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LOOPS IN PARTIAL WAVE EXPANSION OF ASYMPTOTIC AMPLITUDES

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A B S T R A C T

We observe that the loops discovered recently on the Argand diagrams for the partial waves of a Regge pole represent a general property of an asymptotic amplitude. They are a direct consequence of the phase factor $\exp(-i\pi\alpha)$, which follows from the crossing symmetry and the asymptotic power behaviour $s^\alpha (\log s)^\beta (\log \log s)^\gamma \dots$ for t fixed near the forward direction. Since these loops are closely connected to the phase factor, they can appear even at the low energy region where the Regge amplitude gives a poor representation of the data. In this region it is conceivable that some of the physical partial waves can resemble the Regge partial waves, but it is difficult to give much significance to this similarity. A selection rule associated with the Regge loops in πN states is also noted.

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1. INTRODUCTION

Recently Schmid ¹⁾ has made partial-wave analysis for the s channel spin flip amplitude [the $B^{(-)}$ amplitude, corresponding to the t channel isospin 1] for the Regge exchange of ρ in πN scattering. He found that these partial wave amplitudes describe counterclockwise loops on the Argand diagrams and suggested that these loops might correspond to the prominent experimental N^* resonances. Collins, Johnson and Squires ²⁾ have further studied the partial wave amplitude for the sum of the crossed channel Regge amplitudes with exchange of the P , P' , ρ , and baryon poles. They also obtained loops in the partial waves, some of them remarkably similar to those found in the partial-wave analyses of experimental data. Collins et al. pointed out that since there are no poles on the unphysical sheet in their Regge partial wave amplitudes, one should not associate these loops with resonances in the direct channel. They observed also that the Regge loops are very different from true resonances; the latter induce structures in total cross-sections while the former give smooth Regge energy dependence for all values of t including $t=0$.

In this note we point out that these loops are not only a consequence of the particular form of the Regge parametrization but also express a more general property of asymptotic amplitudes. Further, we discuss some questions concerning the loops and their relation to existing resonances. In Section 2 we discuss the origin and the qualitative features of these loops. As we shall see, the loops have their origin in the phase factor $e^{-i\pi\alpha}$ together with the monotonic decrease of α with $|t|$ for negative t . For a Regge amplitude, $\alpha(t)$ is the trajectory and the residue functions together with the s^α factor damp the circular motion at higher energies. The loops essentially originate from the crossing symmetry and the power behaviour s^α at fixed t in the asymptotic amplitude.

In Section 3 we recall that in the intermediate energy region for πN scattering only the B_s amplitude is approximately equal to the physical amplitude $B^{(-)}$, whereas the $A^{(-)}$ amplitude cannot be described by $A_s^{(-)}$. So we remark that it is more useful to discuss the implication of the loops in B_s than talking about the loops associated with the exchange as a whole. Further we discuss some arguments for the existence of the intermediate energy region, and finally we note a selection rule for resonances in this region.

2. THE GENERAL NATURE AND ORIGIN OF THE SCHMID LOOPS

2.1. Partial wave expansion of a Regge pole term

We begin with the simple example of the development of the signature factor into partial waves

$$1 \pm e^{-i\pi\alpha} = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\vartheta_s) [\delta_{l0} + a_l] \quad (2.1)$$

with

$$a_l = \pm e^{-i\pi\alpha_0} i^l e^{2q^2\pi\alpha_1 i} j_l(-2q^2\pi\alpha_1), \quad (2.2)$$

$$\begin{aligned} \alpha &= \alpha(t) = \alpha_0 + \alpha_1 t, \\ t &= -2q^2 (1 - \cos\vartheta_s) \end{aligned} \quad (2.3)$$

and q being the c.m.s. momentum. The expression a_l consists of four factors. The first two factors represent a constant phase. The most crucial is the third factor which generates anticlockwise circular motion on the Argand diagram with angular velocity depending on α_1 . Finally, the fourth factor determines the instantaneous radius of the circle. Due to the damped oscillations of j_l with periods slightly different from those of the third factor, the circular motion tends to eventually spiral and is accompanied by a very slow precession. The behaviour of a_0 , a_1 and a_2 , for an odd signature factor, is shown in Fig. 1 where we have taken $\alpha = 0.5 + t$.

