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RADIATIVE CORRECTIONS TO THE LEPTONIC DECAY MODESOF THE NEUTRAL VECTOR MESONS ρ^0, ω, φ

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The radiative corrections to the leptonic decay rate $\Gamma(V \rightarrow \ell^+ \ell^-)$ of a neutral vector meson $V \equiv \rho^0, \omega$ or φ are evaluated for $\ell^\pm \equiv e^\pm$ or $\mu^\pm$ including the contributions due to the emission of hard photons. Admitting a radiative energy loss of at least 10% of the total c.m. energy of the decaying system, the radiative corrections are found to reduce $\Gamma(V \rightarrow e^+ e^-)$ by at most 10% and to hardly change $\Gamma(V \rightarrow \mu^+ \mu^-)$ in the range of c.m. energies from 500 to 1,500 MeV.

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Several experiments have recently been reported on, which had been set up in order to measure the rare electromagnetic decay modes of neutral vector mesons into lepton pairs ¹⁾, i.e., $V \rightarrow \ell^+ + \ell^-$. Here V stands for ρ^0, ω, ϕ and $\ell^\pm$ for $e^\pm$ or $\mu^\pm$. The results of such investigations can give valuable information about the limits of validity of present-day quantum electrodynamics and/or the possible non-electromagnetic structure of the muon ²⁾. Furthermore, the existing theoretical predictions of the vector-meson-photon coupling strength can be checked in the region of large timelike momentum transfer, thus enlarging our knowledge of the nucleon structure ³⁾.

In most experiments which were so far performed the vector mesons were produced in the reactions

or

where the protons p may be also bound in light target elements. In what follows we shall restrict our discussion to reaction (1).

With increasing accuracy of the experiments it will become necessary for the reliable interpretation of the accumulating data to know the radiative corrections of which care has to be taken in reaction (1). Such corrections are known to be relatively large for colliding beam experiments, in particular, if the colliding particles are electrons of several hundred MeV c.m. energy ⁴⁾.

2. - THE CALCULATIONS

We assume that near resonance the cross-section of the reaction (1) is approximated by the Breit-Wigner formula ^{4),5)}

$$\sigma(E) = \pi \chi^2 \frac{3}{4} \frac{\Gamma_P \Gamma_{V \rightarrow e^+ e^-}}{(2E - m_V)^2 + \Gamma^2/4} \quad (3)$$

where Γ_P is the production rate for the vector particle having mass m_V and total decay width Γ while $\Gamma_{V \rightarrow e^+ e^-}$ is the leptonic decay rate of the meson.

In this paper we shall be concerned with the evaluation of the decay rate $\Gamma_{V \rightarrow e^+ e^-}$ (simply called Γ hereafter) including the radiative corrections to lowest order of the fine structure constant α . For the basic decay process (see Fig. 1a below) we will also admit possible lepton structure and deviations from quantum electrodynamics whereas the radiative corrections will be calculated under the assumption of pointlike leptons interacting in the usual way with the radiation field. These corrections will correspond to the diagrams of Fig. 1b-g.

- Figure 1 -

We shall neglect in our investigation all corrections to the decay

$$V \rightarrow e^+ + e^- \quad (4)$$

which would give rise to a radiative dependence of the decay rate Γ on the particular production process of the vector meson like (1) or (2). A typical diagram leading to such a correction is shown in Fig. 2.

- Figure 2 -

Furthermore we do not consider the radiative corrections to the production rate Γ_P . The evaluation of these corrections would require detailed assumptions about the production mechanism of the vector particles. But it is usually argued that the radiative corrections to Γ_P are comparatively small since the masses of the particles involved in the production are much larger than the lepton masses (which is certainly true for electrons).

