general unambiguous method for their removal. This fact should be borne in mind when comparing the data to the result of our calculation which has not taken sharp resonances into account. The effects of resonance production will be discussed briefly in the next Section.

As explained in the Introduction, we are particularly interested in understanding the dependence of single particle distributions on:

- a. the multiplicity of the reaction, b. the incoming energy and
- c. the quantum numbers of the final particles. We shall now describe each in turn.

a. Dependence on multiplicity

When the multiplicity is low, the amplitude tends to be fully reggeised, and particle distributions are expected to be strongly peripheral. Moreover, since the "partial energies" s_i in (6) are large on the average, the exchange of low lying trajectories is strongly suppressed. Thus in the sequence shown in Fig. 2, graphs containing more meson exchanges will dominate. As the multiplicity increases, cluster formation sets in and there is a general tendency for distributions to be less peripheral. At the same time, as the average values of the "partial energies" s_i decrease, baryon exchange graphs become more important, giving rise to clearly visible effects. The distributions of final particles get closer to statistical distributions as one goes up in multiplicity. It should be noted, however, that even at very high multiplicities, such as $n=9$ for πp reactions at 8 GeV/c, although the average s_i is indeed very small, there are still regions of phase space where some of the s_i 's are large. This is sufficient to reggeise part of the amplitude and give peripheral characteristics to the particle distributions, which are thus still markedly different from purely statistical ones.

In Fig. 8, we show the c.m. angular distributions of protons in the reactions (a) from experiment at 8 GeV/c ²³⁾ together with those obtained in our calculation. There is in both cases a sharp "backward" ²⁸⁾ peak of the proton which decreases with increasing multiplicity but persists even at $n=9$. The same effect is reflected in

the longitudinal momentum distributions of Fig. 9, which are seen to be different from statistical even at the highest multiplicities but are reasonably well reproduced by our model. Our calculations, however, have failed significantly in reproducing both the height of the backward peaks in the angular distributions, and the sharp spikes in the longitudinal momentum distributions at lower multiplicities. This failure, we believe, is due to the production of resonances. For the reaction with $n=3$, we shall show in the next Section that this is indeed the case.

The mechanism for pion production, according to our model, is basically the same as that for protons. However, there being several pions with most of them coming from middle vertices, the total distributions are necessarily less peripheral. For this reason, as can be seen in Fig. 10, pion angular distributions for higher multiplicities become rapidly isotropic. That the mechanism is still basically multiperipheral is clearly illustrated by the structure seen in the longitudinal momentum distributions for lower multiplicities as shown in Fig. 11. For $n=3$, for example, the distribution consists of a forward peak plus a hump around $p_L^*=0$. In terms of a double Regge graph, the "forward" peak corresponds to the pion emitted from the top vertex, and the secondary hump to the pion from the middle. As the multiplicity increases, however, the distributions in Fig. 11 rapidly lose their structure, but the deviation from pure phase space is still marked.

The fact that our vertices are peaked in four-momentum transfers limits naturally the transverse momenta of the final particles. We have studied the distributions in transverse momenta of both the protons and pions in reactions (a) and found good agreement between our calculation and experiment. Figure 12 is an example. However, since the distributions in transverse momentum have always the same shape and are insensitive to almost all changes both experimentally and theoretically, we have chosen instead to present the results in terms of the average values $\langle p_T \rangle$. In Fig. 13 are presented the average

transverse momenta of protons and pions as functions of the multiplicity both from experiment and from our calculation. We note the difference in behaviour between the protons and the pions. Whereas for the pions $\langle p_T \rangle$ decreases rapidly with increasing multiplicity, for the protons, there is a tendency for $\langle p_T \rangle$ to rise slightly before it is depressed at extreme multiplicities by the limitation of phase space. This behaviour is explained in our model by the increasing importance at higher multiplicities of baryon exchanges, which force the protons to have higher momentum transfers, and hence also higher transverse momenta. There is again some discrepancy between our calculation and experiment at lower multiplicities, which is a reflection of the fact that the protons in our calculation are insufficiently peripheral. We ascribe the discrepancy mainly to resonance production; this claim is supported by the values of $\langle p_T \rangle$ also shown in Fig. 13, which are obtained (i), by artificial removal of resonance events from data; (ii), by the inclusion of resonance production in our calculation to be presented in the next Section.

