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PION NUCLEAR FORWARD DISPERSION RELATIONS

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A B S T R A C T

Forward dispersion relations are applied to $\pi^{12}\text{C}$ (and $\pi^4\text{He}$) elastic scattering. The unphysical region is estimated to have small influence. From $\pi^{-9}\text{Be}$ scattering we find an effective $\pi^9\text{Be}$ coupling constant of order $(f^2)_{\text{eff}} \approx 0.06$.

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The application of dispersion relations to complex nuclei is usually considered (with some reason) to be difficult and complicated because of the importance of structure effects (unphysical region !) arising from the composite nature of nuclei. To investigate this problem in a very simple, controllable case, we consider here the π nuclear elastic scattering in the forward direction for nuclei of isospin $I=0$ and $\frac{1}{2}$. Our main conclusion is that in the forward direction nuclei look much like elementary particles to pions : for $I=0$ nuclei, cuts in the unphysical region are weak using a reasonable phenomenological estimate and pole terms are small, too. For $I=\frac{1}{2}$ nuclei, the charge anti-symmetric amplitude is in practice (as for πN scattering) a good tool to determine an effective coupling constant which seems close to the nucleonic one. Although experimental material is scarce, it is clear that definite conclusions about the unphysical region can be reached by comparing predictions and experiments. If the general unimportance of the unphysical region for forward scattering is confirmed, this will provide an important stimulus for extending dispersive techniques to more ambitious problems in nuclei.

The relevant dispersion relations for forward πX scattering can be taken over from standard πN literature with minor modifications ¹⁾. After a subtraction at threshold $\omega = \mu$, the real part of the forward charge averaged amplitude $T^{(+)}(\omega) = \frac{1}{2}(T_{\pi^+X}(\omega) + T_{\pi^-X}(\omega))$ is given by ^{*)} :

$$\operatorname{Re} T^{(+)}(\omega) = \operatorname{Re} T^{(+)}(\mu) - \sum_i \frac{2\omega_i f_i^2}{i(\mu^2 - \omega_i^2)} \frac{q^2}{(\omega^2 - \omega_i^2)} + \frac{2}{\pi} q^2 \mathcal{P} \int_{\omega_0}^{\infty} \frac{\omega' \operatorname{Im} T^{(+)}(\omega') d\omega'}{q'^2 (\omega'^2 - \omega^2)}, \quad (1)$$

*) Notation : ω , q and μ are the pion energy, momentum and mass, respectively. M is the mass of X . The direct channel pole X_i has mass $M_i = M + \epsilon_i$, where ϵ_i is its excitation energy above the ground state of X . The $(X X_i \pi)$ coupling constant is f_i . All quantities are in the lab. system. The forward cross-section is $(d\sigma(\omega)/d\Omega)_{\theta=0} = |T(\omega)|^2$.

where $\omega_i = -\mu^2/2M + \varepsilon_i \approx \varepsilon_i$ is the position of the i th pole. Equation (1) is appropriate for isospin $I=0$ and the I independent part of $I=\frac{1}{2}$ nuclei^{*)}. The branch point ω_0 is approximately at $\omega_0 = 0$. For $I=\frac{1}{2}$ nuclei we have, in addition, the charge antisymmetric amplitude^{**)} $T^{(-)}(\omega) = \frac{1}{2}(T_{\pi^+X}(\omega) - T_{\pi^-X}(\omega))$ which needs no subtraction (Pomeranchuk theorem):

$$\operatorname{Re} T^{(-)}(\omega) = \sum_i \frac{2f_i^2 \omega}{\omega^2 - \omega_i^2} + \frac{2}{\pi} \omega \mathcal{P} \int_{\omega_0}^{\infty} \frac{\operatorname{Im} T^{(-)}(\omega')}{\omega'^2 - \omega^2} d\omega' \quad (2)$$

