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NUCLEON-NUCLEUS FORWARD DISPERSION RELATIONS

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ABSTRACT

Elastic forward dispersion relations are illustrated by $n-\alpha$ scattering and preliminary results are given. Exchange effects are of great importance at low energies. This result should be valid in general for light nuclei.

Stimulated by our previous results ¹⁾ on the effective π -nucleus coupling constant in $T=\frac{1}{2}$ nuclei, we presently apply forward dispersion relations (FDR) to nucleon-nucleus elastic scattering. Since total cross-sections are very similar above 20 MeV for all light elements we have chosen the technically simplest case of $n-\alpha$ scattering to demonstrate the method [one amplitude (spin and isospin zero), no negative energy resonances in the compound state, no Coulomb effects]. Our main conclusion is that the nucleus behaves like an elementary particle at high energies, whereas at low energies very important contributions come from the exchange channel, especially from the exchange of deeply bound systems (in the particular case of $n-\alpha$ scattering from ^3He exchange).

The once subtracted dispersion relation ²⁾ for the forward $n-\alpha$ laboratory scattering amplitude is ^{*})

$$\text{Re } f(E) = \text{Re } f(0) + \frac{G^2 E}{(E_0 - E)E_0} - \frac{E}{\pi} P \int_{-2m}^E dE' \frac{\text{Im } \bar{f}(E')}{(E' - E)E'} + \frac{k^2}{2\pi^2} \int_0^\infty dk' \frac{\sigma(k')}{k'^2 - k^2} - \frac{E}{4\pi^2} \int_0^\infty dE' \frac{[\bar{G}(E') - G(E)]k'}{(E' + E + 2m)(E' + 2m)} \quad (1)$$

This exact formula is relativistic and completely general except for the pole term. For non-relativistic approximations one can neglect the last term ($\sim \bar{G} - G$).

The singularity structure of the unphysical region for $n-\alpha$ scattering is the following :

The pole term corresponds to ^3He exchange, with a position $E_0 = -15.4$ MeV. The sign of the pole is characteristic of the positive parity of ^3He . [In an elementary particle treatment of $n(\frac{1}{2}^+) ^3\text{He}(\frac{1}{2}^+) ^4\text{He}(0^+)$ the pole is analogous to that in Kp scattering ³⁾. Note that the correspondence is to pseudoscalar Kp scattering, because ^3He is a negative energy intermediate state. Our definition of G^2 includes the mass factors that are responsible for this sign dependence on parity.]

^{*}) Notation : E is the laboratory kinetic energy of the neutron (mass m), $k = (E^2 + 2mE)^{\frac{1}{2}}$ its momentum, f and $\bar{f}$ the $n-\alpha$ and $\bar{n}-\alpha$ amplitudes, σ and $\bar{\sigma}$ the corresponding total cross-sections, E_0 the ^3He pole position, G the pole residue, E_b the branch point of the unphysical cut. $\text{Re } f(0) = (1 + m/m_{^4\text{He}}) \times (\text{scattering length})$, $d\sigma/d\Omega(0^0) = |f|^2$; $c = 1$.

The exchange of more than one particle leads to cuts. The nearest branch point coming from pd exchange starts at $E_b = -19.4$ MeV. The exchange of ppn begins at $E_b = -21.1$ MeV, whereas the cut corresponding to ${}^3\text{He}\pi^0$ exchange starts at -120 MeV.

A number of accurate phase shift analyses ⁴⁾ being available for $0 < E < 50$ MeV, we have studied Eq. (1) mainly in this energy region. To exhibit the importance of the unphysical region, we have compared the dispersion integral from the physical region only, including the subtraction constant (dashed line in Fig. 1) to the real part of the amplitude from phase shift analysis ⁴⁾ (solid line).

Fig. 1 : The full points are Ref in $n-\alpha$ scattering from phase shifts ⁴⁾. The solid line is a smooth interpolation. The error bar is the estimated systematic error at 40 MeV. It is less at lower energies. The dashed line is the contribution to Ref from the physical region (see text) with $\text{Ref}(0) = -3.04$ F.

The total cross-sections are taken from Ref. ⁵⁾. The integral dependent on $(\bar{G} - G)$ has been neglected since it contributes less than 1% for $E < 50$ MeV under all reasonable parametrizations (note the suppression due to the large denominator $> 4 \text{ m}^2$). The difference Δ (= solid minus dashed line) is very large and is displayed in Fig. 2. If the ${}^3\text{He}$ pole of Eq. (1) is the dominating singularity, we expect the quantity E/Δ to be linear in E . Indeed, the linearity is striking (dashed line). Comparison with the pole term in (1) fixes the parameters as :

$$E_0 = -10 \text{ MeV} \quad G^2 \approx 1.3 \quad \text{or} \quad \frac{G^2}{E_0} \approx 25 \text{ F}$$

Note that the sign of G^2 is characteristic of the exchange of a particle of positive parity (cf., the remark further above). The shift of the pole position by 5 MeV, if it is a real effect, could be due to a negative contribution from the nearby cut. A comparatively large cancellation of the order of 50 percent would be necessary then.

Fig. 2 : See text. The full points correspond to phase shift.

An evaluation of $\text{Re } f$ around 100 MeV, pole term included, still indicates missing contributions of the order of $-4F$ which vary slowly with E . These presumably come from more distant singularities like exchange of ${}^3\text{He } \pi$, ${}^3\text{He } \rho$ (or ω or φ) etc., and are of expected magnitude and sign. However, the experimental real part begins to be unreliable at these energies. In addition, the physical region of antiparticle cross-sections begins to contribute a few per cent to the dispersion integral. Therefore these statements must remain qualitative.

The exchange amplitude from ${}^3\text{He}$ of about $25F$ can be related to the reduced width for ${}^4\text{He} \rightarrow n + {}^3\text{He}$. With an interaction radius $R \simeq 2F$ this is compatible with a reduced width $\gamma^2 \sim 1$; it should be noted, however, that the exponential penetration factor $\exp\{-\alpha R\}$ is very large.

Since the total cross-sections have similar shape as for ${}^4\text{He}$ in other light elements, similar dispersion integrals occur leading to the general conclusion of very important exchange terms in nucleon-nucleus forward scattering amplitudes. It is interesting to ask how this is reconcilable with ordinary potential scattering. The explanation is the following theorem: scattering from an external real potential with some bound states occupied is identical to that from an unoccupied well, provided that the incident particle is identical to the bound ones and that the particles are non-interacting. Thus, the exchange contribution is exactly the contribution from the states which are occupied and hence blocked.

To establish the corresponding importance of exchange for other light nuclei, we presently investigate $n^{12}\text{C}$ scattering, which has the additional complication of a few negative energy compound states.

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For energies above 100 MeV, nucleon-¹²C total cross-sections with scaling factors have also been used.