When the Regge contribution

$$\beta(t) \left(\frac{s}{s_0}\right)^\alpha \frac{1 \pm e^{-i\pi\alpha}}{\Gamma(\alpha) \sin\pi\alpha} \quad (2.4)$$

is developed, the factors $\beta(t)$ and $\Gamma(\alpha)\sin\pi\alpha \equiv \pi/\Gamma(1-\alpha)$, which are smooth for negative t may be considered as constant for an appropriate choice of s_0 . To include the factor $(s/s_0)^\alpha$ one has to replace in Eq. (2.2) the expression $2\pi q^2 \alpha_1$ by $2(\pi + i \log \frac{s}{s_0}) \alpha_1 q^2$ and in Eq. (2.1) the term δ_{e_0} by a more complicated functions. This has the general effect of damping the circular motion with this motion terminated after one or two periods.

2.2 Partial wave expansion of the spin-flip meson-nucleon amplitude

We define the partial wave projection of the B amplitude ^{*)}

by

$$B_\ell = \frac{1}{2} \int_{-1}^1 (1-x^2) P'_\ell(x) B dx \quad (2.5)$$

where $P'_\ell(x)$ is the derivative of the Legendre polynomial and $x = \cos \vartheta_s$. The inverse formula is

$$B = \sum_\ell c_\ell B_\ell P'_\ell \quad \text{with} \quad c_\ell = \frac{2\ell+1}{\ell(\ell+1)}. \quad (2.6)$$

Using above formulae one finds

$$B_\ell = \frac{4\pi}{c_\ell(E+M)} (f_{\ell-1}^+ - f_{\ell+1}^-) + \frac{4\pi}{c_\ell(E-M)} (f_\ell^- - f_\ell^+) \quad (2.7)$$

where E and M denote the nucleon c.m.s. energy and the mass. The functions $f_\ell^\pm$ are the standard meson-nucleon partial waves ³⁾ and the + or - superscripts mean then $J = \ell + \frac{1}{2}$ or $\ell - \frac{1}{2}$.

^{*)} This definition differs a little from that used by Schmid in Ref. ¹⁾.

When one develops the signature factor in terms of $P'_\ell(\cos\Theta)$ instead of $P_\ell(\cos\Theta)$ the situation is quite similar to that discussed in Section 2.1. The corresponding formula is

$$1 \pm e^{-i\pi\alpha} = \sum_{\ell=1}^{\infty} (2\ell+1) P'_\ell(\cos\Theta_s) \times$$

$$\left[\frac{1}{3} \delta_{\ell 1} \pm e^{-i\pi\alpha_0} (-i)^{\ell-1} e^{2q^2\pi\alpha_1 i} \frac{j_\ell(2\pi\alpha_1 q^2)}{2\pi\alpha_1 q^2} \right]. \quad (2.8)$$

The only relevant difference is in the factor $(2\pi\alpha_1 q^2)^{-1}$ which makes the loops shrink more rapidly as the energy increases.

If a damping factor e^{at} , (coming from s^α), is included the size of the loops diminishes; the tops associated with the first loops are shifted toward lower energies and the circular motion, in particular that associated with the secondary loops, is greatly retarded. The imaginary part of a few partial waves with various damping factors, together with that for the Regge amplitude of Eq. (2.9) is shown in Fig. 2.

We have also studied how sensitive are these phenomena to the shape of the trajectory. It appears that if the trajectory flattens out for large negative t then there remains only the first, or in some cases the first few loops. The position of the top of the first loop is however fairly stable for a given trajectory intercept and slope at $t=0$.

In Fig. 3 we show the partial wave for the spin flip amplitude

$$B = \left(\frac{E_L}{E_0} \right)^{\alpha-1} \frac{1 - e^{-i\pi\alpha}}{\sin\pi\alpha \Gamma(\alpha)} \quad (2.9)$$

with $E_0 = 0.7 \text{ GeV}$ and E_L being the total lab. energy of the incident pion ¹⁾. The dashed line corresponds to the straight-line trajectory $\alpha = 0.5 + t$. The solid line is for the trajectory function defined as follows

$$\alpha = \begin{cases} 0.5 + t & \text{for } t \geq -1 \text{ GeV}^2 \\ 0.5 - \sqrt{2 - (t+2)^2} & \text{for } -2 \leq t < -1 \text{ GeV}^2 \\ 0.5 - \sqrt{2} & \text{for } t < -2 \text{ GeV}^2 \end{cases} \quad (2.10)$$

For this Regge amplitude the position of the top of the loop is also not sensitive to the shape of the trajectory. It is given by the approximate expression

$$q_e^2 \approx \frac{\ell + 2\alpha_0 - 2}{4\alpha_1} \quad (2.11)$$

and the actual energy value for $\ell \leq 5$ is somewhat smaller than that determined by Eq. (2.11).

