CM-P00057761

Ref. TH. 847

Archives

RADIATIVE PION CAPTURE AND AXIAL CURRENTS IN NUCLEI

M. Ericson

Institut de Physique Nucléaire, Lyon
and
CERN - Geneva

and

A. Figureau

Institut de Physique Nucléaire, Lyon

A B S T R A C T

The PCAC hypothesis is used to generalize the Kroll-Ruderman theorem of threshold photoproduction on nucleons to light nuclei and to express the result in terms of nuclear axial matrix elements. The ($^3\text{He}, \text{T}$) system is studied in more detail including various higher order effects. A Panofsky ratio of 2.70 is obtained as compared to the experimental value of 2.28 ± 0.18 .

The radiative pion capture in complex nuclei is kinematically similar to μ capture. It has been pointed out that it is also formally similar to the π capture axial vector transition ¹⁾⁻⁴⁾. This conclusion was reached in the impulse approximation assuming the elementary processes to be : $\mu + N \rightarrow \nu + N$; $\pi + N \rightarrow \gamma + N$.

The consequence of this similarity is that the ordinary theory of μ capture using an effective Hamiltonian is immediately applicable to radiative π capture in complex nuclei.

The purpose of this letter is to demonstrate our suggestion ⁵⁾ that there is more than a formal analogy between the two processes : in the soft pion limit both are governed by the same model independent axial vector matrix element. We illustrate this approach by a detailed application to the reactions $\pi^- + {}^3\text{He} \rightarrow \gamma + T$ and $\pi^+ + T \rightarrow \gamma + {}^3\text{He}$ with the initial and final nuclei treated as elementary particles.

In weak interaction processes the treatment of nuclei as elementary particles has been used as an alternate method to the impulse approximation ^{6),7),8)}; the transition matrix elements are then expressed in terms of form factors for the nucleus as a whole. This method relates processes which are governed by the same form factors, for instance the axial vector transitions in μ capture and in β decay. But the momentum transfers are very different in the two processes and the axial muon transition cannot be directly deduced from the corresponding β decay transition: it is necessary to postulate the dependence of the axial form factor on q^2 .

We show here that a third reaction, the radiative pion capture can be related to the same axial matrix element. This relation is obtained by the use of the partially conserved axial current hypothesis (PCAC) which has proved to be successful in the similar case of low energy photo-production of pions on nucleons ^{9),10),11)}.

Consider the process

$$\gamma + A \rightleftharpoons \pi + B$$

2.

where A and B are two nuclear states. The amplitude for negative (positive) pion photoproduction $S_{fi}^{\mp}$ is written using reduction techniques:

$$S_{fi}^{\mp} = \langle B \pi^{\mp} | A \gamma \rangle = i \frac{(2\pi)^4}{\sqrt{2q_0}} \delta^4(p + k - p' - q) \langle B | j_{\pi}^{\pm}(0) | A \gamma \rangle \quad (1)$$

where j_{π} is the pion source, $j^{\pm} = (j_1 \pm i j_2)/2$; p and p' are the four momenta of the initial and final nuclei, k and q those of the γ ray and of the pion. We consider s wave pions $\underline{q} = 0$ (in the c.m. system $\underline{p}' = -\underline{q} = 0$, $\underline{p} = -\underline{k}$).

The PCAC hypothesis relates the pion field ϕ to the divergence of the axial current $\partial_{\lambda} J_{\alpha}^{\lambda}$ (12), (13). Including electromagnetic interactions to first order in e it writes (14):

$$(\partial_{\lambda} + i e A_{\lambda}) J_{\alpha}^{\lambda \pm} = i C \phi^{\pm} \quad (2)$$

Here A_{λ} is the electromagnetic potential, the constant C is

$$C = \frac{\sqrt{2} m \mu^2 f_A(0)}{g_r K(0)}$$

where m and μ are the nucleon and pion masses, f_A is the axial form factor of the nucleon, g_r is the pion-nucleon coupling constant, $K(0)$ the off-mass shell pionic form factor of the nucleon [we neglect its momentum dependence and take $K(0) = K(-\mu^2) = 1$].

