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A HYBRID MODEL FOR ELASTIC SCATTERING

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A B S T R A C T

We present a model for the large-energy, small angle scattering of hadrons, which is a combination of the optical and the Regge models. In the infinite-energy limit, our model reduces to the model suggested by Chou and Yang, but at finite energies there are corrections arising from the exchange of Regge poles (excluding the Pomeron) and cuts. With a very simple form of this model - we include only one Regge trajectory, parametrized as simply as possible - we attempt to understand pp scattering. We find that the structure observed in the pp differential cross-section is reproduced. Our calculated cross-section shows breaks at $-t \approx 1.2 \text{ GeV}^2$ and 5.5 GeV^2 , and has correct energy dependence. In further applications of this model, we reproduce the cross-over in the pp and pp differential cross-sections near the forward directions, and predict a dip in the pp differential cross-section near $-t \approx 0.7 \text{ GeV}^2$ at high energy.

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"We must deal with poles and cuts."

- Grodsky and Streater, PRL 20, 695 (1968).

1. INTRODUCTION

Chou and Yang have recently proposed a model for the high-energy, small-angle elastic scattering of hadrons¹⁾. In this model, hadrons are considered to be spatially extended objects, and the hadron-hadron interaction in the infinite-energy limit takes place locally between the two hadron matter densities. Durand and Lipes²⁾ have shown that this model is capable of producing a proton-proton scattering amplitude which has zeros at values of momentum transfer ($t \approx -1.3 \text{ GeV}^2$ and -6 GeV^2) close to those at which the measured differential cross-sections show some indication of having structure³⁾.

The Chou-Yang model is intended to describe the (presumably finite) infinite-energy limit of the differential cross-section, and it is not obvious how one should compute finite-energy corrections to it. This question is important, since at presently available energies, elastic cross-sections at fixed values of t outside of the forward peak seem to decrease very rapidly as the energy increases, which means that in this region the energy-dependent parts of the amplitude are much larger than the energy-independent asymptotic limit. Since energy dependence is the forte of the Regge model, we suggest that a description of the approach to the infinite-energy limit may be furnished by Regge theory. The asymptotic amplitude proposed by Chou and Yang represents what otherwise might be described by a (flat) Pomeron trajectory; therefore we would add to the Chou-Yang model the effects of trajectories other than the Pomeron. We shall refer to these as "proper" Regge trajectories.

For the sake of simplicity, we ignore the presence of spin. Suppose we write the S-matrix in the impact-parameter representation as:

$$S(s, b) = \exp \left[2i B(s, b) \right] \quad (1)$$

or, equivalently, the full amplitude as

$$A(s,t) = B(s,t) + \frac{i}{4\pi} \int dt_1 dt_2 B(s,t_1) B(s,t_2) \tau^{-1/2}(t,t_1,t_2) \theta(\tau) \\ + (\text{terms involving more than two } B\text{'s}). \quad (2)$$

Here τ is the triangle function; see the Appendix of Ref. 4. Our amplitude A is normalized so that $\sigma^T = 4\pi \operatorname{Im} A(s,0)$. Equations (1) or (2) may be considered to be a definition of B . By analogy to Glauber theory⁵⁾, we shall refer to $B(s,t)$ as the "single-scattering" term and to the other terms on the right-hand side of Eq. (2) as "multiple-scattering" terms. This terminology is motivated not only by the formal similarity of Eqs. (1) and (2) to equations of Glauber theory, but also by the geometric interpretation of $B(s = \infty, b)$ suggested by Chou and Yang. The Chou-Yang model consists of the interpretation of the single-scattering term as the point-interaction between extended matter densities. For pp scattering, the matter density may be similar to the charge density; then the Chou-Yang model can be defined by Eq. (2) together with

$$B(s,t) = iK [F_1(t)]^2, \quad (s \rightarrow \infty) \quad (3)$$

where $F_1(t)$ is the charge form factor of the proton and K is a constant. We shall continue to write the asymptotic form of the single scattering term as in Eq. (3), with the understanding that the identification of the matter density with the charge density is certainly not meant to be precise.