We also note that the distance between points corresponding to equal energy intervals decreases with the energy. This behaviour is different from that obtained with a simple Breit-Wigner form.

Finally we remark that the circular motion on the Argand diagram is essentially caused by the phase factor $e^{-i\pi\alpha}$. Such a factor is, however, a general feature of the amplitudes behaving asymptotically like s^α or, more generally, like $s^\alpha (\log s)^\beta (\log \log s)^\gamma \dots$ for fixed t [cf., review papers quoted in Ref. ⁴⁾]. It is, therefore, not a result of the particular form of a Regge pole contribution.

3. REGGE LOOPS AND RESONANCES

There are several remarks which we would like to make in connection with the Regge loops.

- 1) Since the loops are closely related to the phase factor $\exp(-i\pi\alpha)$, they can appear even at rather low energy where the Regge amplitude gives still a poor representation of the physical amplitude. If one wants to associate the loops coming from a given Regge parametrization with the experimentally observed structures, one should restrict oneself to the energy region where the Regge parametrization describes well the physical amplitude. For example, let us suppose that there exists an energy region in πN scattering, where $B_\rho \sim B^{(-)}$, but where A'_ρ is a poor approximation to $A'^{(-)}$, then one can learn about the behaviour of $B_e^{(-)}$ from B_ρ but not about $A'_e^{(-)}$ from A'_ρ . We call such a region the "intermediate energy region" and shall discuss it later. A priori, looking at the behaviour B_e^ρ alone, one does not know whether it corresponds approximately to a "resonance" or not. It is also difficult to decide if this structure could be accompanied by a bump in the total cross-section, as the total cross-section is entirely determined by A' . Even when B_e is well approximated by B_ρ , there might be some structures in σ_T related to the loops in B_e^ρ , since in the intermediate energy region, by definition A' cannot be described by A'_ρ . We find, therefore, that it is more useful to treat B_ρ and A'_ρ separately instead of talking about the ρ exchange contribution as a whole, especially in the intermediate energy region. By the same token, we feel it is difficult to discuss the implications of the Regge loops associated with the sum of P , P' and ρ exchange [cf., Ref. 2)], in the region where the whole Regge amplitude gives a poor representation of the data.

- 2) Let us now discuss the experimental situation with regard to the existence of this intermediate energy region.

We first recall the results of Dolen, Horn and Schmid ⁵⁾. They demonstrated that Regge parameters calculated from the finite energy sum rule for the $B^{(-)}$ amplitude are not sensitive to the cut-off for $P_L \gtrsim 1$ GeV/c. This is due to the fact that most of the resonance contributions to the sum rule integrand $\text{Im} B^{(-)}$ have the same sign. On the other hand, when evaluating Regge parameters for the $A'^{(-)}$ amplitude, it is necessary for them to choose much higher cut-off energy ($P_L \gtrsim 3$ GeV/c). In the latter case, the resonances contribute with different signs depending whether $I = \frac{1}{2}$ or $I = \frac{3}{2}$. This suggests that the intermediate energy region could plausibly be somewhere within 1 to 3 GeV/c, because the Regge behaviour for $B^{(-)}$ seems to set in around 1 GeV/c while for $A'^{(-)}$ it does not seem to set in until about 3 GeV/c.

Phase shifts solutions ⁶⁾ suggest a smooth variation for $|B^{(-)}|$ as a function of P_L at $t=0$ for $P_L \gtrsim 1.2$ GeV/c. There is also some crude resemblance between $B^{(-)}$ obtained from phase shifts and the B_0 amplitudes plotted on the Argand diagram as a function of t for $|t| < 0.8$ GeV². On the other hand, the $\text{Im} B^{(-)}$ calculated from phase shift solutions shows structures at $t=0$. Due to the large uncertainties associated with present solutions, no definite conclusion can be drawn about the existence of this intermediate energy region from the phase shifts. However, the proposal of this region is not inconsistent with the data. This intermediate region could occur for $1.2 \lesssim P_L \lesssim 2.4$ GeV/c.

