CM-P00057776

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Ref. TH. 862

MULTIPLE-SCATTERING CORRECTIONS TO THE JOHNSON-TREIMAN RELATION
IN THE QUARK MODEL

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A B S T R A C T

We apply the Glauber formalism for multiple scattering to meson-nucleon interactions, using a quark model for the mesons. Assuming only that the quark-nucleon interactions maintain isosymmetry, we find immediately an equation relating the Johnson - Treiman sum of amplitudes to correction terms arising from double-scattering effects. The correction terms turn out to be very small, consistent with the excellent agreement of the Johnson-Treiman relation with experiment.

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Of the various predictions obtained in the independent quark model, the most nearly ubiquitous is the Johnson-Treiman relation ¹⁾ among the meson-nucleon scattering amplitudes

$$F(\pi^+p) + F(K^+N) + F(K^-p) = F(\pi^-p) + F(K^+p) + F(K^-N) \quad (1)$$

The ease with which this result is obtained follows from the fact that it depends only on two basic assumptions, namely, quark structure for the mesons and the additivity of quark-nucleon amplitudes.

Franco ²⁾, however, has recently pointed out that there may be sizeable contributions to the scattering amplitude coming from double-scattering effects, and that these contributions are not additive. In this paper we shall apply the formalism developed by Franco and Glauber ³⁾ for including double-scattering effects in deuteron collisions to the quark model calculation of meson-nucleon scattering amplitudes. We shall find that within reasonable approximations it is still possible to derive the Johnson-Treiman relation, despite the failure of the additivity assumption, because the double-scattering corrections are small.

We begin by assuming that the meson-nucleon scattering amplitude can be written

$$F_{MN}(\vec{k}, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2b \, e^{i\vec{\Delta} \cdot \vec{b}} T_{MN}(\vec{b}, \vec{k}) \quad (2)$$

where $\vec{k}$ is the nucleon momentum in the rest frame of the meson, $\vec{\Delta}$ the momentum transfer, and F_{MN} and T_{MN} are matrices in the isospace of the meson-nucleon system. We neglect complications due to spin, since we shall only consider forward scattering. Resolving the meson into its component quarks, we write analogously to the treatment of nucleon-deuteron scattering

2.

$$T_{Nq}(b, k) = \int d^3 \vec{s} \phi_M^*(\vec{s}) \left[T_{qN}(\vec{b} + \frac{\vec{s}}{2}) + T_{\bar{q}N}(\vec{b} - \frac{\vec{s}}{2}) - \frac{1}{2} \{ T_{qN}(\vec{b} + \frac{\vec{s}}{2}), T_{\bar{q}N}(\vec{b} - \frac{\vec{s}}{2}) \}_+ \right] \phi_M(\vec{s}) \quad (3)$$

where $\phi_M(\vec{s})$ is the bound-state spatial wave function for a quark-antiquark pair at positions $\vec{s}/2$ and $-\vec{s}/2$ respectively, and the anti-commutator has been taken to symmetrize the double scattering term. The nucleon could also be considered in terms of its quark structure, of course, but we shall not find it necessary to do so.

The most general form of the function T_{qN} describing the interaction between a nucleon and an isodoublet quark is conveniently written

$$T_{qN}(\vec{b}, \vec{k}) = A(\vec{b}, \vec{k}) + 4B(\vec{b}, \vec{k}) \vec{T}_q \cdot \vec{T}_N \quad (4a)$$

where $\vec{T}_q$ and $\vec{T}_N$ are the isospin operators for the quark and nucleon respectively. For the isosinglet λ quark there is only one function, which we define as

$$T_{\lambda N}(\vec{b}, \vec{k}) = C(\vec{b}, \vec{k}) \quad (4b)$$

We define similarly functions $\bar{A}(\vec{b}, \vec{k})$, $\bar{B}(\vec{b}, \vec{k})$ and $\bar{C}(\vec{b}, \vec{k})$ describing nucleon-antiquark interactions. Using these definitions in Eq. (3) we easily find the following results:

$$\Gamma_{\pi^+p}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{\pi^+}^*(\vec{s}) \{A+B+\bar{A}+\bar{B} - A\bar{A} - A\bar{B} - B\bar{A} - B\bar{B}\} \phi_{\pi^+}(\vec{s}) \quad (5a)$$

$$\Gamma_{\pi^-p}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{\pi^-}^*(\vec{s}) \{A-B+\bar{A}-\bar{B} - A\bar{A} + A\bar{B} + \bar{A}B - B\bar{B}\} \phi_{\pi^-}(\vec{s}) \quad (5b)$$

$$\Gamma_{K^+p}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{K^+}^*(\vec{s}) \{A+B+\bar{C} - A\bar{C} - B\bar{C}\} \phi_{K^+}(\vec{s}) \quad (5c)$$

$$\Gamma_{K^+N}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{K^+}^*(\vec{s}) \{A-B+\bar{C} - A\bar{C} + B\bar{C}\} \phi_{K^+}(\vec{s}) \quad (5d)$$

$$\Gamma_{K^-p}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{K^-}^*(\vec{s}) \{C+\bar{A}-\bar{B} - C\bar{A} + C\bar{B}\} \phi_{K^-}(\vec{s}) \quad (5e)$$

$$\Gamma_{K^-N}(\vec{b}, \vec{k}) = \int d^3\vec{s} \phi_{K^-}^*(\vec{s}) \{C+\bar{A}+\bar{B} - C\bar{A} - C\bar{B}\} \phi_{K^-}(\vec{s}) \quad (5f)$$

where the argument of A , B , C is $(\vec{b} + \frac{\vec{s}}{2})$, and of $\bar{A}$, $\bar{B}$, $\bar{C}$ is $(\vec{b} - \frac{\vec{s}}{2})$.