From relation (2) it follows :

$$\frac{C}{\mu^2 - q^2} \langle B | j_{\pi}^{\pm}(0) | A \gamma \rangle = -q_{\lambda} \langle B | J_{\alpha}^{\lambda \pm}(0) | A \gamma \rangle + e \langle B | A_{\lambda}(0) J_{\alpha}^{\lambda \pm}(0) | A \gamma \rangle \quad (3)$$

We separate out the pion pole terms on both sides :

$$\langle B | j_{\pi}^{\pm}(0) | A \rangle = \frac{\mu^2}{C} \left\{ -q_{\lambda} [\langle B | j_{\pi}^{\lambda \pm}(0) | A \rangle]' \mp e \langle B | A_{\lambda}(0) j_{\pi}^{\lambda \pm}(0) | A \rangle \right\} \quad (4)$$

where the prime indicates that the pion pole term has been subtracted. To lowest order in e : $A_{\lambda}(x) = A_{\lambda}^{\text{in}}(x)$.

The second term on the right-hand side of Eq. (4) is then :

$$\frac{\mu^2}{C} e \langle B | A_{\lambda}(0) j_{\pi}^{\lambda \pm}(0) | A \rangle = \frac{\mu^2}{C} \frac{e}{\sqrt{2}k_0} \epsilon_{\lambda} \langle B | j_{\pi}^{\lambda \pm}(0) | A \rangle \quad (5)$$

where ϵ_{λ} is the polarization vector of the γ ray.

In the soft pion limit $[q_{\lambda} = 0]$ this is the only non-vanishing contribution to the transition amplitude. For the real pion mass, this term dominates over the other one for light nuclei. This dominant term gives the equivalent of the Kroll-Ruderman theorem¹⁵⁾ for complex nuclei :

$$S_{fi}^{\pm} = \mp i \frac{(2\pi)^4 e \mu^2}{2C \sqrt{k_0} q_0} \delta^4(p+k-p'-q) \epsilon_{\lambda} \langle B | j_{\pi}^{\lambda \pm}(0) | A \rangle \quad (6)$$

In this approximation the radiative pion capture is completely equivalent to an axial weak interaction transition and can be treated with standard multipole or form factor techniques.

The correction term is more complex since it includes Born terms with exchange of excited nuclei. It is discussed in detail below in an application to radiative capture by ${}^3\text{He}$ or T : $\pi^{-} + {}^3\text{He} \rightarrow \gamma + T$; $\pi^{+} + T \rightarrow \gamma + {}^3\text{He}$. The nuclei ${}^3\text{He}$ and T are treated as two isospin states of a nucleus of mass M .

The dominant term is first expanded in form factors :

$$\epsilon_{\lambda} \langle B | J_a^{\lambda \pm}(0) | A \rangle = \epsilon_{\lambda} \bar{u}(p') \left[\gamma_5 \gamma_a F((q-k)^2) + (k-q) \gamma_5 \frac{2M F_A((q-k)^2)}{\mu^2 - (q-k)^2} \right] \gamma_5 u(p) \quad (7)$$

where $u(p)$ and $u(p')$ are the Dirac spinors of the initial and final nuclei, and F_A is the axial form factor. We have here assumed that the axial form factor appears with the same momentum transfer in the induced pseudoscalar term.

With this term only, the cross-sections are equal for positive and negative pions (in the same approximation neutral pions would not be radiatively absorbed since this term then is inexistent).

We now discuss the corrective term which has Born terms with nuclei in the intermediate state. Ignoring for the moment the possibility of having intermediate excited states (their influence is discussed later) we separate the Born terms from the rest :

$$\begin{aligned} q_{\lambda} \langle B | J_a^{\lambda \pm}(0) | A \rangle' &= \frac{e}{\sqrt{2}k_0} \left\{ F_A(q^2) \bar{u}(p') \left[\not{q} \gamma_5 \gamma_a \frac{\not{p} + \not{k} + M}{(p+k)^2 - M^2} + \frac{3 + \gamma_5}{2} \right. \right. \\ &\quad \left. \left. + \not{q} \frac{3 + \gamma_5}{2} \frac{\not{p} - \not{k} + M}{(p-k)^2 - M^2} \not{q} \gamma_5 \gamma_a \right] u(p) \right. \\ &\quad \left. + \frac{q_{\lambda} \epsilon}{(q-k)^2 - \mu^2} \frac{e}{\mu^2} \sqrt{2} G((q-k)^2) \bar{u}(p') \gamma_5 \gamma_a u(p) + q^{\mu} \epsilon^{\nu} N_{\mu\nu}(q^2, qk) \right\} \end{aligned} \quad (8)$$

We have not included the anomalous magnetic moment terms in this expression since they contribute only to order $\mathcal{O}(\mu^2/M^2)$ in the amplitude for s waves and are gauge invariant.

We have written the remainder as $q^{\mu} \epsilon^{\nu} N_{\mu\nu}(q^2, qk)$. With our present assumption (neglect of excited intermediate states) $N_{\mu\nu}$ has no pole and we can expand $N_{\mu\nu}$ in powers of q^2 and qk .