As soon as we consider the decay (4) and include the radiative corrections we must also discuss the contributions from the decay

$$V \rightarrow e^+ + e^- + \gamma \quad (5)$$

to the decay rate Γ since this leptonic decay with the simultaneous emission of hard bremsstrahlung is essential for the correct interpretation of the experimental data. These data are obtained by measuring the invariant mass of the emitted lepton pair while the total energy which

is involved in the reaction (1) or (2) can only be estimated very roughly ⁶⁾. Consequently rather hard bremsstrahlung can be emitted in the leptonic decay of the meson without conflicting with the experimental information.

In fact, as we shall see below, in the approximation just described the decay rate Γ may be written in the form

$$\Gamma = \Gamma_0 (1 + \delta_R + \delta_H) \quad (6)$$

where Γ_0 is the decay rate as calculated from Fig. 1a while δ_R stands for the radiative corrections of order α which are determined by Fig. 1b-g thus including the finite contributions of virtual photons as well as of real soft photons whose energy is less or equal to a cut-off energy $\mathcal{E} \ll m$ (m being the lepton mass) and δ_H accounts for the emission of hard bremsstrahlung in the decay (5). It is only through the addition of δ_R and δ_H that the catastrophic $(\ln E/m)^2$ term, known from other calculations to appear in δ_R , can be eliminated so that the result (6) remains finite and makes sense even at high energies E ^{4),7)}.

Let us assume that in general the decay (4) can be described by a current-current type coupling ³⁾. Then the matrix element will be of the form

$$M_i = ie j^\mu(p', p) \int_\nu(q) g_{\mu\nu} \frac{D(q^2)}{q^2} \quad (7)$$

Here $D(q^2)$ is introduced to account for a non-electromagnetic modification of the meson-lepton coupling as well as to denote the change of the photon propagator due to the finite part of the lowest order vacuum polarization corresponding to Fig. 1e ⁸⁾. Hence we write

$$D(q^2) = D_0(q^2) \left(1 - \frac{\alpha}{2\pi} P(q^2)\right) \quad (8)$$

and for a pure electromagnetic interaction we have $D_0(q^2) = 1$. $J_\nu(q)$ apparently is the vector meson current and we put ⁹⁾

$$J_\nu(q) = e_\nu \bar{F}(q^2) \quad (9)$$

with the polarization vector e_ν subjected to the condition $(q \cdot e) = 0$ and $\bar{F}(q^2)$ is the form factor including the meson-photon coupling strength while q is the timelike four-momentum of the decaying system so that $q^2 = -m_V^2$ at resonance ^{*}). Finally for the lepton current $j^\mu(p', p)$ we will take

$$j^\mu(p', p) = j_0^\mu + j_s^\mu + j_v^\mu \quad (10)$$

where j_0^μ is the current corresponding to pointlike leptons and j_s^μ and j_v^μ denote the possible non-electromagnetic and lowest order electromagnetic vertex corrections respectively. The latter is given by the finite part of Fig. 1b,c,d. p' and p are the four-momenta of the lepton and antilepton respectively so that $p'^2 = p^2 = -m^2$ and $p' + p = q$ by momentum conservation. The current corrected for lepton structure will be described by the ansatz ¹⁰⁾

$$j_0^\mu + j_s^\mu = (\bar{u}(p') \gamma^\mu v(p)) (1 + L(q^2)) + (\bar{u}(p') \sigma^{\mu\nu} q_\nu v(p)) \frac{\kappa}{2m} M(q^2) \quad (11)$$

where $L(q^2)$ and $M(q^2)$ are the charge and magnetic moment corrections respectively. Similarly the current due to the electromagnetic vertex correction is known to be of the form ¹¹⁾

^{*}) We put throughout $\hbar = c = 1$ and employ the metric $(a \cdot b) = (\vec{a} \cdot \vec{b}) - a_0 b_0$, a and b being four-vectors. See Ref. ⁸⁾.

$$\begin{aligned}
j_V^\mu &= \bar{u}(p') \Delta_c^\mu(q^2) v(p) \\
&= -\frac{\alpha}{2\pi} [(\bar{u} \not{p} \not{v}) K_E(q^2) + \frac{1}{m} (\bar{u} \sigma^{\mu\nu} q_\nu v) K_M(q^2)] \quad (12)
\end{aligned}$$

with the charge correction $K_E(q^2)$ and magnetic moment correction $K_M(q^2)$ to be given below.