Apart from differences due to different quantum numbers which shall be discussed later, the general dependence on multiplicity of particle distributions in the reactions (b) is similar. We mention here only one particularly interesting example. In Fig. 14 we show the angular distributions of Λ 's from the reactions (b) obtained from experiment at 6 GeV/c²⁴), together with the result of our calculation. In addition to a "backward" peak similar to that of the protons in the reactions (a), one sees also a conspicuous "forward" peak which increases in relative significance with increasing multiplicity. In terms of our model, this effect is simply explained by the increasing importance as one goes up in multiplicity of baryon exchanges, which are just rendered more conspicuous by the lower intercepts of strange meson Regge poles.

b. Dependence on the incoming energy

As the incoming energy increases at a fixed multiplicity, the partial energies s_i have a chance to attain larger values on the average. The amplitude thus becomes more fully reggeised giving rise to more peripheral distributions. At the same time, the relative importance of baryon exchange graphs is suppressed.

In Fig. 15, we show the c.m. angular distributions of protons and pions from the reactions:

$$\pi^\pm p \rightarrow p \, 3\pi^\pm \, 2\pi^\mp \quad (14)$$

at 5.5²⁹⁾, 8²³⁾ and 16 GeV/c³⁰⁾, with the result of our calculation also shown in comparison. The sharpening observed experimentally of the proton "backward" peak with increasing energy is well reproduced by our model. For pions, the change in angular distributions, though less spectacular, is still clearly seen. Similar effects would be expected for reactions at other multiplicities, but no data are as yet available for comparison.

To illustrate the decrease in importance of baryon exchanges with increasing energy, we have studied the angular distributions of Λ 's from the reactions (b) at 10 GeV/c²⁵⁾. The experimental distributions have considerably lower statistics than those at 6 GeV/c, and are quantitatively not well reproduced by our calculations. However, the general suppression of the "forward" peak as compared with the 6 GeV/c data is clearly observed. We quote here only one particularly favourable example in Fig. 16.

c. Dependence on the quantum numbers of the final particles

In our model, as in the Regge model for two-body reactions, the strongest dependence on quantum numbers comes from the difference in intercepts of Regge trajectories, which are characteristic of the quantum numbers of exchange. We have mentioned already one example, namely the difference in angular distributions between the protons of reactions (a) and the Λ 's of reactions (b), which is reflected also in the longitudinal momentum distributions as seen in Figs. 9 and 17. In the present calculation, this difference comes only from the difference in intercepts between the Pomeron and strange meson trajectories, since all other parameters have deliberately been kept the same for both reactions. For the reactions (a), because of the high intercept $\alpha_P = 1$, most of the incoming energy goes into the Pomeron exchanges, leaving little chance for the baryon exchanges to reggeise. Whereas for the reactions (b), the much lower strange meson Regge poles make baryon exchanges more favourable.

Another example is afforded by the difference in the reactions (b) between distributions of pions of different charges. As already discussed in Section 2, the absence of any high-lying strange meson trajectory with $I > \frac{1}{2}$ implies a strong selection rule on the multi-Regge graphs. Thus for the reaction (10) we have only the graphs of Fig. 5. This leads immediately to very different distributions for the π^+ and the π^- from this reaction. Whereas the π^- tends to go strongly forwards, the π^+ tends to be isotropic. This effect can be seen in the c.m. angular distributions shown in Fig. 18 as well as in the longitudinal momentum distributions shown in Fig. 19. It is particularly instructive to compare the distributions a and b of Fig. 19 with Fig. 11a. The last distribution is almost identical in shape to the sum of the former two. For the reaction (11) with $n=4$, the difference between pions of different charges, though exaggerated by our calculation, is still visible in the data, as can be seen in Fig. 20. As the multiplicity increases beyond four, however, the pion distributions become so close to phase space at present energies that such differences are no longer noticeable.