Neglecting second order terms in the collective part, the amplitude is :

$$\begin{aligned} \langle B | j_{\pi}^{\pm}(0) | A \rangle = & - \frac{2Mq\mu^2}{C\sqrt{2k}} F_A(q^2 - 2qR) \left(\bar{u}(p) \left[\frac{q}{2M} - \frac{qE}{qR} \right] \epsilon^{\pm} + \left(\frac{qR}{2pR} + \frac{1}{2M} - \frac{pE}{pR} \right) \epsilon^{\pm} \frac{3+\epsilon^2}{2} \right. \\ & \left. + \left(\frac{p'E}{pR} - \frac{qR}{2pR} - \frac{1}{2M} \right) \frac{3+\epsilon^2}{2} \epsilon^{\pm} \right) i \gamma_5 u(p) + \frac{1}{2M} q^{\mu} \epsilon^{\nu} N_{\mu\nu}(q^2=0, q.R=q) \} \end{aligned} \quad (9)$$

Gauge invariance requires ¹¹⁾ : $N_{\mu\nu}(q^2=0, k \cdot q=0) = 0$. The cross-sections for the photoproduction of negatively or positively charged pions is then :

$$\frac{d\sigma^{\mp}}{dR} = A^{\mp} \frac{q}{R} \quad (10)$$

with

$$A^{\mp} = \frac{q^2}{4\pi} \frac{e^2}{4\pi} \frac{F_A^2(-\mu^2)}{2m^2} \frac{1 \pm \frac{3\mu}{M}}{\left(1 + \frac{\mu}{M}\right)^2} \quad (11)$$

The axial form factor $F_A(-\mu^2)$ has been estimated from the muon capture. The square momentum transfers in pion and muon capture are respectively 0.47 and 0.27 F^{-2} and $F_A(-0.27) = 0.88$ as given by Drechsler ⁶⁾. We have made a linear extrapolation of F_A in q^2 giving $F_A(-0.47) = 0.80$. The numerical values are then : $A^- = 15.6 \mu\text{b}$; $A^+ = 11.8 \mu\text{b}$. The corresponding radiative absorption probability : $\pi^- + {}^3\text{He} \rightarrow \gamma + \text{T}$ from the 1s orbit is $\omega_{1s} = 3.46 \times 10^{15} \text{s}^{-1}$ ^{*}). It is proportional to $|\phi(0)|^2$, the pion wave function square at the origin. The π nucleus interaction

^{*}) This result is in disagreement with the values given by Fujii ¹⁾ $\omega_{1s} = 8.32 \times 10^{15} \text{s}^{-1}$ and by Divakaran ¹⁶⁾ $\omega_{1s} = 0.97 \times 10^{15} \text{s}^{-1}$, both obtained in the impulse approximation. We believe that the difference in approach is not responsible for these discrepancies.

introduces in principle a deviation of the wave function from its Bohr value. In the present case this distortion effect is negligible because of a cancellation in the local part of the optical potential due to isospin effects ¹⁷⁾. A small correction (4%) due to the finite size of ${}^3\text{He}$ has been taken into account in the numerical result for ω_{1s} .

No direct experimental value is available for these quantities, but the Panofsky ratio has been measured in ${}^3\text{He}$ ¹⁸⁾

$$P = \frac{\pi^- + \text{He}^3 \rightarrow \pi^0 + \text{T}}{\pi^+ + \text{He}^3 \rightarrow \pi^- + \text{T}} = 2,28 \pm 0,18$$

Assuming for the moment that the pions are captured from the $1s$ orbit of the mesic atom, we can compare this value to a theoretical Panofsky ratio for $\ell = 0$. The charge exchange cross-section for s wave pions is not presently measured. It is given in the impulse approximation by :

$$\frac{d\sigma}{d\Omega} = \frac{2}{9} (\alpha_1 - \alpha_3)^2 \frac{q_{\pi_0}}{q_{\pi^-}} \left(\frac{M/\mu}{M+\mu} \right)^2 \left(\frac{m+\mu}{m\mu} \right)^2 \quad (12)$$

where α_1 and α_3 are the pion-nucleon isospin $\frac{1}{2}$ and $\frac{3}{2}$ scattering lengths, q_{π_0} and q_{π^-} the c.m. momenta of the neutral and charged pion. The value for $(\alpha_1 - \alpha_3)$ is taken from Hamilton ¹⁹⁾ : $\alpha_1 - \alpha_3 = (0.259 \pm 0.07) \mu^{-1}$ [however the uncertainty on this value is likely to be higher than quoted ²⁰⁾]. From the two cross-sections (11) and (12) we deduce a Panofsky ratio $P = 2.70$ which is also the value for bound pions in the $1s$ orbit. This ratio is independent of the distortion effects mentioned above.