In the physical region the pole terms in Eqs. (1) and (2) contribute in practice as single effective poles close to $\omega = 0$, since $\omega_i \ll \mu$. A quick estimate of the effective pole in Eq. (1) is the coherent sum of A nucleon poles:

$$T_{\text{pole}}^{(+)}(\omega) \approx - \frac{\sum_i 2\omega_i f_i^2}{\mu^2} \frac{g^2}{\omega^2} \sim A \frac{f^2}{m} \frac{g^2}{\omega^2} = 0.011 A \frac{g^2}{\omega^2}, \quad (1.a)$$

where $f^2 = 0.08$ is the πN coupling constant and m the nucleon mass. A justification of this intuitive estimate is obtained by relating f_i to axial matrix elements assuming PCAC. In impulse approximation (used as in β decay and μ capture) the leading terms for small q^2 are

*) If X has spin $s > \frac{1}{2}$ (e.g., ${}^9\text{Be}$), $T(\omega)$ stands for an appropriately spin weighted amplitude.

**) We assume that X has neutron excess. Otherwise the charge signs of the pions must be reversed.

spin wave dipole matrix elements. The spin wave generalization of the Thomas-Reiche-Kuhn sum rule gives indeed the estimate (1.a). In view of the smallness of the term (1.a) in nuclei, we have presently dropped it in our discussion of Eq. (1) ^{*}). The pole terms in Eq. (2) will be discussed later [see Eq. (3)].

A special feature of Eqs. (1) and (2) is that $\text{Im} T^{(\pm)}(\omega) \neq 0$ also at and below the physical threshold $\omega = \mu$: pions can be absorbed in this region, because nuclei are composite. The corresponding cuts start close to $\omega_0 = 0$, which is a normal threshold in our application to ^4He and ^{12}C , and an anomalous threshold ^{2),3)} very close to this point for ^9Be . The extrapolation from the elastic threshold $\omega = \mu$ to $\omega_0 = 0$ has been made phenomenologically using the parametrization $\text{Im} T^{(\pm)}(\omega) = \alpha^{(\pm)} + \beta^{(\pm)} q^2 + \dots$. The coefficients $\alpha^{(\pm)} \approx \text{Im} a_0^{(\pm)}$, $\beta^{(\pm)} \approx 3 \text{Im} a_1^{(\pm)}$ are given by the appropriate imaginary parts of the s and p wave scattering lengths $a_0^{(\pm)}$ and $a_1^{(\pm)}$ (see the Table). This procedure gives us control over the magnitude of contributions from the cuts in the unphysical region ($< 4\%$ for $I=0$ nuclei).

The imaginary part above threshold has been obtained from total cross-sections using the optical theorem. A smooth polynomial fit through the experimental points ⁴⁾ has been joined to the threshold behaviour. The concrete numerical data in the applications are given in the figure captions. Although $\text{Re} T^{(\pm)}(\omega)$, for $\omega < 500$ MeV is not strongly influenced by $\sigma_{\text{tot}}^{(\pm)}(\omega)$ at energies above 2 GeV, some reasonable input values are needed. These were obtained from πN amplitudes ⁵⁾ using the Glauber high energy multipole scattering model ⁶⁾ to determine $\text{Im} T^{(+)}(\omega)$.

^{*}) In the corresponding πN case the main effect of the pole term is a 6% contribution to the appropriate p wave πN amplitude $T^{(+)}(\omega)$ at threshold.

The results for $\text{Re } T^{(+)}(\omega)$ of ^{12}C are given in Fig. 1. The general behaviour is nearly entirely governed by the 33 resonance for $40 < \omega - \mu < 500$ MeV. (Compare the solid and the dashed curves.) Various drastic changes in the input data show in particular great insensitivity to the extrapolation into the unphysical region. The general behaviour of $\text{Re } T^{(+)}(\omega)$ with a zero at about 190 MeV is qualitatively confirmed as a change of sign in the Coulomb nuclear interference in the $\pi^{-12}\text{C}$ differential cross-section of a CERN experiment ⁷⁾ in progress at $150 < \omega - \mu < 280$ MeV. Results for ^4He are closely similar but reduced by a factor ≈ 2.5 and are therefore not shown.