Denoting the matrix element corresponding to the basic diagram (Fig. 1a) by $\mathcal{M}_0$ we will have to add to the probability $|\mathcal{M}_1|^2$ calculated from (7) through (12) the term

$$|\mathcal{M}_0|^2 \left(-\frac{\alpha}{2\pi} K_S(q^2) \right) \quad (13)$$

which accounts for the emission of real soft photons represented by Fig. 1f,g¹²⁾. The function $K_S(q^2)$ is defined by

$$K_S(q^2) = \frac{2}{\pi} \int_{|\vec{k}| \leq \varepsilon} \frac{d^3 k}{\sqrt{\vec{k}^2 + \lambda^2}} \left[\frac{m^2}{(2p' \cdot k)^2} + \frac{m^2}{(2p \cdot k)^2} + \frac{2(p' \cdot p)}{(2p' \cdot k)(2p \cdot k)} \right] \quad (14)$$

and the result of the integration will be given later. In Eq. (14) k denotes the photon four-momentum and λ is the small photon mass usually introduced to by-pass the infra-red divergence difficulties.

The leptonic decay rate can now be evaluated in a straightforward manner by means of the formula

$$\Gamma = \frac{1}{2\pi} \int \delta(p + p' - q) d^3 p d^3 p' \frac{m^2}{p_0 p'_0} |\mathcal{M}|^2 \quad (15)$$

where the probability $|\overline{m\epsilon}|^2$ is summed over the spin orientations of the emitted leptons and over the polarization of the decaying meson. Using the Eqs. (7) to (13) we obtain after integration of (15) in the rest frame of the decaying meson ($\vec{p}' + \vec{p} = 0$)

$$\begin{aligned}\Gamma &= 2 m^2 \sqrt{1-\eta^2} |\overline{m\epsilon}|^2 = 2 m^2 \sqrt{1-\eta^2} (|\overline{m\epsilon}_1|^2 - \frac{\alpha}{2\pi} K_f |\overline{m\epsilon}_0|^2) \\ &= \Gamma_0 \left[1 - \frac{\alpha}{\pi} (\text{Re } K_E + \frac{1}{1+\eta^2/2} \text{Re } K_M + \text{Re } P + \frac{K_S}{2}) \right]\end{aligned}\quad (16)$$

with $\eta = \frac{2m}{m_V}$ and

$$\begin{aligned}\Gamma_0 &= 2 m^2 \sqrt{1-\eta^2} |\overline{m\epsilon}_0|^2 = 4\pi\alpha \frac{\sqrt{1-\eta^2}}{m_V^2} |F|^2 |D_0|^2 \times \\ &\times \left[(2+\eta^2)|1+L|^2 - 4m \text{Re}(M+LM^*) + (1+2\eta^2)m_V^2 |M|^2 \right]^{*})\end{aligned}\quad (17)$$

The expression (16) is correct in so far as terms of order $\frac{\alpha}{\pi} \times$ (non-electromagnetic corrections) are disregarded and in this approximation the decay rate Γ has the predicted form (6).

