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ABSORPTIVE CORRECTIONS AND REGGE SINGULARITIESJ. Finkelstein^{*} and M. Jacob

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A B S T R A C T

Two possible alternative methods of computing unitarity corrections to Regge pole amplitudes are discussed. One method is to correct for rescattering, as motivated directly from the unitarity relation; the other is to correct for absorption, in order to account for the presence of competing channels. It is shown that these two corrections have the same singularity structure, but need not be the same; in the simplest cases the corrections have the same magnitude, but differ in sign.

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1. INTRODUCTION

If the Regge pole model is presently offering the best theoretical framework for high energy phenomenology, it is nevertheless widely recognized that some particular phenomena, such as the observed polarization effect in pion-nucleon charge exchange scattering, are not amenable to an analysis in terms of leading Regge poles ¹⁾. Described in terms of singularities in the angular momentum plane, these effects could be interpreted as being due to the presence of cuts, which could provide small corrections to the Regge pole terms. Recent work ^{2), 3)} has shown in particular that the analysis of pion-nucleon elastic and charge exchange scattering in terms of poles and cuts could provide a good fit to present data and retain at the same time an important predictive value.

It is known that cuts in the angular momentum plane should be present due to the third double spectral function as first shown by Mandelstam ⁴⁾. Nevertheless it has been already attempted several times to calculate such cuts through rescattering corrections to amplitudes which are assumed to have a Regge pole behaviour ⁵⁾. The cuts which are thus easily obtained have, at least, the same positions as those predicted on more general grounds ^{4), 6)}. Also, following the peripheral model calculations with absorptive corrections ⁷⁾, attempts have been made to correct Regge pole amplitudes for absorption. Even though such a procedure may seem a priori to depart from the general Regge pole philosophy, it has been shown ³⁾ that the asymptotic form of the amplitudes thus obtained is similar to the one which would arise from the presence of poles and cuts. The success of the peripheral model, and now the Regge pole model, used with absorptive corrections has stirred interest in trying to justify the simple procedure followed in correcting for absorption in terms of rescattering corrections, which have a more direct interpretation in terms of mass shell amplitudes ⁸⁾.

The purpose of this paper is to stress that such a connection is not unambiguous. In particular we show that the rescattering approach for Regge pole amplitudes, as introduced by Amati, Fubini and Stanghellini⁵⁾ and the absorption approach, as usually applied⁷⁾, lead to the same analytic structure but to corrections which differ at least in sign. Thus, even the sign of the unitarity corrections which should be added to the Regge pole amplitude is model dependent. In the second part of this paper we display the corrections to a Regge amplitude which one would write, either correcting for rescattering⁵⁾ or for absorption⁷⁾. We show how they are related but also how they differ. One cannot conclude at this point which should be the better approach since, even though the expressions one starts with are formally the same, they actually correspond to different approximations to the complete amplitude. However, in the third section, we show that the sign of the correction, as obtained from the absorption procedure, is better suited to present analysis of experimental results and in particular to the polarization observed in pion nucleon charge exchange scattering⁹⁾. We also show that the rescattering approach will in certain cases lead to a conflict with crossing symmetry, a difficulty which is avoided in the absorptive approach. We collect in an Appendix several relations between the partial wave, momentum transfer, and impact parameter representations which we will need in the second section.

2. RESCATTERING AND ABSORPTION CORRECTIONS

We begin by recalling the relation written by Amati, Fubini and Stanghellini ⁵⁾ for the correction to the elastic Regge amplitude due to rescattering. This correction corresponds to the graph shown in the Figure, with states a and b the same. In the framework of their multiperipheral model for production processes, the Regge pole appears as the overlap function. That is, its contribution to the unitarity relation contains all the important intermediate states but the elastic one. Therefore the relation which they write for the imaginary part of an elastic amplitude T_{aa} reads:

$$\text{Im } T_{aa} = \text{Im } R_{aa} + T_{aa} T_{aa}^* \quad (1)$$

where R_{aa} is the Regge term, the phase of which is given by the signature factor. This equation can be interpreted as it is in terms of partial wave amplitudes. It may also be looked at as a relation for the full amplitude if multiplication is interpreted as integration over phase space; this point is discussed further in the Appendix. Solving Eq. (1) by iteration one first gets

$$\text{Im } T_{aa} = \text{Im } R_{aa} + R_{aa} R_{aa}^* \quad (2)$$

We can extend this idea to inelastic processes by continuing to identify the Regge pole with the overlap function. This is straightforward in the framework of the multiperipheral model in which the production process is described in a well defined way ¹⁰⁾. We then write, for an inelastic scattering amplitude T_{ab} ,

$$\text{Im } T_{ab} = \text{Im } R_{ab} + T_{aa} T_{ab}^* + T_{ab} T_{bb}^* \quad (3a)$$

4.