Besides the general dependence of single particle distributions on multiplicity, energy and quantum numbers, which we have described above in some detail, we have studied in certain cases also the dependence of $\langle p_T \rangle$ on p_L^* as a simple example of two-dimensional distributions. Some results are shown in Fig. 21. It is seen that reasonable agreement is obtained. This, however, cannot be claimed as a strong support for the model since much of the structure is due to phase space - our model serves only to depress in general the average value of the transverse momentum.

4. EFFECTS OF RESONANCE PRODUCTION

The most conspicuous shortcoming of our calculation presented above is the failure to produce sufficiently peripheral baryons, especially for low multiplicity reactions at high energy. This is apparent in the angular distributions of Fig. 8 as well as in the longitudinal momentum distributions of Fig. 9. That this discrepancy can be largely remedied by the inclusion of resonance effects is easily seen. The formation of resonant states between the final particles reduces the effective multiplicity of the reaction, giving thus more peripheral baryons. It is interesting to note that if one had compared the experimental distributions of Fig. 8 with the result of our calculation not for reactions with the same multiplicities but for reactions with multiplicities reduced by one, we get much better agreement all the way. Resonance production of course would have similar effects on meson distributions, but these are rendered less conspicuous by averaging the distributions over the final mesons.

The parametrization (6) of the multiparticle amplitude assumes that the structure of low mass clusters is governed only by phase space. This assumption is clearly not applicable when a cluster can form a sharp resonance. Since it is known experimentally that resonances are copiously produced essentially at all multiplicities and there does not exist a general unambiguous criterion for their removal, it would be desirable from a phenomenological viewpoint to take account of them in our calculation. We have, however, not succeeded as yet in finding an easy way for doing this in general and can only offer as an example the reaction

$$\pi^+ p \rightarrow p \pi^+ \pi^0 \quad (15)$$

at 8 GeV/c where such a calculation has been attempted.

The following discussion serves also as an illustration for possible elaborations of the model relevant for future study, such as the distinction in the reactions (a) between pions with different charges, which has so far been ignored. We should stress, however, that it is not meant to be an accurate parametrization of the reaction (15), which we feel can only be made effectively by experimenters with detailed knowledge of the data.

- Figure 22 -

First we calculate the graphs in Fig. 22 again with the parametrization (6). Among the various possible meson exchanges, we distinguish only two types: (i) the Pomeron P with $G=+$, $I=0$; (ii) other mesons M with $G=+$, $I=1$, or $G=-$, $I=0, 1$. Since we now distinguish between π^+ and π^0 , the ratio between the couplings g_P/g_M is taken to be a parameter. Also, since the low effective mass region is to be partly filled by resonances, a smaller value of c than that given in (13) is used. The actual distributions shown below have been calculated with $\alpha_M \sim \alpha_P = 0.57$, $g_P/g_M = 0.7$, $c = 1$, all other parameters being identical in value to those given in (13).

To the graphs in Fig. 22 are now added the diagrams in Fig. 23 for production of $N^*(1236)$ and ρ . Assuming $I=1$ exchange, the

- Figure 23 -

two graphs for N^* production are simply related by Clebsch-Gordan coefficients. For convenience in computation, an amplitude for resonance production is parametrized in a way similar to (6) except that we replace by a Breit-Wigner shape that term in the product (6) which corresponds to the non-resonant interaction between the components of the resonance. Parameters for resonances are taken from the Rosenfeld tables and the amounts for their production have been adjusted to fit roughly the corresponding effective mass distributions as shown in Fig. 24. All graphs have again been added incoherently.

