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SOME ELECTROMAGNETIC POLARIZATION EXPRESSIONS

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A B S T R A C T

The matrix element squared
two spin half fermions (both may have an anomalous magnetic moment) is calculated in the first Born approximation, as a function of two polarizations.

In connection with the study of polarization properties of particle beams, sometimes one has to use expressions for the cross-section of electromagnetic scattering of polarized particles, with anomalous magnetic moments. It seems that some of these expressions do not exist in the literature and it is the purpose of this paper to give a number of these formulae. The calculation is done in the first Born approximation. The derivation is straightforward. A few intermediate steps are given to make it easier to find any possible algebraic mistakes.

The expressions to be given are valid for the electromagnetic scattering of any two spin $\frac{1}{2}$ fermion, whose interaction Lagrangian with the electromagnetic field can be characterized by two form factors $\Gamma^{(1)}$ and $\Gamma^{(3)}$

$$\mathcal{L}_{int} = e \sum_{\mu} A^{\mu} \bar{\Psi} [\Gamma^{(1)} (p^{\mu} + p'^{\mu}) + \Gamma^{(3)} \gamma^{\mu}] \Psi \quad (1)$$

Here Ψ absorbs the fermion, A^{μ} creates a photon, the ingoing and outgoing momenta of the fermion are p and p' , and e is the electron charge. The $\Gamma^{(1)}$ and $\Gamma^{(3)}$ are some functions of $(p'-p)^2$ only.

All known elementary particles satisfy the above condition, which follows from quite general principles, for example from Lorentz invariance and current conservation alone. (In addition, they all have charge e , but we shall not use this fact.)

The momenta of the two elementary fermions will be denoted by p_1 and p_2 . The form factors of particle i are $\Gamma_i^{(1)}$, $\Gamma_i^{(3)}$, its covariant spin vector is S_i , and its mass M_i . Quantities referring to final particles are denoted by a prime.

2.

In the first Born approximation we can write for the matrix element squared

$$\begin{aligned}
 & |\langle p_1', s_1'; p_2', s_2' | S | p_1, s_1; p_2, s_2 \rangle|^2 = \\
 & = \frac{e^4}{64 q^4} \frac{1}{M_1^2 M_2^2} \sum_{\mu\nu} T_1^{\mu\nu} \left\{ [1 + i\gamma^5 (\gamma s_1')] [1 + i\gamma^5 (\gamma s_1)] \right\} \cdot \\
 & \quad \cdot T_2^{\mu\nu} \left\{ [1 + i\gamma^5 (\gamma s_2')] [1 + i\gamma^5 (\gamma s_2)] \right\}
 \end{aligned} \tag{2}$$

where

$$\begin{aligned}
 T_i^{\mu\nu} \{y, x\} &= \text{Tr} \left\{ \gamma (\Gamma_i^{(1)} P_i^\mu + \Gamma_i^{(3)} \gamma^\mu) (M_i - i\gamma P_i) x \beta \cdot \right. \\
 & \quad \left. (\Gamma_i^{(1)} P_i^\mu + \Gamma_i^{(3)} \gamma^\mu) \beta (M_i - i\gamma P_i') \right\} \\
 q &= p' - p = p_2 - p_2' \\
 P_i &= p_i' + p_i
 \end{aligned} \tag{3}$$

Time reversal invariance implies

$$\Gamma_i^{(3)} = \Gamma_i^{(3)*} \quad , \quad \Gamma_i^{(1)} = -\Gamma_i^{(1)*} \tag{4}$$

and we find

$$\begin{aligned}
 T_i^{\mu\nu} \{1, 1\} &= 4(-1)^{1+\delta_{\nu 4}} \left\{ -(\Gamma_i^{(1)})^2 (M_i^2 - p_i p_i') P_i^\mu P_i^\nu + \right. \\
 & \quad + (\Gamma_i^{(3)})^2 [\delta_{\mu\nu} (M_i^2 + p_i p_i') - P_i^\mu P_i'^\nu - P_i^\nu P_i'^\mu] - \\
 & \quad \left. - i \Gamma_i^{(1)} \Gamma_i^{(3)} 2 M_i P_i^\mu P_i^\nu \right\}
 \end{aligned} \tag{5a}$$

