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MULTIPLE POMERANCHUK EXCHANGE VIOLATES UNITARITY

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In this note we will show that, in the multi-Regge model (MRM) ¹⁾⁻⁷⁾ repeated exchange of the Pomeranchuk trajectory would lead to a violation of the Froissart bound, even if the Pomeranchuk has finite slope. This result is known ^{5),6)} in the case of a flat Pomeranchuk. A stronger result, that the total cross-section would increase faster than any power of the energy, has been claimed by Verdiev et al., ⁷⁾ but it can be shown that the expression for the total cross-section considered in Ref. ⁷⁾ is in fact bounded at least by some power of the energy. Using this fact, it is easy to prove [contrary to the implication of the claim made in Ref. ⁷⁾] that if one considers only those trajectories with intercepts less than one, one need never violate the Froissart bound. Further discussion of this point, as well as other results on integrated cross-sections in the MRM, are presented elsewhere ⁸⁾.

Consider the process $p_a + p_b \rightarrow p_1 + p_2 + \dots + p_n$, where the final particles may, for simplicity, be taken to be scalar states with common mass m . Then, in the kinematic region in which the squared sub-energies $s_i = (p_i + p_{i+1})^2$ are all large and the momentum transfers $t_i = (p_a - p_1 - \dots - p_i)^2$ are all small, the matrix element in the MRM corresponding to repeated exchange of the Pomeranchuk is

$$M_n(s_1, \dots, s_{n-1}, t_1, \dots, t_{n-1}, \omega_1, \dots, \omega_{n-1}) = F(t_1) F(t_n) \prod_{i=2}^{n-1} G(t_{i-1}, t_i, \omega_i) \prod_{j=1}^{n-1} (s_j/s_0)^{\alpha(t_j)} \quad (1)$$

where F is the vertex function for the trajectory and two particles, G the vertex function for a particle and two trajectories, and $\alpha(t_i)$ the Pomeranchuk trajectory function. Here we have let G depend on ω_i , the angular variable introduced by Bali, Chew, and Pignotti ^{4),5)}. We take it to be part of the MRM that the vertex functions do not vanish when the momentum transfers are zero.

2.

We can obtain a lower bound for the integrated production cross-section $\sigma_n(s)$ [where $s = (p_a + p_b)^2$] by integrating the square of the matrix element (1) over just those kinematic regions where it is supposed to be valid; this bound is, neglecting t_i and m^2 in comparison with s_i and s ,

$$\sigma_n(s) \geq \frac{\Gamma^n}{s^2} \int_0^\infty dx_1 \dots dx_{n-1} \int_{-t_0}^0 dt_1 \dots dt_{n-1} \int_0^{2\pi} d\omega_2 \dots d\omega_{n-1} \frac{\sqrt{\lambda_2 \dots \lambda_{n-1}}}{L_2 \dots L_{n-1}} \cdot \delta(x_1 + \dots + x_{n-1} - x - h(t_1, \dots, t_{n-1}, \omega_2, \dots, \omega_{n-1})) |M_n|^2, \quad (2)$$

with the notation

$\Gamma_n =$ some positive constant, depending on the normalization of M_n

$$x_i = \ln(s_i/s_0)$$

$$x = \ln(s/s_0)$$

$$L_i = 2\sqrt{t_{i-1}t_i} \cos \omega_i + m^2 - t_{i-1} - t_i$$

$$\lambda_i = \lambda(m^2, t_{i-1}, t_i)$$

$$h(t_1, \dots, t_{n-1}, \omega_2, \dots, \omega_{n-1}) = \ln \frac{\lambda_2 \dots \lambda_{n-1}}{L_2 \dots L_{n-1} s_0^{n-2}}$$

and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. In (2) we have chosen our unit s_0 so large that $s_i = s_0$ is within the limit of validity of the MRM; t_0 is some fixed positive number.

It may be that the vertex functions have zeros for $t_{i-1}, t_i \in [-t_0, 0]$ and $\omega_i \in [0, 2\pi]$, but if they do, we could take a smaller t_0 , and/or a smaller interval in ω_i , to avoid these zeros, so we might as well assume that the vertex functions do not vanish. If $\Gamma_2 > 0$ is the smallest value of $|G|^2$ or $|F|^2$ in our restricted integration region, we may in the lower bound replace the product of squares of the vertex functions by Γ_2^n ; treating the kinematic factor $\sqrt{\lambda_2 \dots \lambda_{n-1}} / L_2 \dots L_{n-1}$ in the same way, we see from (1) and (2) that

$$\sigma_n(s) \geq \frac{\Gamma_2^n}{s^2} \int_0^\infty dx_1 \dots dx_{n-1} \int_{-t_0}^0 dt_1 \dots dt_{n-1} \int_0^{2\pi} d\omega_1 \dots d\omega_{n-1} \delta(x_1 + \dots + x_{n-1} - x - h) \prod_{i=1}^{n-1} e^{2\alpha(t_i)x_i} \quad (3)$$

Some $\Gamma_3 > 0$

Define α' (roughly, the slope of the trajectory) to be such that $1 + \alpha't \leq \alpha(t)$ for all $t \in [-t_0, 0]$, and replace $\alpha(t_i)$ in Eq. (3) by $1 + \alpha't_i$. Using the δ function to integrate over x_{n-1} , we get

$$\sigma_n(s) \geq \frac{\Gamma_2^n}{s^2} \int_0^\infty dx_1 \dots dx_{n-2} \int_{-t_0}^0 dt_1 \dots dt_{n-1} \int_0^{2\pi} d\omega_1 \dots d\omega_{n-1} \Theta(x + h - x_1 - \dots - x_{n-2}) \cdot e^{-2h} e^{2\alpha't_{n-1}(x+h-x_1-\dots-x_{n-2})} \prod_{i=1}^{n-2} e^{2\alpha't_i x_i} \quad (4)$$

4.

