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DETERMINATION OF PARTICLE SCATTERING AMPLITUDES FROM NUCLEAR
PHOTOPRODUCTION

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A B S T R A C T

Recent work on the extraction of resonance nucleon scattering amplitudes from the cross-section for the production of resonances from helium is extended to include photoproduction and also to include heavier target nuclei. It is found that from the reactions

and

the rho-nucleon scattering amplitude can be deduced, provided that the energy resolution of the experiment is good enough to fix the final state of the nucleus unambiguously.

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1. INTRODUCTION

In a recent paper ¹⁾ (hereafter referred to as I.) it was shown that one could obtain the magnitude and phase of a resonance nucleon scattering amplitude by looking at the process in which the resonance was produced coherently on a ${}^4\text{He}$ target. As an example, the reaction

was considered in some detail. The essential point of the argument was that the possibility of the production process occurring via the double scattering term shown in Fig. 1.a introduces $f_{\rho N}$ (the ρ -N scattering amplitude) into the calculation of reaction (1.1). In this way, by varying $\sigma_{\rho N}$ (the ρ -N total cross-section) and ρ ($\equiv \text{Re} f_{\rho N} / \text{Im} f_{\rho N}$), one can see corresponding variation in $d\sigma/dt(\pi + {}^4\text{He} \rightarrow \rho + {}^4\text{He})$ as a function of t . One could then use the experimental results for reaction (1.1) to pick out the values of $\sigma_{\rho N}$ and ρ which give the best fit, and thus determine $f_{\rho N}$ completely.

In I., however, two problems were noted. First, the presence of a double scattering term of the type in Fig. 1.b introduces $f_{\pi N}$, the pion-nucleon amplitude, into the expression for $d\sigma/dt$. This means that it is necessary to have an accurate determination of $f_{\pi N}$ in order to find $f_{\rho N}$. One can ask then, whether one could get a determination of $f_{\rho N}$ which does not depend on our knowledge of other scattering processes. Clearly, such a process is

where, for example, the only double scattering terms are of the type shown in Fig. 1.c.

In addition, when one uses isospin zero nuclei as a target, production by pions can proceed only by $I=0$ exchange (at least for single scattering). This means that, for example, ρ^0 meson production must be considered at high enough energies for the π exchange to be neglected. This difficulty does not arise in photoproduction, which goes by diffraction processes.

An additional value of considering photoproduction processes is that we can investigate the properties of neutral resonances (ρ^0 , φ etc.) which cannot be produced from π or P beams. In addition, by looking at reaction (1.2) for nuclei other than ${}^4\text{He}$, one can deduce σ_N from the A dependence of $(d\sigma/dt)|_{t=0}^2$, and this provides a useful check of at least one of the parameters in $f_{\rho N}$.

In I., we found that in order to get a determination of ρ it would be necessary to measure a $d\sigma/dt$ of $\sim 1\text{--}5 \mu\text{b}(\text{GeV}/c)^2$. We would like to raise this cross-section into a more easily measurable range. The obvious way to do this, of course, is simply to go to higher A nuclei. Consequently, we shall look in some detail at the reaction

Unfortunately, when we do so, we find ourselves on the horns of a rather typical dilemma. ${}^4\text{He}$ has the disadvantage that in the critical momentum transfer range, $d\sigma/dt$ is small. On the other hand, as was pointed out in I., the fact that ${}^4\text{He}$ has no stable excited states means that it is very easy to determine the final state of the nucleus. If one detects the recoil ${}^4\text{He}$, one can be sure that it is in the ground state.

With oxygen, although there is a somewhat larger cross-section at the minimum than one finds for ^4He , the first excited state is 6.6 MeV above the ground state, so that we need very fine energy resolution on both the initial and final particles to guarantee that we know the nuclear wave functions. While resolutions of ~ 3 MeV have been achieved in elastic p nucleus scattering at 1.67 GeV/c ³⁾, the typical resolution for photoproduction reactions of the type (1.1) is ~ 10 MeV ⁴⁾, which means that one must sum over many final states of the target nucleus. To show the effect of the energy resolution problem we shall consider a relatively simple case where the final nucleus can be either in its ground state or one excited state.