Equation (2) makes it in principle possible to determine an effective coupling constant $(f^2)_{\text{eff}} = \sum_i f_i^2$ for the antisymmetric amplitude. Neglecting $(\omega_i/\mu)^2 \ll 1$ we solve Eq. (2) for $(f^2)_{\text{eff}}$ at $\omega = \mu$:

$$(f^2)_{\text{eff}} = \frac{\mu}{2} \text{Re } T^{(-)}(\mu) - \frac{\mu^2}{\pi} \mathcal{P} \int_0^\infty \frac{\text{Im } T^{(-)}(\omega')}{\omega'^2} d\omega' \quad (2.a)$$

We are acutely aware of the unsatisfactory nature of present input data in this relation, but we believe a magnitude estimate of $(f^2)_{\text{eff}}$ is possible for ^9Be using indirect evidence, as follows.:

The total cross-sections needed for the evaluation of the integral in (2.a) are known only for $\pi^{-9}\text{Be}$ ⁴⁾. Observing that $\sigma^{(+)}(\omega)$ for the $I=0$ nuclei ^{12}C and ^4He have closely the same shape, with a scale factor ≈ 2.5 , we have assumed a power behaviour A^n for $\sigma^{(+)}$, thus determining $\sigma_{\text{Be}}^{(+)}$ and hence the cross-section difference $\sigma_{\text{Be}}^{(-)} = \frac{1}{2}(\sigma_{\pi^+\text{Be}} - \sigma_{\pi^-\text{Be}}) = (\sigma_{\text{Be}}^{(+)} - \sigma_{\pi^-\text{Be}})$. With the above 2.5 ratio the power $n=0.83$ and the dispersion integral contributes 0.04 to $(f^2)_{\text{eff}}$. Thus $\sigma_{\pi^-\text{Be}}$ is 50-100 mb above $\sigma_{\text{Be}}^{(+)}$ in the dominant 33 resonance region giving a positive contribution (see Fig. 2). Corrections from the unphysical region have been obtained by estimating $\text{Im } a_0^{(-)}$ and $\text{Im } a_1^{(-)}$ from a multiple scattering theory ⁸⁾ which adequately describes low energy π nuclear reactions. Small contributions, of the order of 0.005, result.

In much the same way, the scattering lengths $\text{Re } T^{(-)}(\mu)$ are deduced from π mesic level shifts in ${}^9\text{Be}$ and in the neighbouring $I=0$ nuclei ${}^9\text{Li}$. The shift for ${}^9\text{Be}$ being more repulsive, $\text{Re } T^{(-)}(\mu) = 0.048 \pm 30\%$. Hence we find $(f^2)_{\text{eff}} = 0.06$. Note that both contributions to $(f^2)_{\text{eff}}$ are positive, so there are no cancellations. This result is remarkably close to the nucleon value $f^2 = 0.08$. Indeed, the arguments leading to the estimate (1.a) give the following results for the pole terms in $T^{(-)}$

$$T_{\text{pole}}^{(-)} \approx \frac{2(f^2)_{\text{eff}}}{\omega} \sim \frac{2f^2}{\omega}. \quad (3)$$

The simplest way of obtaining $(f^2)_{\text{eff}} = f^2$ is thus to form the coherent sum of πN poles. (The antisymmetric part is proportional to the total isospin I , which is $\frac{1}{2}$ as for the nucleon.) This hides the true nature of the result, however.

Relating the pole strength f_i to axial matrix elements by PCAC as for the $T^{(+)}$ pole (1.a), it might be thought that the ground state isospin $\frac{1}{2}$ multiplet ${}^9\text{Be}(\frac{3}{2}^-) - {}^9\text{B}(\frac{3}{2}^-)$ would give the dominant contribution, taking the place of the (n,p) system. Although the strength of the allowed Gamow-Teller β transition is unknown, the similar β transition for ${}^{11}\text{C}(\frac{3}{2}^-) - {}^{11}\text{B}(\frac{3}{2}^-)$ has only 13% of the nucleon strength. Consequently we expect the ${}^9\text{Be} - {}^9\text{B}$ ground state multiplet to contribute a similar small fraction of f^2 to $(f^2)_{\text{eff}}$. This is at variance with the observed value above which must therefore be understood as a sum rule. Indeed, if we retain only the allowed axial transitions and neglect form factor effects, closure over all states gives $(f^2)_{\text{eff}} = f^2$. We have used the standard β decay impulse approximation to obtain this result. Clearly some changes in the sum rule would result with realistic form factors.