We have evaluated the real part of the functions K_E , K_M and P by somewhat different techniques than those usually employed and we compared our expressions with those scattered in the literature ¹³⁾. The same was done with K_S though the integration of (14) is rather standard. Our calculations yield

*) Of course one could incorporate D into L and M in order to reduce the number of unknown form factors.

$$\begin{aligned} \operatorname{Re} K_E &= 2 \left(\frac{1+2\rho}{2(1+\rho)} T(\rho) - 1 \right) \left(\ln \frac{m}{\lambda} - 1 \right) + \frac{\rho}{2(1+\rho)} T(\rho) + \frac{1+2\rho}{2(1+\rho)} \gamma (G(\rho) - \pi^2) \\ &= 2(2u \operatorname{ctgh} 2u - 1) \left(\ln \frac{m}{\lambda} - 1 \right) + u \operatorname{ctgh} u + (\operatorname{ctgh} 2u) (F(u) - \pi^2) \end{aligned}$$

$$\operatorname{Re} K_M = \frac{T(\rho)}{4(1+\rho)} = -\frac{u}{\sinh u}$$

$$\operatorname{Re} P = \frac{2}{3} \left[\frac{5}{3} - \frac{1}{\rho} - \left(1 - \frac{1}{2\rho} \right) T(\rho) \right] = \frac{2(1+2\cosh^2 u)}{3\cosh^2 u} (1 - u \operatorname{tgh} u) - \frac{2}{9}$$

$$\begin{aligned} K_S &= 2 \left[2 \left(\frac{1+2\rho}{2(1+\rho)} T(\rho) - 1 \right) \ln \frac{\lambda}{2\varepsilon} - \frac{\rho}{1+\rho} T(\rho) + \frac{1+2\rho}{2(1+\rho)} \gamma G(\rho) \right] \\ &= 2 \left[2(2u \operatorname{ctgh} 2u - 1) \ln \frac{\lambda}{2\varepsilon} - 2u \operatorname{ctgh} u + (\operatorname{ctgh} 2u) F(u) \right] \end{aligned}$$

$$\begin{aligned} G(\rho) &= \frac{1}{\rho} T(\rho) \left(\ln(-4\rho\rho^2) - \frac{1}{2\rho} T(\rho) \right) + \frac{\pi^2}{3} - 2 \mathcal{L}_2 \left(\frac{1-\rho}{1+\rho} \right) \\ &= F(u) = 2u \left(\ln(4 \sinh^2 u) - u \right) + \frac{\pi^2}{3} - 2 \mathcal{L}_2(e^{-2u}) \end{aligned}$$

(18)

We have also expressed our results in the variable u frequently used in the literature ¹³⁾ and which is related to our variable ρ by

$$\rho = \frac{1}{4m^2} (\vec{q}^2 - q_0^2) = -\cosh^2 u$$

$$T(\rho) = \gamma \ln \left| \frac{1+\rho}{1-\rho} \right| ; \quad \gamma = \sqrt{1+1/\rho} \quad \text{for } \rho > 0 \text{ and } \rho < -1$$

$$T(\rho) = 2\gamma \operatorname{arctg} \frac{1}{\gamma} ; \quad \gamma = \sqrt{-1-1/\rho} \quad \text{for } -1 < \rho < 0 \quad (19)$$