We shall see that even in this case the ability to distinguish between different choices of $f_{\phi N}$ by looking at nuclear production data is lost, and that the characteristic features of scattering to a single final nuclear state are completely washed out.

This problem is much more serious than it seems if one considers the excellent energy resolution obtained in Ref. ³⁾. It must be remembered that our use of the Glauber approximation ⁵⁾ means that our results will be best at high energies. Also, experiments proposed at CERN ⁶⁾ plan to look at processes of type (1.1) in the 10 GeV/c range. At energies of this magnitude, the problem of getting a 6 MeV resolution becomes very difficult indeed, so that some device, like the detection of the recoil helium, is needed to determine the final state of the nucleus.

In Part 2, we present the calculations for reaction (1.2) and discuss the effect of using different wave functions on the results. In Part 3, we extend the method to oxygen, first for elastic p - ^{16}O scattering, to check the method by comparing to the data of Ref. ³⁾ and then to photoproduction. In both parts we find that for momentum transfers up to the first minimum, the results are insensitive to the details of the nuclear structure. For helium, this insensitivity continues to higher momentum transfers, but for oxygen it is found necessary to include all of the higher orbital angular momentum components of the single particle wave functions in order to get a good fit to the data.

4.

Finally, in Part 4, we look at the problem of including the type of process which leaves the nucleus in an excited state and show how our method breaks down in this case.

2. HELIUM TARGET

The formalism which we shall use to handle reaction (1.2) is precisely the same as the one used in I., namely, the Glauber model for high energy scattering. The amplitude for (1.2) is written

$$\langle \psi_f | F | \psi_i \rangle = \frac{i p}{2\pi} \langle \psi_f | \int d^2 b e^{i \vec{\Delta} \cdot \vec{b}} \left\{ 1 - \prod_{j=1}^A \left[1 - \frac{1}{2\pi i k_j} \int d^2 \delta_j e^{-i \vec{\delta}_j \cdot (\vec{b} - \vec{s}_j + \vec{s})} f_j(\delta_j) \right] \right\} | \psi_i \rangle \quad (2.1)$$

where ψ_f and ψ_i refer to the final and initial states, $\vec{\delta}_j$ is the momentum transfer in the j^{th} scattering, p and k are the incident momentum in the c.m. of the projectile-nucleus and the projectile-nucleon system, respectively, $f_j(\delta_j)$ is the scattering amplitude of the j^{th} scattering, $\vec{\Delta}$ is the total momentum transfer. The position vector of the i^{th} nucleon is written as $\vec{r}_i = \vec{s}_i + \vec{z}_i$, where $\vec{s}_i$ is a vector in the plane perpendicular to the beam axis, and $\vec{b}$ is the impact parameter.

As in I. we start by considering the case when ^4He in the process (1.2) remains in its ground state, and we describe this state by the wave function for which

$$|\psi_0|^2 = \rho_0^4 \prod_{i=1}^4 e^{-r_i^2/R^2} \quad (2.2)$$

where $R^2 = 2/3 \langle r^2 \rangle$ ($\sqrt{\langle r^2 \rangle}$ is the r.m.s. radius of ${}^4\text{He}$) and $\int_0^4$ is the normalization factor.

If we then write

$$f_{\rho N}(\delta_j) = \frac{i + \rho}{4\pi} \tilde{\sigma}_{\rho N} k_j e^{-B/2 \delta_j^2} \quad (2.3)$$

for ρ -N elastic scattering from the j^{th} nucleon and

$$f_i(\delta_i) = \frac{i + \gamma}{4\pi} G k_i e^{-C/2 \delta_i^2} \quad (2.4)$$

for the photoproduction of a ρ from the i^{th} nucleon, and if we drop terms of $O(G^3)$, we find