We suggest that the generalization of Eq. (3) appropriate to finite energy should be

$$B(s,t) = iK [F_1(t)]^2 + \sum_i R_i(s,t), \quad (4)$$

where the summation extends over proper Regge pole terms $R_i =$

$\beta_i(t) s^{\alpha_i(t)-1}$. These additional terms might be interpreted as arising from a finite-range interaction, which disappears as the energy increases. It is essential for our model that the Regge poles R_i be added to the single-scattering term, and not directly to the amplitude; this point will be discussed further below.

In this paper, we shall be especially interested in pp scattering. We suggest that for this process, the data at high energies may be well understood by including in Eq. (4) just the trajectories on which lie the ω and the $f^0(1250)$, which we will approximate by one exchange degenerate trajectory. Our expression for the single-scattering term is thus formally similar to the expression for the amplitude proposed by Abarbanel, Drell and Gilman⁶⁾, but our model is intended to be valid at small angles, and theirs at large angles.

In the next section we discuss in more detail the motivation for our model, and explain its relation to more conventional Regge theories. In Section 3 we present an analysis of pp scattering at high energy on the basis of the form-factor-plus-one-pole model mentioned above. Section 4 is an extension of our model to other processes, and in Section 5 we summarize our main conclusions.

2. MOTIVATION

The motivation for the asymptotic part of our model has already been discussed by Chou and Yang¹⁾. We wish here to try to motivate our treatment of Regge poles for elastic scattering; to this end, we will describe our model first as an extension of the Chou-Yang model, as suggested by Regge theory, and then, alternatively, as an extension of Regge theory, as suggested by the Chou-Yang model.

2.1 Corrections to the Chou-Yang model

Since we are describing the asymptotic scattering amplitude by an optical diffraction picture, it might seem that there is no room in our model for the inclusion of Regge poles. Indeed, the successes of Regge theory have been for inelastic (or backward elastic) scattering⁷⁾, so we should first state why it is that we believe Regge theory to be relevant for near-forward elastic scattering. It is reasonably well established that Regge theory can provide a good description of some inelastic processes; for example, the πN charge-exchange amplitude does seem⁸⁾ to have an important term which behaves (in our normalization) like

$\beta_p(t) s^{\alpha_p(t)-1}$. But once we accept this, isospin symmetry tells us that either the $\pi^+ p$ or the $\pi^- p$ elastic amplitude (or both) has a similar

term; thus it is extremely plausible that (at least some) elastic amplitudes have terms corresponding to the exchange of proper Regge trajectories.

Suppose we define a quantity $Y(s,t)$ by

$$B(s,t) \equiv i K[F_1(t)]^2 + Y(s,t) . \quad (5)$$

We expect $Y(s,t)$ to vanish as $s \rightarrow \infty$ so that we recover the Chou-Yang formula Eq. (3). In the case of pp scattering, the observed large real part of the amplitude in the forward direction⁹⁾ implies that Y is substantial at presently-available energies. The fact that the pp differential cross-section is strongly energy dependent beyond the forward peak is another indication that deviations from the asymptotic formula (3) are important. Thus elastic scattering data indicate that $B(s,t)$ must contain an important term in addition to $iK F_1^2$, while the analogy with πN charge exchange scattering indicates that there should be a term corresponding to the exchange of proper Regge poles. Our model consists in identifying these two terms; that is, we suggest that the term Y in Eq. (5) is none other but the proper Regge term, and thus we arrive at Eq. (4). Below we shall see that by including one Regge trajectory we are able to correlate and explain the experimentally observed features of pp and $\bar{p}p$ elastic scattering at high energy.

In the next section, we will indicate why we think that the Regge pole should be put into the single scattering term, instead of directly into the amplitude; now we point out that this is necessary in order for the model to be able to explain the structure observed in the pp differential cross-section. The asymptotic cross-section calculated by Durand and Lipes²⁾ has the structure suggested by the data, but lies several orders of magnitude below the data taken at energies (such as 9.2 GeV) at which the structure has already appeared. Thus if one adds to the Durand-Lipes amplitude a term which is a smooth function of t , then the dips in the asymptotic amplitude would, at such energies, be filled in to such an extent that nobody would ever see them. Put another way, it is the asymptotically-vanishing part of the amplitude that explains the data. One could, of course, concoct an asymptotically-vanishing part which itself has dips, but unless there were some good

reason why the asymptotically-vanishing part had the same structure as the asymptotically-constant part, the significance of the Durand-Lipes calculation would be lost. As we shall see, such a reason is present in our model.