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decay into	$r_{\max}$ 2E(MeV)	0.10			0.25			0.50			1.00		
		δ_R	δ_H	δ	δ_R	δ_H	δ	δ_R	δ_H	δ	δ_R	δ_H	δ
electrons $m = m_e$ $\epsilon = 0.01$ m	500.0	- 0.571	0.498	- 0.074	- 0.571	0.543	- 0.028	- 0.571	0.570	- 0.001	- 0.571	0.589	0.018
	600.0	- 0.599	0.523	- 0.076	- 0.599	0.569	- 0.029	- 0.599	0.598	- 0.001	- 0.599	0.617	0.018
	700.0	- 0.623	0.545	- 0.078	- 0.623	0.593	- 0.030	- 0.623	0.621	- 0.001	- 0.623	0.641	0.019
	ρ 765.0	- 0.636	0.558	- 0.079	- 0.636	0.606	- 0.031	- 0.636	0.635	- 0.001	- 0.636	0.655	0.019
	ω 782.0	- 0.640	0.561	- 0.079	- 0.640	0.609	- 0.031	- 0.640	0.637	- 0.001	- 0.640	0.659	0.019
	800.0	- 0.644	0.564	- 0.079	- 0.644	0.613	- 0.031	- 0.644	0.642	- 0.001	- 0.644	0.663	0.019
	900.0	- 0.662	0.582	- 0.081	- 0.662	0.631	- 0.031	- 0.662	0.661	- 0.001	- 0.662	0.682	0.019
	1000.0	- 0.679	0.597	- 0.082	- 0.679	0.648	- 0.032	- 0.679	0.678	- 0.001	- 0.679	0.699	0.020
	ϕ 1019.5	- 0.682	0.600	- 0.082	- 0.682	0.651	- 0.032	- 0.682	0.681	- 0.001	- 0.682	0.702	0.020
	1100.0	- 0.695	0.612	- 0.083	- 0.695	0.663	- 0.032	- 0.695	0.694	- 0.001	- 0.695	0.715	0.020
	1200.0	- 0.709	0.625	- 0.084	- 0.709	0.677	- 0.033	- 0.709	0.708	- 0.002	- 0.709	0.730	0.020
	1300.0	- 0.723	0.637	- 0.085	- 0.723	0.689	- 0.033	- 0.723	0.721	- 0.002	- 0.723	0.743	0.021
	1400.0	- 0.735	0.649	- 0.086	- 0.735	0.702	- 0.034	- 0.735	0.734	- 0.002	- 0.735	0.756	0.021
	1500.0	- 0.747	0.660	- 0.087	- 0.747	0.713	- 0.034	- 0.747	0.745	- 0.002	- 0.747	0.768	0.021
muons $m = m_\mu$ $\epsilon = 0.01$ m	500.0	- 0.019	0.029	0.011	- 0.019	0.036	0.017	- 0.019	0.039	0.020	- 0.019	0.039	0.020
	600.0	- 0.028	0.037	0.009	- 0.028	0.044	0.017	- 0.028	0.048	0.021	- 0.028	0.049	0.021
	700.0	- 0.036	0.043	0.007	- 0.036	0.052	0.016	- 0.036	0.057	0.021	- 0.036	0.058	0.022
	ρ 765.0	- 0.041	0.047	0.007	- 0.041	0.057	0.016	- 0.041	0.062	0.021	- 0.041	0.063	0.023
	ω 782.0	- 0.042	0.048	0.006	- 0.042	0.058	0.016	- 0.042	0.063	0.021	- 0.042	0.064	0.023
	800.0	- 0.043	0.049	0.006	- 0.043	0.059	0.016	- 0.043	0.064	0.021	- 0.043	0.066	0.023
	900.0	- 0.050	0.055	0.005	- 0.050	0.066	0.016	- 0.050	0.071	0.022	- 0.050	0.073	0.024
	1000.0	- 0.056	0.060	0.004	- 0.056	0.072	0.016	- 0.056	0.078	0.022	- 0.056	0.080	0.024
	ϕ 1019.5	- 0.057	0.061	0.004	- 0.057	0.073	0.016	- 0.057	0.079	0.022	- 0.057	0.082	0.024
	1100.0	- 0.062	0.065	0.003	- 0.062	0.077	0.016	- 0.062	0.084	0.022	- 0.062	0.087	0.025
	1200.0	- 0.067	0.070	0.003	- 0.067	0.083	0.015	- 0.067	0.090	0.022	- 0.067	0.093	0.025
	1300.0	- 0.073	0.074	0.002	- 0.073	0.088	0.015	- 0.073	0.095	0.023	- 0.073	0.098	0.026
	1400.0	- 0.077	0.079	0.001	- 0.077	0.093	0.015	- 0.077	0.100	0.023	- 0.077	0.104	0.026
	1500.0	- 0.082	0.083	0.001	- 0.082	0.097	0.015	- 0.082	0.105	0.023	- 0.082	0.109	0.027

Table: Radiative corrections to the leptonic decay of vector mesons.

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