$$\langle \psi_0 | F | \psi_0 \rangle = \frac{i\rho}{2} e^{\frac{\Delta^2 R^2}{4A}} \frac{G(1-i\gamma)}{2\pi(R^2+2C)} \cdot \quad (2.5)$$

$$\sum_k \binom{A}{k} \left[\frac{(i\rho-1)\tilde{\sigma}_{\rho N}}{2\pi(R^2+2B)} \right]^{k-1} \frac{e^{-\frac{\Delta^2}{4E}}}{E}$$

where

$$E = \frac{k-1}{R^2+2B} + \frac{1}{R^2+2C}$$

Now we can get the parameters for Eq. (2.4) from photoproduction experiments ⁷⁾. We use

$$C = 7.45 \text{ (GeV/c)}^{-2} \quad (2.6)$$

and

$$\frac{|G(1-i\gamma)|^2}{16\pi} = 100 \text{ } \mu\text{b}/(\text{GeV/c})^2 \quad (2.7)$$

In Fig. 2, we show how $d\sigma/dt(\gamma + {}^4\text{He} \rightarrow \rho + {}^4\text{He})$ changes as a function of $\tilde{\sigma}_{\rho N}$ and ρ (we take $B=C$). The cross-section at the dip is typically of the order of $\sim 0.1 \text{ } \mu\text{b}/(\text{GeV/c})^2$. Such a cross-section is at the limit of feasibility with present techniques ⁸⁾.

There are several points to make. First, unlike the case of the resonance production by hadrons, we can determine only the magnitude, and not the sign, of ρ . This can be seen directly from Eq. (2.5) since the real part of $F/(1+i\gamma)$ contains only even powers of ρ , while the imaginary part contains only odd powers. This, of course, is the disadvantage of photoproduction which corresponds to the advantage of having the amplitude independent of the π -N amplitude. On the other hand, the variation of the curves at minimum is quite rapid, as in production by pions. For example, changing ρ from 0.3 to 0.6 changes the curve from a dip to a shoulder.

In principle, the parameter B in Eq. (2.3) should be considered as a variable. However, since it always appears in the combination $R^2 + 2B$ and $R^2 \sim 50(\text{GeV/c})^{-2}$ we do not expect much of an effect from variations in B ⁹⁾. We have taken

$$B_{\rho N} = B_{\pi N} = C = 7.45 \text{ (GeV/c)}^{-2}$$

for our calculations as a reasonable first approximation.

We see then that, as expected, one can obtain a good deal of information about $f_{\rho N}$ by looking at the reaction (1.2) as a function of Δ^2 . In I., the stability of this type of result against changes in the elementary amplitudes was discussed in some detail, and it was found that except for the variations of ρ and $\tilde{\rho}$ mentioned above, the results are not sensitive to such variations. In addition, it has been shown ^{1), 10)}, that for hadron initiated reactions, the results are fairly insensitive to the type of wave function used for helium. If we require that our wave function reproduces the measured r.m.s. radius of helium and, in addition, that it gives a zero in the form factor ¹¹⁾ at the correct position, then there are two types of wave functions which have been proposed. The first ¹²⁾ is written

$$|\psi_0|^2 = \int \prod_{j=1}^A \left[e^{-r_j^2/R^2} - C e^{-\gamma r_j^2/R^2} \right] \quad (2.8)$$

so that the wave function remains separable, but each individual wave function is flattened at the origin. The second ¹³⁾ introduces the idea of short range correlation effects and writes

$$|\psi_0|^2 = \int \prod_{j=1}^A \frac{e^{-r_j^2/R^2} \left[1 - \sum_{\ell < k} e^{-\frac{\gamma}{R^2} |\vec{r}_\ell - \vec{r}_k|^2} \right]}{1 - \frac{6}{(1+2\gamma)^{3/2}}} \quad (2.9)$$

In the Table, we show the effect of these different wave functions on the position and magnitudes of the minima and maxima in the cross-section. The choice of $C=1$ in Eq. (2.8) corresponds to the value used in Ref. ¹²⁾, while the choice $C=\chi^2$ corresponds to the requirement that the wave function reaches its maximum value at the origin. We see that the differences between different types of wave functions, or between different choices of parameters for the same wave function, are really quite small. It is probable that these differences will be found to be of the same order of magnitude as the experimental errors when process (1.2) is measured.