3) Note that in the expression (2.7) the second term is enhanced in comparison with the first one by a factor of $(E+M)/(E-M) = (E+M)^2/q^2$. For pion-nucleon scattering at P_L about 2 GeV/c this factor is of the order of 1/7. We can neglect, as it was done in Ref. ¹⁾, the first factor [this assumption is also supported by the present phase-shift data, Ref. ⁶⁾] and obtain

$$B_\ell \approx \frac{4\pi}{c_\ell(E-M)} (f_\ell^- - f_\ell^+). \quad (3.1)$$

Writing the isospin indices explicitly one has

$$B_\ell^{(-)} \approx \frac{2\pi}{(E-M)c_\ell} \{ f_\ell^-(\frac{1}{2}) - f_\ell^+(\frac{1}{2}) - f_\ell^-(\frac{3}{2}) + f_\ell^+(\frac{3}{2}) \} \quad (3.2)$$

Should one want to associate the energy value near the top of the loop of $B_\ell^{(-)}$ with the "resonance" position [cf. Eq. (2.11)], in the region where the first term of Eq. (2.7) can be ignored, one finds the following selection rule ⁷⁾, that only

$$\begin{aligned} I = \frac{1}{2}, \quad J = \ell - \frac{1}{2} \\ \text{and} \quad I = \frac{3}{2}, \quad J = \ell + \frac{1}{2} \end{aligned} \quad (3.3)$$

resonances can participate. This rule is satisfied for prominent πN "resonances" and is in agreement with a more general rule for $S=0$ prominent baryon resonances ⁸⁾

$$J - \ell = I - 1. \quad (3.4)$$

4. CONCLUSION

The loops on the Argand diagram appear because of the asymptotic phase factor $\exp(-i\pi\alpha)$ and are not necessarily related to the particular form of the Regge contribution. They are not accompanied, in general, by poles in the complex s plane (although it is possible that the asymptotic expansions are good only for real values of s , as suggested by Schmid), but under some circumstances can be accompanied by bumps in the total cross-section. The natural question is therefore, how to distinguish these structures from the true resonances (poles in the complex s plane). The true resonance should appear in all the communicating channels, as has already been stressed by Collins et al.²⁾, including those governed by the exchange of different Regge poles or, more generally, with different asymptotics. On the other hand, structures discovered by Schmid and discussed by other authors depend crucially on the parameters of the Regge trajectories. For example, for linear trajectory the energy corresponding to the top of the loop strongly depends on α_0 and α_1 [cf. Eq. (2.11)]. Therefore, if a structure is found at the same position and with the same quantum numbers in two or more reactions with different asymptotic behaviours, we consider this as a strong argument that the structure is a resonance.

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- 7) The sign of $\text{Im} B_3$ at $t=0$ is determined to be positive from the high-energy polarization data of πp elastic scattering.
- 8) We thank Dr. Bertocchi for informing us about this rule, which was reported by V. Barger at Oregon conference, Eugene, Oregon (1968).

FIGURE CAPTIONS

- Figure 1 : Partial waves for a signature factor as defined by Eq. (2.1) with $\alpha_0 = 0.5$ and $\alpha_1 = 1.0$. (a) $\ell = 0$, (b) $\ell = 1$ and (c) $\ell = 2$.
- Figure 2 : Imaginary parts of the partial waves for $e^{at}(1 - e^{-i\pi\alpha})$ for various values of a . Here $\alpha = 0.5 + t$. The dotted line indicates the corresponding partial waves for the amplitude given by Eq. (2.9).
- Figure 3 : Partial waves for the spin-flip Regge amplitude from Eq. (2.9). Broken curves correspond to a linear trajectory $\alpha = 0.5 + t$ and the solid curves correspond to a trajectory which is flat for large negative t [Eq. (2.10)]. (a) $\ell = 2$, (b) $\ell = 3$, (c) $\ell = 4$ and (d) $\ell = 5$.

FIG. 1a

FIG.1c

FIG.2

FIG. 3a

FIG. 3b

FIG. 3c

FIG. 3d