In fact not all the pions are captured from the $1s$ orbit but a sizable fraction seems to be captured from the $2p$ orbit. This fraction can be estimated from the corresponding number in ${}^4\text{He}$. It has been observed that the transition yield of the pionic $2p-1s$ transition in

^4He is 0.31 and not 0.54 as for μ mesons ²¹⁾. If we ascribe this difference to pion absorption in the $2p$ orbit, 42% of the pions are absorbed there ^{*)}, from which we deduce an absorption rate $\omega_{2p} = 2.0 \times 10^{12}$ by comparison with the radiative transition rate. Assuming a two nucleon capture model, we have deduced the equivalent number for ^3He and we find $\omega_{2p} = 5.4 \times 10^{11} \text{ s}^{-1}$. [This is in agreement with the value we have found by applying directly the two nucleon capture model to ^3He ²³⁾; it is sizably higher than the value given in Ref. 21).] This capture rate means that 16% if the pions reaching the $2p$ orbit are absorbed from this orbit.

This appreciable proportion could affect the Panofsky ratio but both the charge exchange and radiative capture rate to the triton are very small as compared to the absorption rate in the $2p$ states, (in the impulse approximation, 10^{-3} for the first, 3×10^{-4} for the second). Therefore the Panofsky ratio is not affected.

A second possible correction to the Panofsky ratio is due to the pion absorption (virtual and real). The charge exchange cross-section is modified by this effect which changes the scattering lengths by $\delta\alpha_1$ and $\delta\alpha_3$. The imaginary parts of these amplitudes are obtained from the optical theorem :

$$\text{Im}(\delta\alpha_3) = \frac{q_{\pi^-}}{4\pi} \sigma_-^+ \quad ; \quad \text{Im}\left(\frac{2\delta\alpha_1 + \delta\alpha_3}{3}\right) = \frac{q_{\pi^-}}{4\pi} \sigma_-^- \quad (13)$$

where σ_+^+ and σ_-^- are the absorption cross-sections of positive and negative pions by ^3He . These absorption cross-sections have been calculated ²³⁾ :

$$q_{\pi^+} \sigma_+^+ = 9.1 \times 10^{-1} F \quad ; \quad q_{\pi^-} \sigma_-^- = 5.7 \times 10^{-2} F$$

*) This high fraction (which is at variance with common belief) is supported by the value which can be deduced from measurements of the imaginary part of the p wave π - ^4He phase shift ²²⁾, $\omega_{2p} = 1.36 \times 10^{12} \text{ s}^{-1}$ which represents 32% of the pions reaching the $2p$ orbit.

8.

Then $\text{Im}(\delta\alpha_1 - \delta\alpha_3) = 2.9 \times 10^{-3} \mu^{-1}$ which has a totally negligible effect in the cross-section (10^{-4}).

As for the influence of the real part, it has been shown that in the case of low energy pion elastic scattering it is reasonable to take the real and imaginary parts of $\delta\alpha_1$ and $\delta\alpha_3$ to have similar magnitudes ^{24), 17)}. Then $|\text{Re}(\delta\alpha_1 - \delta\alpha_3)| = 2.9 \times 10^{-3} \mu^{-1}$, as compared to $\alpha_1 - \alpha_3 = 0.259 \mu^{-1}$. The correction is 1%, which is less than the uncertainty in the amplitude and may be ignored. The pion absorption also influences the radiative pion capture. We have as above assumed that the real and imaginary contributions of this process to the amplitude have the same magnitude.

A rough estimate of the amplitude of process (a) is obtained as follows : we relate it to that of process (b) by replacing the (π^0, T) vertex by a (γ, T) vertex. With the photodisintegration cross-section of T taken from the quasi-deuteron model, we find that pion absorption contributes to the photoproduction amplitude by $\sim 2\%$.

The whole effect of pion absorption on the Panofsky ratio is thus expected to be of the order of 5%. The experimental and theoretical values for the Panofsky ratio, 2.28 and 2.70, are in satisfactory agreement. It would be very valuable to have direct measurements of the radiative capture to eliminate the uncertainties in the charge exchange cross-section.

To conclude thus the partially conserved axial current hypothesis can be usefully applied to soft pion problems not only in elementary particle physics but also in complex nuclei. It enhances the interest of the treatment of complex nuclei as elementary particles by relating weak interaction processes to processes involving strongly interacting particles without the intermediate stage of the impulse approximation.