In addition, it must be stressed that the ${}^4\text{He}$ wave function is not a free function. This single wave function must fit $e^{-4}\text{He}$ and $\pi^{-4}\text{He}$ elastic cross-sections, so that in principle, the determination of $f_{\Sigma N}$ will not depend on an arbitrary choice of $|\psi_0|^2$.

With these results on the reaction (1.2), we can turn to the question of whether the cross-section at minimum can be raised by going to higher A nuclei.

3. THE USE OF ${}^{16}\text{O}$ AS A TARGET

To apply the method of the previous section to oxygen, one has only to replace the helium wave functions in Eq. (2.1) by the corresponding wave functions for oxygen and let the product run to $A=16$ rather than $A=4$. The choice of the ${}^{16}\text{O}$ wave function can be made by looking at how well the theory matches the data of Ref. ³⁾ on elastic $p-{}^{16}\text{O}$ cross-sections. We will try two types of ground states. First, for simplicity, we neglect not only the (possible) dynamical correlations but also correlations caused by Fermi statistics and we put all nucleons in the $1s$ state. The corresponding density function is

$$|\psi_0|^2 = \int_0^{16} \frac{A}{j} e^{-r_j^2/R^2} \quad (3.1)$$

(This corresponds to a Gaussian nucleon mass distribution.)

Second, we try to be a bit more realistic by using harmonic oscillator wave functions in the usual shell model prescription

$$\psi_0 = \rho' \sum_P \varepsilon(P) \prod_{j=1}^A \varphi_{n_j, l_j, m_j}(\vec{r}_j) \quad (3.2)$$

where $\sum_P$ means sum over all permutations $\{i_1 \dots i_A\}$ and $\varepsilon(p) = +1(-1)$ if $\{i_1 \dots i_A\}$ is an even (odd) permutation of $\{1 \dots A\}$ and

$$\varphi_{n l m}(\vec{r}_j) = R_{n l}(r_j) Y_{l m}(\hat{r}_j) \quad (3.3)$$

and where

$$R_{n l}(r_j) = \sqrt{\frac{2 n!}{\Gamma(l+n+3/2)}} \frac{1}{R^{3/2}} \left(\frac{r_j}{R}\right)^l e^{-r_j^2/2R^2} L_n^{l+1/2}(r_j^2/R^2) \quad (3.4)$$

with $L_n^m(x)$ being the usual Laguerre polynomial ¹⁴⁾.

In the ground state of $^{16}_0$, the 1s and the 1p shells are completely filled for both protons and neutrons, so that in Eq. (3.2) we have $n=0$, $l=0, 1$, and $m=0, \pm 1$. Proceeding exactly as in Section 2, we find that the amplitude for the photoproduction of a rho meson from $^{16}_0$ in which the nucleus is not excited is

$$\begin{aligned}
\langle \psi_0 | F | \psi_0 \rangle &= \frac{iP}{2} (R^2 + 2B) \left(\frac{(1-i\chi)G}{2\pi(R^2 + 2B)} \right) e^{\frac{\Delta^2 R^2}{4A}} (1 - \int \\
&\sum_{v=0}^8 \binom{8}{v} v! \left(\frac{2B}{R^2 + 2B} \right)^v \left[1 - \frac{\Delta^2 R^2}{4A^2} \right]^{8-v} \sum_{l=0}^v \frac{l!}{(v-l)!} \\
&\sum_{k=0}^8 (1 - \int_{v+k,0}) \binom{8}{k} \left(\frac{(1-i\rho)\tilde{\sigma}_{pN}}{2\pi(R^2 + 2B)} \right)^{v+k-1} e^{-\frac{\Delta^2}{4} \frac{R^2 + 2B}{v+k}} \quad (3.5) \\
&\left(\frac{R^2}{2B(v+k)} \right)^l \sum_{s=0}^l \frac{(-1)^s}{(l-s)!(s!)^2} \left(\frac{\Delta^2 (A - k - v)^2 (R^2 + 2B)}{4A^2 (k+v)} \right)^s
\end{aligned}$$

where $v(k)$ counts the number of scatterings on nucleons with $|m| = 0(1)$.