The results of the calculation are shown in Figs. 9a, 24, 25 and 26. We need note only three points:

- (i) with the inclusion of resonance effects the agreement of the calculated proton distributions with the data is much improved. Thus, e.g., the calculated angular distribution in Fig. 25a is scarcely distinguishable from the data. Also, we are able to reproduce the sharp backward spike in the longitudinal momentum distribution in Fig. 9a; this is clearly due to the quasi-two-body production of the ρ resonance;
- (ii) the experimental distributions for π^+ are more peaked forward than those for π^0 as can be seen in Figs. 25 and 26. This is readily explained in our model by the higher intercept of the Pomeron compared with the ρ trajectory. In fact the parameter g_P/g_M in our calculation has to be taken < 1 , so as not to exaggerate the difference observed between the π^+ and π^0 ;
- (iii) the calculated effective mass distributions are qualitatively correct, as can be seen in Fig. 24. This goes somewhat beyond the scope of the present paper which studies so far only single particle distributions.

5. CONCLUDING REMARKS

We wish first to stress again the extreme crudeness of our present calculation. It is intended only as an illustration of the attractive picture for production processes afforded by a reggeised multiperipheral model. The calculation would have to be much elaborated before it can be expected to give an accurate description of any particular reaction. However, though we are far from a direct test of the full implications of the Regge theory, the features A.-C. discussed in the Introduction, have entered essentially in our interpretation of the data. Thus, for example, A. and C. are necessary for explaining respectively (i) the difference between π^+ and π^- in the reaction (10); (ii) the difference between the protons of the reactions (a) and the Λ 's of the reactions (b). The other assumption B. is, of course, implicit everywhere in limiting the transverse momenta of the final particles.

Next, we list a number of important open questions left unanswered by our investigation:

- (i) a general method for including resonance formation in the calculation;
- (ii) correlation effects between final particles; so far only single particle distributions have been studied;
- (iii) absolute values of the cross-sections, and not just normalised distributions.

All these questions are, at least in principle, within the reach of the present model, and are being investigated.

Finally, we point out a serious shortcoming in our present computational technique. We have relied exclusively on Monte Carlo calculations, which practically forbid a proper optimization of our parameters. These have thus been determined very crudely just by trial and error. However, we know of no practical way at present by which this technical difficulty can be overcome.

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FIGURE CAPTIONS

- Figure 8 : Percentage distributions of protons in the cosine of the c.m. production angle θ^* from the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV/c for n ranging from three to nine. The solid histograms are experimental; the dashed histograms are calculated with the parameters (13).
- Figure 9 : Percentage distributions of protons in the c.m. longitudinal momentum p_L^* from the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV/c for n ranging from three to nine. The histograms are experimental; the solid curves are calculated with the parameters (13); the dashed-dotted curve in (a) is obtained from a calculation which includes the effect of ρ and N^* productions (see Section 4). All calculated curves here, as elsewhere in the paper, have been smoothed by hand from Monte Carlo calculations.
- Figure 10 : Percentage distributions of pions in the cosine of the c.m. production angle θ^* from the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV/c for n ranging from three to eight. The solid histograms are experimental; the dashed histograms are calculated with the parameters (13).
- Figure 11 : Percentage distributions of pions in the c.m. longitudinal momentum p_L^* from the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV/c for n ranging from three to eight. The histograms are experimental; the curves are calculated with the parameters (13).

Figure 12 : Percentage distributions of pions in transverse momentum p_T from the reaction $\pi^+p \rightarrow p3\pi^+2\pi^-$ at 8 GeV/c. The histogram is experimental, the curve is calculated with the parameters (13).