It would obviously be of great interest to have an accurate determination of $(f^2)_{\text{eff}}$ using comparison of $\pi^\pm$ interactions. Prime candidates (apart from ${}^9\text{Be}$) are the nuclei ${}^3\text{He}(\frac{1}{2}^+)$, ${}^{13}\text{C}(\frac{1}{2}^-)$ and ${}^{15}\text{N}(\frac{1}{2}^-)$.

TABLE - Using the leading term of an expansion ⁹⁾, the complex energy shift $\Delta E = \text{Re } \Delta E + i \Gamma/2$ in π mesic atoms is related to s and p wave $\pi\pi$ scattering lengths by

$$a_0 \approx \frac{1}{4Z\alpha} \frac{\Delta E_{1s}}{E_{1s}} \mu^{-1}; \quad a_1 \approx \frac{2}{3(Z\alpha)^3} \frac{\Delta E_{2p}}{E_{2p}} \mu^{-3}.$$

$\Delta E_{1s} = (-6.1 \pm 0.15) + i(1.48 \pm 0.12) \text{ keV}$	$\Delta E_{2p} = (?) + i 0.3 \text{ eV}^*)$
$a_0 = (-0.26 \pm 0.03) + i(0.063 \pm 0.006) \mu^{-1}$	$a_1 = (?) + i(0.07 \pm 0.2) \mu^{-3}$

*) Deduced from X-ray attenuation.

R E F E R E N C E S

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FIGURE CAPTIONS

- Figure 1 : $\text{Re } T^{(+)}(\omega)$ for $\pi^{12}\text{C}$ versus energy. The solid line is obtained from Eq. (1) with the following input data :
- a) $\text{Im } T^{(+)}(\omega) = 0.063 + 0.22 q^2(\mu^{-1})$ for $0 < \omega < (\mu + 20)\text{MeV}$ from the scattering lengths of the Table;
 - b) a smooth polynomial fit to total cross-sections for $(\mu + 20) < \omega < 1350 \text{ MeV}$ ⁴⁾;
 - c) $\sigma_{\text{tot}}(\omega) = [200 + (63/\omega \text{ MeV})^{\frac{1}{2}}]$ mb for $\omega > 4 \text{ GeV}$, obtained as a good fit to our calculation using the Glauber model ⁶⁾ with πN amplitudes ⁵⁾;
 - d) subtraction constant $\text{Re } T^{(+)}(\omega) = (-0.26 \pm 0.03)\mu^{-1}$ from $\text{Re } a_0$ in the Table.

Continuous matching between regions a)-c) is enforced. A change of the coefficients of $\text{Im } T^{(+)}(\omega)$ in a) by 30% is almost imperceptible on this scale. The dashed curve is the effect of increasing $\sigma_{\text{tot}}(\omega)$ by three standard deviations in the resonance region. The three triangular points are from an optical fit to experiments ¹⁰⁾.

- Figure 2 : Experimental ⁴⁾ total cross-sections $\sigma_{\pi^{9}\text{Be}}(\omega)$ and $\sigma_{\pi^{12}\text{C}}(\omega)$ versus energy. The solid line represents smoothed $\sigma_{\pi^{9}\text{Be}}$ cross-sections, while the dashed line is $\sigma_{9\text{Be}}^{(+)}(\omega) \approx (9/12)^{0.83} \sigma_{12\text{C}}^{(+)}(\omega)$. The difference (shaded area) is $\sigma_{9\text{Be}}^{(-)}$ needed in the evaluation of (3). For non-renormalized values of $\sigma_{12\text{C}}^{(+)}$ use the right-hand scale.

FIG.1