We conclude this section by emphasizing that both the Chou-Yang and the Regge theories are based on small-angle approximations, and so our model cannot be expected to be valid at very large angles. We hope, however, that it is valid beyond the exponential forward peak, so that we may at least analyse the structure which appears for values of $-t$ around 1 or 2 GeV^2 at present energies.

2.2 Relation to Regge theory

In a Regge model for elastic scattering, we would start with Regge poles, including both the Pomeranchuk pole and also the proper poles. In addition to the pole contributions, we might wish to take into account the cuts that are thought to exist in the complex J plane. Very little can be proven about these cuts, but models, which involve either the analysis of Feynman diagrams which contain third double spectral functions¹⁰⁾, or a continuation of multiparticle discontinuity formulae¹¹⁾ lead us to expect that the cuts have the following four properties:

- i) The position of the leading branch-point corresponding to the exchange of the n trajectories $\alpha_1, \dots, \alpha_n$ is given, for $t \leq 0$, by

$$\alpha_{\text{cut}}^{(n)}(t) = \text{Max} [\alpha_1(t_1) + \dots + \alpha_n(t_n)] - n + 1, \quad (6)$$

where the maximum is over that set of the t_i for which the quantities $(\sqrt{-t}, \sqrt{-t_1}, \dots, \sqrt{-t_n})$ form a closed polygon. When the n trajectory functions α_i are all the same (and of reasonable shape), Eq. (6) reduces to

$$\alpha_{\text{cut}}^{(n)}(t) = n \alpha \left(\frac{t}{n^2} \right) - n + 1. \quad (7)$$

- ii) At fixed t , the discontinuity of the cut in the J plane behaves, as $J \rightarrow \alpha_{\text{cut}}^{(n)}$ like $\Delta_J A(t, J) \sim \Gamma_{(n)}(t) [\alpha_{\text{cut}}^{(n)}(t) - J]^{n-2}$; this means that the contribution of the cut to the amplitude at large s behaves like

$$A_{cut}^{(n)}(s, t) \longrightarrow \frac{\Gamma_{(n)}(t) s^{\alpha_{cut}^{(n)}(t)-1}}{(\log s)^{n-1}} \left[1 + O\left(\frac{1}{\log s}\right) \right]. \quad (8)$$

iii) Each cut has definite signature, which is the product of the signatures of the n poles; this determines the phase of each $\Gamma_{(n)}(t)$ to within multiples of π . However, the correction term of order $1/\log s$ need not be real.

iv) When each of the n poles is the Pomeron, the signs of the discontinuities alternate with n ; that is, $\Gamma_{(n)}$ contains a factor $(-1)^n$.

Anselm and Dyatlov¹²⁾ have shown that properties (i), (iii), and (iv), together with some simplicity assumptions, suffice to determine the general form of the amplitude in the region $s \gg -t$, $-t \ln s \gg 1$ (i.e. small angle but outside of the shrinking forward peak); this form is $A \sim \exp(-\sqrt{-t \ln s})$ with oscillations superimposed. To get more concrete results, we need a specific prescription for parameterizing the cuts, but we may certainly demand of any prescription that it satisfy properties (i), (ii), (iii) and (iv). One such prescription is to calculate the cuts by using Eq. (2), with B as the sum of Regge pole terms [as has been suggested by Arnold¹³⁾]. This prescription is similar to, but not identical with, the Amati-Fubini-Stanghellini proposal¹⁴⁾ for calculating cuts. This point is discussed further in Ref. 4.

That is, we may regard our model, as defined in Eq. (4), as a device to produce an amplitude which contains poles and cuts, all of which satisfy properties (i) through (iv). It is now clear why we should put the proper Regge trajectory into the single-scattering term, instead of directly into the amplitude: in this way, the amplitude also contains all of the cuts in which the proper trajectories participate. The most important of these will be the cuts formed by the proper trajectories together with any number of Pomerons; since for these cuts α_{cut} is larger than α_{pole} except at $t = 0$, we expect these cuts to be very important, except perhaps for very small t , at large energies.