Figure 13 : The average transverse momenta $\langle p_T \rangle$ of (a) protons and (b) pions from the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV/c as functions of the multiplicity n . The full circles are experimental points including all events; the full triangles are experimental but with ρ and N^* events removed; the open circles are calculated with the parameters (13); the open triangles for $n=3$ are obtained from the calculation of Section 4 in which the effects of ρ and N^* production are included; the curves represent the predictions of pure phase space.

Figure 14 : Percentage distributions of Λ 's in the cosine of the c.m. production angle θ^* from the reactions $K^-p \rightarrow \Lambda+(n-1)\pi$ at 6 GeV/c for n ranging from three to six. The solid histograms are experimental; the dashed histograms are calculated with the parameters (13).

Figure 15 : Percentage distributions of protons and pions in the cosine of the c.m. production angle θ^* from the reactions $\pi^\pm p \rightarrow p3\pi^\pm 2\pi^\mp$ at 5.5, 8, and 16 GeV/c. The solid histograms are experimental; the dashed histograms are calculated with the parameters (13).

- Figure 16 : Percentage distributions of Λ 's in the cosine of the c.m. production angle θ^* from the reaction $K^-p \rightarrow \Lambda 2\pi^+ 2\pi^- \pi^0$ at 6 and 10 GeV/c. The solid histograms are experimental; the dashed histograms are calculated from the parameters (13).
- Figure 17 : Percentage distributions of Λ 's in the c.m. longitudinal momentum p_L^* from the reactions $K^-p \rightarrow \Lambda + (n-1)\pi$ at 6 GeV/c for n ranging from three to six. The histograms are experimental; the curves are calculated with the parameters (13).
- Figure 18 : Percentage distributions of (a) π^+ , (b) π^- in the cosine of the c.m. production angle θ^* from the reaction $K^-p \rightarrow \Lambda \pi^+ \pi^-$ at 6 GeV/c. The solid histograms are experimental; the dashed histograms are calculated with the parameters (13).
- Figure 19 : Percentage distributions of (a) π^+ and (b) π^- in the c.m. longitudinal momentum p_L^* from the reaction $K^-p \rightarrow \Lambda \pi^+ \pi^-$ at 6 GeV/c. The histograms are experimental; the curves are calculated with the parameters (13).
- Figure 20 : Percentage distributions of (a) π^+ , (b) π^0 and (c) π^- in the c.m. longitudinal momentum p_L^* from the reaction $K^-p \rightarrow \Lambda \pi^+ \pi^- \pi^0$ at 6 GeV/c. The histograms are experimental; the curves are calculated with the parameters (13).

- Figure 21 : The average transverse momentum $\langle p_T \rangle$ of (a) protons from the reaction $\pi^+ p \rightarrow p 2\pi^+ \pi^-$, (b) pions from the reaction $\pi^+ p \rightarrow p 2\pi^+ \pi^- \pi^0$, at 8 GeV/c, as functions of their c.m. longitudinal momenta p_L^* . The points with statistical errors are experimental; the solid curves are calculated with the parameters (13); the dashed curve in (b) is the prediction of phase space.
- Figure 24 : Percentage effective mass distributions for the reaction $\pi^+ p \rightarrow \pi^+ \pi^0 p$ at 8 GeV/c. The histograms are experimental; the curves are the result of a calculation which includes ρ and N^* production (see Section 4).
- Figure 25 : Percentage distributions of (a) protons, (b) π^+ , and (c) π^0 in the cosine of the c.m. angle θ^* from the reaction $\pi^+ p \rightarrow p \pi^+ \pi^0$ at 8 GeV/c. The solid histograms are experimental; the dashed histograms are the result of the calculation described in Section 4.
- Figure 26 : Percentage distributions of (a) π^+ and (b) π^0 in the c.m. longitudinal momentum p_L^* from the reaction $\pi^+ p \rightarrow p \pi^+ \pi^0$ at 8 GeV/c. The histograms are experimental; the curves are the result of the calculation described in Section 4.

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