If we define "quark-nucleon scattering amplitudes" by

$$f_A(\vec{k}, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{\Delta} \cdot \vec{b}} A(\vec{b}, \vec{k}) \quad (6)$$

and similarly for f_B , f_C , $f_{\bar{A}}$, $f_{\bar{B}}$, $f_{\bar{C}}$, then it follows that the contribution of single scattering terms is of the form

$$\frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{\Delta} \cdot \vec{b}} \int d^3\vec{s} \phi_M^*(\vec{s}) A(\vec{b} + \frac{\vec{s}}{2}) \phi_M(\vec{s}) = f_A(\vec{k}, \vec{\Delta}) S_M(-\frac{\vec{\Delta}}{2}) \quad (7)$$

4.

where

$$S_M(-\frac{\vec{\Delta}}{2}) = \int d^3\vec{s} \phi_M^*(\vec{s}) e^{-i\vec{s} \cdot \frac{\vec{\Delta}}{2}} \phi_M(\vec{s}) \quad (8)$$

is the meson form factor, and in particular $S_M(0) = 1$. There are, of course, analogous contributions from B , C , $\bar{A}$, $\bar{B}$ and $\bar{C}$. The double scattering terms are of the form

$$\begin{aligned} \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{\Delta} \cdot \vec{b}} \int d^3\vec{s} \phi_M^*(\vec{s}) A(\vec{b} + \frac{\vec{s}}{2}) \bar{A}(\vec{b} - \frac{\vec{s}}{2}) \phi_M(\vec{s}) = \\ = \frac{1}{2\pi ik} \int d^2\vec{q} S_M(\vec{q}) f_A(\vec{k}, \frac{\vec{\Delta}}{2} + \vec{q}) f_{\bar{A}}(\vec{k}, \frac{\vec{\Delta}}{2} - \vec{q}) \end{aligned} \quad (9)$$

In order to evaluate these expressions we now make the assumption that at sufficiently high energies the integral in (9) is well approximated by using exponential diffraction peaks and the so-called strong-binding limit $S(\vec{q}) \approx 1$. Then the right-hand side of (9) becomes for $\vec{\Delta} = 0$

$$\begin{aligned} \frac{1}{2\pi ik} \int d^2\vec{q} S_M(\vec{q}) f_A(\vec{k}, \vec{q}) f_{\bar{A}}(\vec{k}, -\vec{q}) = \\ = \frac{1}{2ik} \int_0^\infty dq^2 f_A(0) f_{\bar{A}}(0) e^{-(r+\bar{r})q^2} \end{aligned} \quad (10)$$

$$= \frac{1}{2ik} \frac{f_A(0) f_{\bar{A}}(0)}{r + \bar{r}}$$

where we have put

$$f_A(\vec{k}, \vec{q}) = f_A(0) e^{-\tau q^2}, \quad f_{\bar{A}}(\vec{k}, \vec{q}) = f_{\bar{A}}(0) e^{-\bar{\tau} q^2}$$

etc. In this approximation the scattering amplitudes resulting from Eqs. (5) can be written

$$F_{\pi^+p}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \lambda \bar{\lambda}] \quad (11a)$$

$$F_{\pi^-p}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \mu \bar{\mu}] \quad (11b)$$

$$F_{K^+p}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \lambda \bar{\nu}] \quad (11c)$$

$$F_{K^+N}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \mu \bar{\nu}] \quad (11d)$$

$$F_{K^-p}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \nu \bar{\mu}] \quad (11e)$$

$$F_{K^-N}(\vec{k}, \vec{0}) = 2ik(\tau + \bar{\tau})[1 - \nu \bar{\lambda}] \quad (11f)$$

where we have defined

$$\lambda = 1 - \frac{f_A(0) + f_{\bar{A}}(0)}{2k(\tau + \bar{\tau})} \quad (12a)$$

6.