We wish to study the case in which T_{aa} and T_{bb} can be considered to be equal (as is exactly true in K nucleon charge exchange and a good approximation in π nucleon charge exchange); then we have

$$\text{Im } T_{ab} = \text{Im } R_{ab} + 2 \text{Re} \{ T_{aa}^* T_{ab} \} \quad (3b)$$

To compute the correction to T_{ab} , we may approximate the inelastic amplitude in the right-hand side of (3b) by R_{ab} ; this gives us

$$\text{Im } T_{ab} = \text{Im } R_{ab} + 2 \text{Re} \{ T_{aa}^* R_{ab} \} \quad (4)$$

This equation coincides with the unitarity relation as written by Squires, in the last paper of Ref. ⁸⁾.

We now turn to the absorptive corrections, still considering the case where $S_{aa} = S_{bb}$, S and T being related by

$$S = I + 2i T \quad (5)$$

where I is the unit matrix. The corrected amplitude is now ⁷⁾

$$T_{ab} = R_{ab} S_{aa} \quad (6)$$

The absorption corrections, also, can be considered to correspond to the graph shown in the Figure. From Eqs. (5) and (6) we get:

$$\text{Im } T_{ab} = \text{Im } R_{ab} + 2 \text{Re} \{ T_{aa} R_{ab} \} \quad (7)$$

This is identical to (4) except for the complex conjugation.

If the connection between the elastic and inelastic cases was simple in the rescattering approach, it is a priori more difficult when dealing with absorption. Nevertheless, a connection can be found within the eikonal approximation, as described by Arnold¹¹⁾. This approximation consists of calculating the eikonal as the Fourier-Bessel transform of the Regge pole amplitude. As shown in the Appendix this is equivalent to writing symbolically S_{aa} as $e^{2iR_{aa}}$ and expanding the exponential using the asymptotic unitarity kernel. The second approximation is now

$$T_{aa} = R_{aa} + i R_{aa} R_{aa}$$

and taking the imaginary part we get

$$\text{Im } T_{aa} = \text{Im } R_{aa} + \text{Re} \{ R_{aa} R_{aa} \} \quad (8)$$

which is identical to (2) except for the complex conjugation; this is the same way in which (4) and (7) are related.

If, as often done, one assumes that the elastic scattering amplitude is purely imaginary, one sees that the first corrections implied by rescattering or by absorption differ only by the sign. Writing the elastic Regge amplitude as iB , with B real and positive, we can replace (2) by

$$\text{Im } T_{aa} = B + B^2 \quad (\text{rescattering}) \quad (9a)$$

whereas (8) now reads

$$\text{Im } T_{aa} = B - B^2 \quad (\text{absorption}) \quad (9b)$$

Pushing forward the iteration, it is clear that all corrections to the imaginary part of the scattering amplitude would add in the case of rescattering and would, to the contrary, alternate in sign in the case of absorption. This is also true for the two methods of correcting inelastic Regge terms. The alternation in signs is consistent with the results of Gribov¹²⁾.

The Regge pole represents therefore two different approximations to the amplitude. These can be characterized, at least in the case of inelastic scattering, by noticing that, while the absorptive formula (6) means that the Regge pole is the amplitude evaluated on the sheet reached by going through the cut associated with the normal threshold of channel *a* (or the cut associated with channel *b*), the rescattering formula (3b) says that the Regge pole is the overlap function. In the rescattering calculation, one adds to the Regge pole that part of the full discontinuity which is due to the elastic intermediate state, while in the absorptive calculation, one adds the discontinuity across the elastic cut.

It is known that rescattering corrections, applied to a Regge pole term, lead to a logarithmic branch point in the complex angular momentum plane⁵⁾. Absorptive corrections will therefore also lead to the same type of branch point, as long as the elastic scattering input is parametrized as if it were a Regge pole (which is as good a parametrization as any). The discontinuities across the cuts differ, but only by constant factors.

There is one more model for unitarity corrections which could be motivated by the Figure, namely the model in which the graph shown in the Figure is interpreted as a (generalized) Feynman diagram. A high energy approximation to this model gives no correction whatsoever⁴⁾!

3. DIFFERING IMPLICATIONS OF RESCATTERING AND ABSORPTION

As shown in the preceding section, the first corrections to the imaginary part of the amplitude differ in sign, depending on whether one uses a rescattering or an absorption approach. We now discuss in one particular example, namely π nucleon charge exchange scattering, how the analysis of the data may shed some light on the proper sign of the correction term to a pure Regge amplitude.

For the purpose of this discussion, we assume that the amplitude is written in terms of the ρ pole and one relatively small cut contribution. The change in sign at $t \simeq -0.2 \text{ GeV}^2$ of the difference of the elastic π^+ and π^- proton differential cross-sections indicates that the imaginary part of the non-spin flip charge exchange contribution has a zero at this point. Unless it should happen that the ρ pole residue vanishes at this point, the zero must arise from a cancellation between the ρ and the cut contributions, implying that the imaginary parts of those contributions have opposite signs. This is what happens in the analysis of Cohen-Tannoudji et al.³⁾ and in solution II of Ref.²⁾. This cancellation is consistent with the absorptive formula (7) but not with the rescattering formula (4). Furthermore, absorption corrections predict the correct sign for the polarization in pion nucleon charge exchange³⁾. This can be understood as follows: since the ρ spin flip amplitude is much larger than the ρ non-spin flip amplitude, the dominant part of the polarization is given by the interference term between the ρ spin flip and the cut non-spin flip contributions¹³⁾. The sign of the ρ spin flip amplitude is known from the elastic polarization; the sign of the ρ non-spin flip amplitude is, on the other hand, known from the difference between the π^+ and π^- total cross-sections. Therefore, the sign of the measured charge exchange polarization determines what must be the relative sign of the non-spin flip pole and cut contributions. As we have seen, rescattering and absorption corrections also make definite (and opposite) predictions for this relative sign. It turns out that the observed result is in favour of the sign obtained from absorption.