One might think of explaining the pp data by approximating the single-scattering term by a Pomeranchuk pole, the energy dependence then

arising from the fact that the pole has a finite slope. However, it is not hard to see why this will not work. At $t = 0$, the most important cut will be the one arising from the exchange of two poles; from properties (i) through (iv), it can be seen that at $t = 0$, the contribution of this cut to the amplitude is primarily negative imaginary, and decreasing with energy. Thus one will be forced to predict an increasing total cross-section. Furthermore, at $t = 0$, this cut has a small positive real part [which asymptotically disappears relative to the imaginary part as $1/\ln s$, in conformity with property (iii)]. To see this, think of the cut as a superposition of positive-signature poles in the J plane, the rightmost of which is at $J = 1$. The average contribution of all these poles will have the phase characteristic of a positive-signature pole displaced slightly to the left of $J = 1$, and so will be in the second or fourth quadrant. Because of property (iv), the cut is approximately negative imaginary, and so the real part is positive. The real part of the amplitude will therefore probably be small, but certainly be positive, while experiment⁹⁾ indicates that the real part is fairly large and negative¹⁵⁾.

We see that at least one additional trajectory is needed, and so are led back to the model proposed in the introduction. The term iKF_1^2 in Eq. (4) represents a fixed Pomeron pole [$\alpha_p(t) = 1$, for all t]. Regge phenomenologists are accustomed to avoiding fixed poles, since fixed poles lead to conflicts with t -channel unitarity which can be resolved only by means which seem unnatural when viewed from within Regge theory¹⁶⁾. This difficulty already exists within the Chou-Yang model, and is in no way exacerbated by our extension of this model. Of course, one is always free to say that the slope of the Pomeron is finite, but very small; this would, for any finite energy, affect the model by less than any desired amount, but it would then require great ingenuity to continue to identify the Pomeron residue function with the square of the charge form factor. We should point out that the cuts in our model have the properties that cuts are expected to have, whether or not the Pomeron is flat.

It is interesting to observe that, if we retain only the terms which are linear in the proper pole contributions, then the piece that we are adding to the asymptotically-constant amplitude corresponds exactly to an

"absorbed" Regge pole [in the sense used by Cohen-Tannoudji, Morel and Navelet¹⁷⁾]. This observation forms the key to understanding why the asymptotically-vanishing part of the amplitude has the same structure as does the asymptotically-constant part. Why is this? Read on.

3. PROTON-PROTON SCATTERING

A very simple parameterization will suffice to illustrate the main features of our model as applied to pp scattering. Let us take for the asymptotic term the form used by Durand and Lipes²⁾, and motivated by the observed proton form factor:

$$B(s, t) \xrightarrow{(s \rightarrow \infty)} \frac{i K \mu^8}{(\mu^2 - t)^4} \quad (9)$$

Following Durand and Lipes, we take $\mu^2 = 1 \text{ GeV}^2$, in order to produce dips in the asymptotic cross-section at more or less the values of t at which they are observed; in any event, μ^2 cannot be varied by very much without significantly changing the slope of the differential cross-section in the forward direction. The constant K is adjusted to produce the correct total cross-section (our total cross-section falls from 39.8 mb at $p_{\text{lab}} = 10 \text{ GeV}/c$ to 37.3 mb as $p_{\text{lab}} \rightarrow \infty$).

Because of the small size of the pn charge-exchange cross-section, we do not bother with any $I = 1$ trajectories. We consider only the trajectories which pass through the ω and the $f^0(1250)$, and approximate these by one exchange degenerate trajectory. The trajectory function we take to be linear, with a slope of 1 GeV^{-2} and an intercept of $1/2$; the residue function we take to be exponential. Thus we write the single-scattering term

$$B(s, t) = \frac{i K \mu^8}{(\mu^2 - t)^4} + \frac{\Gamma}{s^{1/2}} \left(\frac{s}{s_0} \right)^t \quad (10)$$

Changing s_0 is equivalent to introducing a residue function which is exponential in t ; Γ is a constant. The Regge term in (10) is real because we assume exchange degeneracy for the residue functions; also, if it had any considerable imaginary part, the pp total cross-section would vary rapidly with energy.

We have no very good justifications for neglecting a possible spin dependence of the Regge term in Eq. (10), except that polarization in pp scattering appears to be small, much smaller than the ratio of the real to the imaginary part; we would not be surprised if spin effects became more important at large angles. We cannot expect our very simple parameterization to provide us with anything like a fit to the data; in particular, the extremely naive t -dependence of our Regge term is certain to get us into trouble at fairly large values of t . Nevertheless, our formula (10) is able to reproduce remarkably well the general features of the data.