To get the expression for the reaction

it is necessary to replace G and $\tilde{\sigma}_{pN}$ by σ_p and χ and ρ by α_{pN} in the above expression. We can then ask how the wave functions (3.1) and (3.2) compare to the data and to each other. In Fig. 3, we show this comparison.

Several points emerge from the comparison. First, we see that for small scattering angles, both wave functions give essentially the same result. This is similar to the case for ${}^4\text{He}$. The main difference comes after the minimum where the Gaussian does not match the data very well, but the harmonic oscillator seems to do very well even out to the highest angle measured. The explanation of this phenomenon is not difficult to see. At high momentum transfers, we are probing the details of the nuclear structure much more than at low momentum transfers when in fact only $\langle r^2 \rangle$ matters. Thus, the inclusion of statistical correlations, i.e., the handling of the higher angular momentum components of the single particle wave functions becomes important at higher momentum transfer. To a certain extent, the fact that the minimum occurs at a smaller momentum transfer for ${}^{16}\text{O}$ than for ${}^4\text{He}$ is also a factor here, since in Ref. ¹⁾ the technique which we are using seems to break down at a scattering angle of about 25° , which corresponded to the secondary maximum for $p - {}^4\text{He}$ scattering. In the case we are considering now, however, all of the structure of interest to us falls well within a smaller angular range, so that one can have much more confidence in the Glauber approximation.

In Fig. 4, we show the results for photoproduction of rho mesons from ${}^{16}\text{O}$. We see that it is indeed possible to determine $f_{\rho N}$ provided that we can be sure that the nucleus is not excited. The cross-section at minimum is now about $3-10 \mu\text{b}/(\text{GeV}/c)^2$ which is, indeed much easier to measure than the $0.1-0.5 \mu\text{b}/(\text{GeV}/c)^2$ which was found for photoproduction from ${}^4\text{He}$. However, for ${}^{16}\text{O}$, the energy resolution problem (see the next paragraph) is so difficult that the photoproduction with ${}^4\text{He}$ target is probably more reasonable from the experimental point of view.

4. THE ENERGY RESOLUTION PROBLEM

In all of the work that has been presented thus far, we have made explicit use of the fact that the nucleus is not excited during the interaction. However, if the energy resolution of the experiment is not sufficiently good, it will be impossible to do this and it will become necessary to sum over all final states which are energetically available to the system. It is well known ^{5),15)} that if the resolution is bad enough to allow a large number of final states to enter a problem, then one can use the closure approximation to calculate the cross-section, and all structure of the type in which we are interested, minima and secondary maxima, disappears.

However, for a typical energy resolution of 10 MeV, one has about 5-10 final states in the problem and it is probably not valid to make the closure approximation. One can then ask whether there is any way of deriving information from data to which only a limited number of final states can contribute. Our conclusion is that this is not possible and that it is absolutely essential to be able to specify the final state of the nucleus exactly.

To show why this is so, let us consider the reaction (1.3) for the case where the final nucleus can either be in its ground state, or in one available excited state. For ease of calculation, let us take this state to be the one in which one of the outer nucleons has been lifted into the 2s level by the interaction.

In the amplitude we ignore those terms in which an inner nucleon is lifted to the 2s shell, since such a final nucleus will be unstable. Furthermore, it is easy to see that the amplitude for an $\ell=1$, $m=0$ nucleon to be lifted to the 2s shell is equal to zero.