A P P E N D I X

We start with the case of equal mass spinless particles. We consider a scattering amplitude f , defined in such a way that the differential cross-section reads:

$$\frac{d\sigma}{d\Omega} = |f(s, \theta)|^2 \quad (\text{A.1})$$

The optical theorem is therefore written as

$$\text{Im } f(s, 0) = \frac{k}{4\pi} \sigma_{\text{tot}} \quad (\text{A.2})$$

where s is the centre-of-mass energy squared and t the momentum transfer squared.

$$s = 4(k^2 + m^2) \quad \cos \theta = 1 + \frac{t}{2k^2}$$

The decomposition into partial wave reads:

$$f(s, \theta) = \sum_l (2l+1) f_l(s) P_l(\cos \theta) \quad (\text{A.3})$$

with

$$f_l(s) = \frac{e^{2i\delta_l(s)} - 1}{2ik} \quad (\text{A.4})$$

We further write the contribution from elastic unitarity to the imaginary part of the amplitude as

$$\text{Im } f(s, \theta) = \frac{k}{4\pi} \int f(s, \theta') f^*(s, \theta'') d\Omega \quad (\text{A.5})$$

where

$$d\Omega = \sin \theta' d\theta' d\varphi$$

and

$$\cos \varphi = \frac{\cos \theta'' - \cos \theta \cos \theta'}{\sin \theta \sin \theta'}$$

One can replace the integration over $d\Omega$ in (A.5) by an integration over the momentum transfers $t' = 2k^2(\cos \theta' - 1)$ and $t'' = 2k^2(\cos \theta'' - 1)$. Then, relation (A.5) reads

$$\text{Im } f(s, t) = \frac{1}{4\pi k} \int_{-\infty}^0 f(s, t') f^*(s, t'') \theta(k) K^{-1/2}(s, t, t', t'') dt' dt'' \quad (\text{A.6})$$

where K is given by

$$K(s, t, t', t'') = -t^2 - t'^2 - t''^2 + 2(tt' + tt'' + t't'') + \frac{tt't''}{k^2}$$

We also introduce a high energy approximation for K , which is justified on the ground that the amplitude f is sharply peaked near $t=0$, so that t/s terms in K can be neglected even in the integration in (A.6):

$$K(s, t, t', t'') \sim \tau(t, t', t'')$$

with

$$\tau(t, t', t'') = -t^2 - t'^2 - t''^2 + 2(tt' + tt'' + t't'')$$

$\mathcal{U}$ is equal to four times the area of the triangle with sides $\sqrt{-t}$, $\sqrt{-t'}$ and $\sqrt{-t''}$. The θ function leads to a zero contribution if the triangle does not close. One sees easily from (A.6) that a Regge pole approximation for f leads to a cut contribution to the corrected amplitude ⁵⁾.

Relation (A.6) is equivalent to the contribution to the imaginary part obtained using partial wave amplitudes namely

$$\text{Im } f_e(s) = k |f_e(s)|^2 \quad (\text{A.7})$$

(A.7) can of course be obtained directly from (A.6) by projection. In addition, (A.6) can be recovered from (A.7) by summing over the partial waves and using ¹⁷⁾

$$\sum_l (2l+1) P_l(\cos \theta) P_l(\cos \theta') P_l(\cos \theta'') = \frac{4k^2}{\pi} \frac{\theta(K)}{\sqrt{K}} \quad (\text{A.8})$$

$$K = 4k^4 (1 - \cos^2 \theta - \cos^2 \theta' - \cos^2 \theta'' + 2 \cos \theta \cos \theta' \cos \theta'')$$

The relation between (A.6) and (A.7) can be generalized from elastic unitarity: multiplication of partial waves, which is the usual formulation of the absorption model, is equivalent to integration over momentum transfers as in (A.6).

A Lorentz invariant amplitude may be defined as $F = \frac{\sqrt{s}}{2} f$ and its expansion into partial waves, as defined by (A.3), now leads to the unitarity relation

$$\text{Im } F_e(s) = \frac{2k}{\sqrt{s}} |F_e(s)|^2$$

- 10) This is consistent with the assumption that the overlap function becomes approximately diagonal at high energy since only the diagonal elements have the energy dependence associated with elastic scattering. All of the off diagonal elements have energy dependences associated with inelastic scattering.
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Correction to a Regge pole approximation

The same figure may be looked at in two different ways, that is, either as a rescattering correction, as due to two-body unitarity, or as an absorption correction, which corrects for the discontinuity across an elastic cut.