From the definition of $h(t_1 \dots t_{n-1}, \omega_2 \dots \omega_{n-1})$, it is clear that there are fixed numbers h_+ and h_- such that $(n-2)h_- \leq h(t_1 \dots t_{n-1}, \omega_2 \dots \omega_{n-1}) \leq (n-2)h_+$, for all $t_i \in [-t_0, 0]$ and $\omega_i \in [0, 2\pi]$. In the lower bound (4), we may replace e^{2h_0} by $e^{2(n-2)h_-}$, and

$$e^{2\alpha' t_{n-1} (x + h - x_1 - \dots - x_{n-2})}$$

by

$$e^{2\alpha' t_{n-1} (x + (n-2)h_-)}$$

The θ function in (4) tells us that the possible values of x_i depend on the t 's and the ω 's, and so it is convenient to restrict ourselves to a smaller region of phase space, in order to eliminate this sort of a constraint. That is, for multiplicities such that $x + (n-2)h_- > 0$, if we restrict each x_i to the interval $[0, h_- + x/(n-2)]$, the θ function is always satisfied, and it is trivial to integrate over the ω_i 's. We can also integrate in (4) over the t_i 's, obtaining expressions of the form $(1 - e^{-2\alpha' t_0 x_i}) / (2\alpha' x_i)$, which we may replace by $t_0 / (2\alpha' t_0 x_i + 1)$, using the inequality

$$\frac{1}{y} (1 - e^{-ay}) \geq \frac{a}{ay+1}, \quad \text{all } a > 0, y \geq 0$$

In this way, we obtain from (4)

$$\sigma_n(s) \geq \frac{\Gamma_5^n}{x + (n-2)h_- + 1} \int_0^{h_- + x/(n-2)} dx_1 \dots dx_{n-2} \prod_{i=1}^{n-2} \frac{1}{2\alpha' t_0 x_i + 1} \quad (5)$$

From this we get immediately

$$\sigma_n(s) \geq \frac{\Gamma_6^n}{x + (n-2)h_- + 1} \left\{ \left[1 + 2\alpha' t_0 (h_- + x/(n-2)) \right] \right\}^{n-2}, \quad \text{Some } \Gamma_6 > 0 \quad (6)$$

valid whenever $x + (n-2)h_- > 0$.

Surely the total cross-section for scattering of particles a and b , σ_{tot} , is no less than σ_n , for any n ; thus (6) is a lower bound for σ_{tot} . Suppose we now let n and s go to infinity together, in such a way that $h_- + x/(n-2) = z$, where z is a fixed positive number. Then from (6)

$$\begin{aligned} \sigma_{\text{tot}} &\geq \sigma_n \left(\ln s = (n-2)(z-h_-) \right) \geq \frac{\Gamma_6^2}{x + (n-2)h_+ + 1} \exp \left[(n-2) \ln \left(\Gamma_6 \ln (1 + 2\alpha' t_0 z) \right) \right] \\ &= \frac{\Gamma_6^2}{\ln(s/s_0) + (n-2)h_+ + 1} \lesssim \frac{1}{z-h_-} \ln \left(\Gamma_6 \ln (1 + 2\alpha' t_0 z) \right) \quad \text{some } \Gamma_6 > 0 \end{aligned} \quad (7)$$

Now, whatever the values of the parameters h_- , Γ_6 , α' , and t_0 , we could have chosen z so large that the exponent in (7) is positive; thus σ_{tot} is bounded from below by some positive power of s . To derive this result, we have used the assumption that we could let n remain proportional to $\log s$ as $s \rightarrow \infty$, and stay within the region of validity of the MRM. But we can obtain a similar result using much smaller values of n : if we set $n = \log x (= \log \log(s/s_0))$ in (6), we still see that σ_{tot} increases faster than any power of $\log s$.

Thus the MRM cannot, without modification, be consistently applied to repeated Pomeron exchange. It may be that such an application could agree well with experimental data at some intermediate energies, but at sufficiently large energy, the calculated cross-sections would be too large. There are several possible ways of avoiding this difficulty, within the framework of the MRM:

- 1) Verdiev et al. ⁷⁾ point out that one can avoid this result by letting the vertex functions vanish as the momentum transfers approach zero. However, it is difficult to see why this should happen, except by dynamical accident; furthermore, if we admit this possibility, we would not know how to apply the model unless we understood the detailed behaviour of the vertex functions near the point of zero momentum transfers.

- 2) It might be necessary to include the effects of cuts. Since the difficulty with the Froissart bound occurs only for the Pomeranchuk, it might be that the Pomeranchuk is different from other trajectories in that it may not be dealt with separately from the cuts without encountering inconsistencies.
- 3) It has been suggested ⁹⁾ that there may not be any Pomeranchuk pole at all, i.e., that the singularity at $J=1$ is just a cut. Now, the plausibility of the MRM, which assumes a factorized dependence on the sub-energies [see Eq. (1)] is based on the expectation that there would be poles (with factorized residues) exchanged. Thus, if the Pomeranchuk were itself a cut, this simplification might not occur; it might even be meaningless to speak about multiple Pomeranchuk exchange.

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