We have two parameters, s_0 and Γ , with which to try to reproduce all of the s -dependence, all of the t -dependence outside of the forward peak, and the real part in the forward direction. We have chosen the values $s_0 = 4.5 \text{ GeV}^2$, $\Gamma = -22 \text{ GeV}^{-1}$; the general features of the data are reproduced for a fairly large range of these parameters. We calculate the ratio of the real to the imaginary part in the forward direction to be -0.20 at $p_{\text{lab}} = 12 \text{ GeV}/c$ and -0.18 at $p_{\text{lab}} = 25 \text{ GeV}/c$.

The differential cross-sections calculated¹⁸⁾ with these parameters are shown in Fig. 1, for $p_{\text{lab}} = 12.4 \text{ GeV}/c$ and $p_{\text{lab}} = 24.9 \text{ GeV}/c$, together with experimental differential cross-sections^{3,19-22)}. The energy dependence is reproduced extremely well, as is the general trend of the t -dependence. There is a pronounced kink at $t \approx -1.2 \text{ GeV}^2$; there is also a slight shoulder at $t \approx -5$ to -6 GeV^2 , which is too slight to be easily seen from the figure. With slightly different values of the parameters, we could make this shoulder more noticeable, but the energy dependence would become slightly worse. Both of these kinks are, at the energies represented on the figure, associated with zeroes of the real part of the amplitude.

Let us denote the asymptotic part of the single-scattering term as P , and the exchange degenerate trajectory as D ; thus $B = P + D$. From Eq. (1) we may write, schematically,

$$\begin{aligned}
 A = & P + P \otimes P + P \otimes P \otimes P + \dots & I \\
 & + D + D \otimes P + D \otimes P \otimes P + \dots & = + \Pi \\
 & + D \otimes D + D \otimes D \otimes P + D \otimes D \otimes P \otimes P + \dots & + \text{II} \\
 & + \dots & + \dots
 \end{aligned} \tag{11}$$

The convolutions are taken in the sense of Eq. (2); we have neglected numerical factors. The first line (I) of Eq. (11) represents the asymptotic amplitude, and is pure imaginary. The second line (II) contains the D pole and all of the cuts (multiple scattering terms) associated with one D and any number of P's, and is real; this line may be considered to be an absorbed Regge pole. The third line (III) contains all cuts associated with two D's, and is imaginary. It turns out that all terms involving more than two D's are small enough to be ignored in the following discussion.

In Fig. (2) we display the contributions to the pp differential cross-sections, at $p_{\text{lab}} = 12.4 \text{ GeV}/c$, of the three lines of Eq. (11). Cross terms between I and III are not shown. The asymptotic term, curve I, is the same as was calculated by Durand and Lipes²); it makes a very small contribution for $-t > 1 \text{ GeV}^2$. The imaginary part of the amplitude has its only zero where curves I and III cross, at $t = -1.8 \text{ GeV}^2$ (remember the curves in the figure represent contributions to the differential cross-section), but here each of I and III is so small that this zero does not lead to any noticeable structure in the cross-section. The structure at $t \approx -1.2 \text{ GeV}^2$ is clearly associated with a vanishing of curve II, that is, a zero of the real part of the amplitude.

It is also interesting to look at the region around $t = -5 \text{ GeV}^2$, although this represents an angle of about 60° , which is not an extremely small angle. Here again the real part vanishes; since this dip is largely filled up by the contribution of the imaginary part (curve III), the main observable effect is a flattening of the calculated differential cross-section. Thus there are three regions of momentum transfer with three different slopes.

We can understand the relation between our result and that of Durand and Lipes if we understand why the zeroes of the asymptotic amplitude (I) and of the term which produces the structure at $12.4 \text{ GeV}/c$ (II) so nearly coincide. The reason for this is that II represents an absorbed Regge pole, and the amplitude doing the absorbing is the one which also gives rise to the diffraction pattern (I). Thus the asymptotic amplitude induces a structure in the real part. Put another way, the zero in the asymptotic amplitude occurs because of cancellations between the various terms on

the first line of Eq. (11); the second line has the same general structure as the first line, and so its zeroes should occur at roughly the same places. This will be approximately true as long as the t -dependence of the P and D terms near the forward direction are not too dissimilar.