$$T_i^{\mu\nu} \{1, \gamma^5(\gamma S_i)\} = T_i^{\mu\nu} \{\gamma^5(\gamma S_i), 1\} =$$

$$(-1)^{1+\delta_{\nu 4}} \left\{ (\Gamma_i^{(3)})^2 M_i \sum_{\alpha\beta} \epsilon_{\mu\nu\alpha\beta} S_i^\alpha q^\beta - \right.$$

$$\left. - \sqrt{(-1)} \Gamma_i^{(1)} \Gamma_i^{(3)} \sum_{\alpha\beta\gamma} (P_i^\mu \epsilon_{\nu\alpha\beta\gamma} - P_i^\nu \epsilon_{\mu\alpha\beta\gamma}) P_i^\alpha P_i^\beta S_i^\gamma \right\} \quad (5b)$$

Observing that under an exchange $\mu \leftrightarrow \nu$, the $T_i^{\mu\nu}(1,1)$ is unchanged, while $T_i^{\mu\nu} \{1, \gamma^5(\gamma S_i)\}$ switches sign, we conclude that

$$|\langle P_1', S_1'; P_2', 0 | S | P_1, 0; P_2, 0 \rangle|^2 =$$

$$= |\langle P_1', 0; P_2', 0 | S | P_1, S_1; P_2, S_2 \rangle|^2 = 0 \quad (6)$$

Thus, in the first Born approximation, the electromagnetic scattering of two unpolarized fermions satisfying Eq. (1) leaves both particles unpolarized; the angular distribution when scattering two such unpolarized particles is the same as when one of the incoming particles is polarized, but the other one is not, etc.

$$|\langle P_1', 0; P_2', 0 | S | P_1, S_1; P_2, S_2 \rangle|^2 = \frac{e^4}{4q^4 M_1^2 M_2^2} \left\{ [4(P_1 P_2)^2 + 2(P_1 P_2)q^2] G_1^2 G_2^2 - \right.$$

$$- q^2 M_1^2 (\Gamma_1^{(3)})^2 G_1^2 - q^2 M_2^2 (\Gamma_2^{(3)})^2 G_2^2 + \frac{q^4}{2} H^4 + 2q^2 M_1 M_2 \cdot$$

$$\cdot \left\{ (\Gamma_1^{(3)})^2 (\Gamma_2^{(3)})^2 [(S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q)] - (\Gamma_1^{(3)})^2 \Gamma_2^{(3)} K_2 F_2^{(2)} [(1+q^2/4M_2^2) \cdot \right.$$

$$\cdot (S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) - \frac{1}{2M_2^2} (S_1 P_2)(S_2 q)] - (\Gamma_1^{(3)})^2 \Gamma_2^{(3)} K_1 F_1^{(2)} \cdot$$

$$\cdot [(1+q^2/4M_1^2)(S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) + \frac{1}{2M_1^2} (S_1 q)(S_2 P_1)] +$$

$$+ \Gamma_1^{(3)} \Gamma_2^{(3)} K_1 F_1^{(2)} K_2 F_2^{(2)} [(1+q^2/4M_1^2)(1+q^2/4M_2^2)(S_1 S_2) +$$

$$+ (1/q^2 - (P_1 P_2)/4M_1^2 M_2^2)(S_1 q)(S_2 q) + \frac{1}{2M_1^2} (1+q^2/4M_2^2)(S_1 q)(S_2 P_1) -$$

$$\left. - \frac{1}{2M_2^2} (1+q^2/4M_1^2)(S_1 P_2)(S_2 q) \right] \} \} \quad (7)$$

4.

where

$$\begin{aligned}
 G_k^2 &= (F_k^{(1)})^2 + \left(K_k F_k^{(2)} \frac{q}{2M_k} \right)^2 \\
 H^4 &= (\Gamma_1^{(3)})^2 (\Gamma_2^{(3)})^2 + \frac{1}{2} [(\Gamma_1^{(3)})^2 - G_1^2] [(\Gamma_2^{(3)})^2 - G_2^2] \\
 F_k^{(1)} &= \Gamma_k^{(3)} - 2M_k |\Gamma_k^{(1)}| \\
 K_k F_k^{(2)} &= -\sqrt{(-1)} 2M_k \Gamma_k^{(1)}
 \end{aligned} \tag{7a}$$

According to this definition, the anomalous magnetic moment of the i^{th} particle is K_i .