$$\mu = 1 - \frac{f_A(0) - f_B(0)}{2k(r+\bar{r})} \quad (12b)$$

$$\nu = 1 - \frac{f_C(0)}{2k(r+\bar{r})} \quad (12c)$$

and similarly for $\bar{\lambda}$, $\bar{\mu}$, and $\bar{\nu}$ in terms of $f_{\bar{A}}(0)$, $f_{\bar{B}}(0)$ and $f_{\bar{C}}(0)$. If we now define a quantity

$$G_{MN}(\vec{k}, 0) = 1 + \frac{i}{2k(r+\bar{r})} F_{MN}(\vec{k}, \vec{0}) \quad (13)$$

then Eqs. (11) imply

$$G_{\pi+p} G_{K^+N} G_{K^-p} = G_{\pi-p} G_{K^+p} G_{K^-N} \quad (14)$$

a relation which becomes after some algebra

$$F_{\pi+p}(\vec{k}, 0) + F_{K^+N}(\vec{k}, 0) + F_{K^-p}(\vec{k}, 0) - F_{\pi-p}(\vec{k}, 0) - F_{K^+p}(\vec{k}, 0) - F_{K^-N}(\vec{k}, 0) = i(S+T)$$

$$S = \frac{1}{2k(r+\bar{r})} \left\{ F_{\pi-p} F_{K^+p} + F_{\pi-p} F_{K^-N} + F_{K^+p} F_{K^-N} - F_{\pi+p} F_{K^+N} - \right. \\ \left. - F_{\pi+p} F_{K^-p} - F_{K^+N} F_{K^-p} \right\} \quad (15)$$

$$T = -i \left(\frac{i}{2k(r+\bar{r})} \right)^2 \left\{ F_{\pi-p} F_{K^+p} F_{K^-N} - F_{\pi+p} F_{K^+N} F_{K^-p} \right\}$$

The left-hand side of (15) is just the combination of amplitudes which vanishes in the Johnson-Treiman relation (1), while S and T are the correction terms which result from the inclusion of multiple scattering effects.

In order to obtain a simple estimate of the magnitudes of the correction terms we assume that all the amplitudes are purely imaginary and use the optical theorem

$$\sigma_{MN}(k) = \frac{4\pi}{k} \text{Im} F_{MN}(\vec{k}, \vec{0})$$

Then (15) becomes

$$\sigma_{\pi^+p} + \sigma_{K^+N} + \sigma_{K^-p} - \sigma_{\pi^-p} - \sigma_{K^+p} - \sigma_{K^-N} = \sigma^{(S)} + \sigma^{(T)} \quad (16)$$

$$\sigma^{(S)} = \frac{1}{8\pi(\gamma + \bar{\gamma})} \left\{ \sigma_{\pi^-p} \sigma_{K^+p} + \sigma_{\pi^-p} \sigma_{K^-N} + \sigma_{K^+p} \sigma_{K^-N} - \right. \\ \left. - \sigma_{\pi^+p} \sigma_{K^+N} - \sigma_{\pi^+p} \sigma_{K^-p} - \sigma_{K^+N} \sigma_{K^-p} \right\} \quad (17a)$$

$$\sigma^{(T)} = \left(\frac{1}{8\pi(\gamma + \bar{\gamma})} \right)^2 \left\{ \sigma_{\pi^-p} \sigma_{K^+p} \sigma_{K^-N} - \sigma_{\pi^+p} \sigma_{K^-p} \sigma_{K^+N} \right\} \quad (17b)$$

We naturally expect the values of γ and $\bar{\gamma}$ to be approximately the same as those of the slopes of the meson-nucleon diffraction peaks, so we take $(\gamma + \bar{\gamma}) = 35$ mb as a reasonable value. Then, we see immediately that since the cross-sections in question are in the range 15-25 mb the terms in $\sigma^{(S)}$ and $\sigma^{(T)}$ are small (~ 0.1 mb) and furthermore tend to cancel. In the Table we present values of the Johnson-Treiman sum [the left-hand side of (16)] and of $\sigma^{(S)}$ and $\sigma^{(T)}$, calculated from the total cross-section data of Galbraith et al.⁴⁾.

$P_{\text{lab}}, \text{Gev}/c$	6	8	10	12	14	16	18
$\sigma(\text{J-T})$	0.3 ± 1.7	1.8 ± 1.6	0.4 ± 1.6	0.0 ± 1.6	0.0 ± 1.6	-0.3 ± 2.0	-0.3 ± 2.9
$\sigma^{(\text{S})}$	0.04	0.1	0.03	0.009	0.007	-0.002	-0.006
$\sigma^{(\text{T})}$	0.0008	0.001	0.0004	0.0002	0.0001	0.00003	0.00004

Values in millibarns of the Johnson-Treiman sum, $\sigma^{(\text{S})}$, and $\sigma^{(\text{T})}$.

The extremely small values of the correction terms result partially from the fact that in the approximation used the double-scattering contribution to the meson-nucleon amplitude is small ($\sim 0.1-0.3$ mb), but even if it were considerably larger the values of $\sigma^{(\text{S})}$ and $\sigma^{(\text{T})}$ would still be quite small because of the cancellation of terms in Eqs. (17a) and (17b).

We, therefore, conclude that the failure of additivity due to the possibility of multiple scattering has negligible effect on the Johnson-Treiman relation.

The author wishes to acknowledge a very helpful conversation with Professor L. Van Hove.

R E F E R E N C E S

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