In our explanation of pp scattering, the real part of the amplitude plays an extremely important role. As can be seen clearly from Fig. (2), we are predicting that the amplitude should be predominantly real in the region between the two breaks, at presently-available energy. As the energy increases, the Regge term dies away. In our calculation, at $p_{\text{lab}} = 50 \text{ GeV}/c$ the real and the imaginary parts play a roughly equal role in producing the structure around $-t \sim 1.2 \text{ GeV}^2$; by $p_{\text{lab}} = 200 \text{ GeV}/c$, the cross-section closely resembles the asymptotic form, with the real part only filling in the bottom of the dips. The region between presently-available energy and roughly 50 GeV is a transition region in which the dips do not get sharper, as the structure is shifting from the real to the imaginary part.

4. EXTENSION TO OTHER PROCESSES

It is quite easy to extend our model to the $\bar{p}p$ amplitude; we have merely to change the sign of the ω -pole term. This corresponds to multiplying the exchange degenerate Regge term in Eq. (10) by $e^{-i\pi\alpha(t)}$. We have not attempted to readjust our two parameters to give the best simultaneous agreement with the pp and $\bar{p}p$ data; our "prediction" for the $\bar{p}p$ differential cross-section at $p_{\text{lab}} = 12.4 \text{ GeV}/c$ is shown in Fig. (3), together with data from Ref. (22). The calculated $\bar{p}p$ differential cross-section falls more steeply with increasing momentum transfer than does the pp , but not enough so; the crossover which should be at $t = -0.2 \text{ GeV}^2$ appears instead at $t = -0.37 \text{ GeV}^2$. The classical Regge description of the crossover involves a zero in the ω residue function. Unfortunately, factorization then implies a dip in the ω exchange contribution to any process, and this seems to be in conflict with the data on ρ production²³). In our model, the crossover occurs because of cancellations between the imaginary parts of those single and multiple scattering terms which are odd under charge conjugation; thus factorization is foiled.

The calculated $\bar{p}p$ cross-section shows a remarkable structure at $t = -0.7 \text{ GeV}^2$. Presumably this is connected with the dip observed near $t = -0.5 \text{ GeV}^2$ at $p_{\text{lab}} = 5.9 \text{ GeV}/c^{24}$, although this dip has not yet been observed at higher energies. Since the data in the small t region falls considerably below the calculation, we find it hard to believe that the differential cross-section will rise after the dip to quite the extent indicated in the figure; however, we feel that the presence of this dip, although not its precise magnitude nor location, should be taken seriously. At extremely high energies, the $\bar{p}p$ differential cross-section should of course have a dip as given by the asymptotic term in Eq. (10). Therefore we might expect the dip to move slowly outward in t with increasing energy.

There is an amusing relation which is obeyed by the pole terms, and approximately obeyed by the multiple scattering terms, in our model, and which is independent of many of the details of our parameterization. In going from the forward pp amplitude to the forward $\bar{p}p$ amplitude, we change the contribution from the exchange degenerate Regge trajectory from negative real to positive imaginary, without changing its magnitude. We can thus immediately predict

$$\frac{\text{Re } A_{pp}(s, 0)}{\text{Im } A_{pp}(s, 0)} = \frac{\sigma_{pp}^T - \sigma_{\bar{p}p}^T}{\sigma_{pp}^T} . \quad (12)$$

This relation should hold at all energies for which our model is valid. We have used this relation, together with experimental values for σ_{pp}^T and $\text{Re } A_{pp}/\text{Im } A_{pp}$ (Ref. 9), to predict $\sigma_{\bar{p}p}^T$, and compared with the values given in Ref. 25. The results are shown in Fig. 4.

As can be seen from Fig. 1, our model predicts that the pp forward peak should shrink (to a finite limit) as the energy increases. In the case of $\bar{p}p$, the Regge term adds to the total cross-section, and this makes the whole diffraction pattern sharper. As the energy increases, the Regge term disappears, the diffraction pattern broadens, and so the forward peak antishrinks. These examples illustrate a general feature of our model: a proper Regge term which is real (of either sign) tends to make the peak shrink, and one that is positive (negative) imaginary tends to make it antishrink (shrink).