The expression for $|\langle p_1', S_1'; p_2', 0 | S | p_1, 0; p_2, S_2 \rangle|^2$ can be obtained from Eq. (7) by the substitution

$$\begin{aligned}
 S_1 &\rightarrow S_1' \\
 (P_1, P_2) &\rightarrow -(P_1, P_2') = -(P_1, P_2) - q^2/2
 \end{aligned} \tag{8}$$

SPECIAL CASES

1) Proton-proton scattering

In this case, for $i = 1, 2$

$$M_i = M, \quad \Gamma_i^{(k)} = \Gamma_P^{(k)}, \quad F_i^{(k)} = F_P^{(k)}, \quad K_i = K_P \tag{9}$$

and

$$\begin{aligned}
 |K_P', 0; P_2', 0 | S | P_1, S_1; P_2, S_2 \rangle|^2 &= \frac{e^4}{4q^4 M^4} \cdot \\
 &\cdot \left\{ \left[4(P_1 P_2)^2 + 2(P_1 P_2) q^2 \right] G_P^4 - 2q^2 M^2 G_P^2 (\Gamma_P^{(3)})^2 + \frac{1}{2} q^4 H_P^4 + \right. \\
 &+ 2 M^2 q^2 (\Gamma_P^{(3)})^2 \left\{ (\Gamma_P^{(3)})^2 \left[(S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) \right] - \right. \\
 &- 2 \Gamma_P^{(3)} K_P F_P^{(2)} \left[(1 + q^2/4 M^2) (S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) + \right. \\
 &+ \frac{1}{4 M^2} \left[(S_1 q)(S_2 p) - (S_1 p_2)(S_2 q) \right] \left. \right] + K_P^2 (F_P^{(2)})^2 \cdot \\
 &\cdot \left[(1 + q^2/4 M^2) (S_1 S_2) + \left(\frac{1}{q^2} - (P_1 P_2)/4 M^4 \right) (S_1 q)(S_2 q) + \right. \\
 &+ \left. \left. \frac{1}{2 M^2} (1 + q^2/4 M^2) \left[(S_1 q)(S_2 p_1) - (S_1 p_2)(S_2 q) \right] \right\} \right\} \quad (10)
 \end{aligned}$$

where

$$\begin{aligned}
 G_P^2 &= (F_P^{(1)})^2 + (K_P F_P^{(2)} q / 2 M)^2 \\
 H^4 &= (\Gamma_P^{(3)})^4 + \frac{1}{2} [(\Gamma_P^{(3)})^2 - G_P^2]^2 \quad (10a)
 \end{aligned}$$

To take into account the Pauli principle, the appropriate antisymmetrization has to be carried out in Eq. (10). For most purposes, when q^2 is small, this gives only a small correction.

2) Proton-electron scattering

The proton we call the first particle, and the electron the second. In this case

$$\begin{aligned}
 M_1 &= M, \quad \Gamma_1^{(k)} = \Gamma_P^{(k)}, \quad F_1^{(k)} = F_P^{(k)}, \quad K_1 = K_P \\
 M_2 &= m, \quad F_2^{(1)} = 1, \quad F_2^{(2)} = 0
 \end{aligned}$$

6.

$$\begin{aligned}
 | \langle P_1', 0, P_2', 0 | S | P_1, S_1; P_2, S_2 \rangle |^2 &= \frac{e^4}{4q^4 M^2 m^2} \cdot \left\{ \left[4(P_1 P_2)^2 + 2(P_1 P_2) q^2 \right] G_P^2 - q^2 \left[M^2 G_P^2 + m^2 (\Gamma_P^{(3)})^2 \right] + \right. \\
 &+ \frac{1}{2} q^4 (\Gamma_P^{(3)})^4 + 2 M m q^2 \Gamma_P^{(3)} \left\{ \Gamma_P^{(3)} \left[(S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) \right] - \right. \\
 &- K_P F_P^{(2)} \left[(1 + q^2/4M^2) (S_1 S_2) - \frac{1}{q^2} (S_1 q)(S_2 q) + \right. \\
 &+ \left. \left. \frac{1}{2M^2} (S_1 q)(S_2 P_1) \right] \right\} \left. \right\} \quad (11)
 \end{aligned}$$

and G_P^2 is defined in Eq. (10a).