In πp elastic scattering, we expect the most important proper Regge trajectory to be that of the f^0 . Since this trajectory has a positive signature and an intercept $\alpha(0) \cong \frac{1}{2}$, its phase at $t = 0$ is near either $+135^\circ$ or -45° . From the fact that the total cross-section decreases as the energy increases²⁶⁾, or, alternatively, from the fact that the real part of the amplitude in the forward direction is negative²⁶⁾, we conclude that the phase of the Regge term is approximately $+135^\circ$. Since the Regge term is therefore half way between real and positive imaginary, it is not surprising that the slope of the forward peak does not change with energy.

For a more detailed discussion of πp scattering, we would have to know, or guess, something about the pion's form factor. Chou and Yang¹⁾ have approached this problem from the other direction: starting from an assumed form for the πp asymptotic differential cross-section, they have solved for the form factor. In our model, the form factor, since it determines the absorptive corrections to the proper Regge pole, can induce a structure in the asymptotically-vanishing part of the amplitude, which as we have seen can be important at fairly large momentum transfer. Therefore it is possible that a careful study of the energy dependence of the πp differential cross-section away from the forward peak could provide more accurate information on the pion form factor.

5. SUMMARY AND CONCLUSIONS

- 1) Proton-proton elastic scattering at presently available energy may be understood by a hybrid diffraction-exchange picture.
- 2) Outside of the forward peak, the energy-dependent part of the pp amplitude is much larger than the energy-independent part; it can be described remarkably well by the addition of one "proper" exchange-degenerate Regge trajectory in the single scattering.
- 3) With this addition, the model predicts that the pp amplitude at $p_{\text{lab}} = 12.4 \text{ GeV}/c$ changes character twice as $-t$ increases from 0 to, say, 8 GeV^2 : the amplitude is first predominantly imaginary, then predominantly real, then again predominantly imaginary, and in each of these three regions the differential cross-section has a different slope.

- 4) This same model is able to comprehend certain features of several different elastic reactions. In particular, it enables us to understand the energy dependence of the forward peaks in pp , $\bar{p}p$, and πp scattering, predicts a dip at small momentum transfer in the $\bar{p}p$ differential cross-section, and supplies a simple but non-trivial relation between the real part of the pp forward amplitude and the pp and $\bar{p}p$ total cross-sections.
- 5) The model may be re-interpreted within Regge theory, by noticing that it corresponds to a specific form for the Pommeranchuk trajectory, together with a specific prescription for computing cuts which have all the properties that Regge cuts are expected to have.

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FOOTNOTES AND REFERENCES

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$$B(s,b) = \frac{i}{96} K \mu^5 b^3 K_3(\mu b) + \left[\frac{\Gamma}{4 s^{1/2} \ln(s/s_0)} \right] \exp\left[\frac{-b^2}{4 \ln(s/s_0)} \right],$$

and integrate numerically

$$A(s,t) = i \int_0^\infty b db J_0(b\sqrt{-t}) \left[1 - \exp(2iB(s,b)) \right].$$

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Figure captions

- Fig. 1 : The pp elastic differential cross-section versus $-t$. The calculated cross-section is illustrated by the solid curves at 12.4 and 24.9 GeV/c. The dashed curve represents the asymptotic cross-section. For further details on the data, see Refs. 3, 19-22.
- Fig. 2 : The pp differential cross-section versus $-t$. The contribution to the differential cross-section by the $P+PP+PPP \dots$, $D+DP+DPP \dots$, and $DD+DDP+DDPP \dots$ terms [see Eq. (11)] are shown by curves I, II and III, respectively. For further details on the data, see Refs. 3, 19 and 20.
- Fig. 3 : The $\bar{p}p$ differential cross-section versus $-t$. The $\bar{p}p$ curve calculated at 12.4 GeV/c crosses the pp curve at the same energy at $t \approx -0.37 \text{ GeV}^2$. The $\bar{p}p$ curve is compared with the available data and the optical point at $P_{\text{lab}} = 11.8 \text{ GeV/c}$. For further details on the data, see Ref. 22.
- Fig. 4 : The $\bar{p}p$ total cross-section versus P_{lab} . The solid points are the $\bar{p}p$ cross-sections predicted by Eq. (12) using the pp data of Foley et al., Ref. 9. The solid line is hand-drawn through the average of the predicted points to guide the eye.

Fig. 1

Fig. 2

Fig. 3

Fig. 4