Thus we need only concern ourselves with the $m = \pm 1$ particles being excited. This means that instead of 16 terms contributing to the single scattering, there will be only eight. Referring to Fig. 1 of Ref. ¹⁾, we see that this means that the position of the minimum for the excited state term will be shifted toward the origin with respect to the ground state term. Thus, we have a situation in which two curves which have minima at different points are added together, and we expect that the structure of both minima will be completely washed out.

In Fig. 5 we show the results for our hypothetical problem. We see that even if we include only one extra state in the problem, the characteristic features of the dip are completely lost. We note that the shape of the combined differential cross-section for photoproduction is very similar to those which have been measured for light nuclei ⁴⁾ with a resolution of 10 MeV. Thus, the size of the energy resolution in those experiments explains why no dips were seen.

To emphasize the problems which arise in determining $f_{\rho N}$ under conditions where the energy resolution is not good enough to isolate ground state transitions, we show in Fig. 6 the combined differential cross-section for various values of $\sigma_{\rho N}$ and σ . It is extremely unlikely that any experiment could make a convincing distinction between the different choices of $f_{\rho N}$.

We see, then, that from an experimental point of view, it is much more important to design the apparatus so that the final state of the nucleus can be completely specified than to try to go to higher A nuclei in the hope of getting bigger cross-sections. That one can live with small cross-sections was pointed out in Ref. ¹⁾, where it was shown that important information could be derived from reaction (1.1) even in the case where the cross-section at minimum fell below the limits of measurement.

5. CONCLUSION

We have seen that nuclear photoproduction processes can be used to determine things like the total rho-nucleon cross-section, and the phase of the rho-nucleon amplitude. With present techniques, the measurement of these quantities with the use of a helium target is probably feasible. Using oxygen as a target instead of helium would raise the cross-section at minimum but it was seen that unless an energy resolution of ~ 6 MeV can be achieved, no knowledge of $f_{\rho N}$ can be extracted from such data. Since it is unlikely that it will be possible to get such a small energy resolution it is felt that the ${}^4\text{He}$ reaction is much more promising as a technique for measuring $f_{\rho N}$.

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$\gamma\text{He}^4 \rightarrow \rho\text{He}^4$ for $P = 6 \text{ GeV}/c$

Type of wave function	C	γ	γ'	r.m.s. f	$\frac{d\sigma}{dt} \Big _{t=0}$ $\mu\text{b}(\text{GeV}/c)^2$	t_1 $(\text{GeV})^2$	$\frac{d\sigma}{dt} \Big _{t=t_1}$ $\mu\text{b}(\text{GeV}/c)^2$	t_2 $(\text{GeV})^2$	$\frac{d\sigma}{dt} \Big _{t=t_2}$ $\mu\text{b}(\text{GeV}/c)^2$
Gauss	-	-	-		1201	0.270	0.10	0.365	0.20
Bessel-Wilkin	1	0.47		1.68	1207	0.265	0.10	0.360	0.20
		0.1832			1203	0.270	0.10	0.360	0.20
			3.5		1179	0.240	0.07	0.310	0.15
Czyż			4.0		1183	0.250	0.07	0.320	0.15
			4.5		1186	0.250	0.08	0.325	0.16
					1140	0.300	0.14	0.405	0.27
Gauss				1.50	1168	0.275	0.11	0.375	0.28
Bessel-Wilkin	1	$\sqrt{0.2}$							

Parameters C, γ are defined by Eq. (2.8).

Parameter γ' is defined by Eq. (2.9).

t_1 is the position of the first minima.

t_2 is the position of the second maxima.

We used $\sigma_{\text{tot}} = 30 \text{ mb}$ $|\alpha_{\rho N}| = 0.3$,
 $\frac{d\sigma}{dt} \Big|_{t=0} = 100 \mu\text{b}/(\text{GeV}/c)^2$

R E F E R E N C E S

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FIGURE CAPTIONS

- Figure 1 : Possible contributions to double scattering.
- Figure 2 : The reaction $\gamma + {}^4\text{He} \rightarrow \rho^0 + {}^4\text{He}$ at 6 GeV/c. We have taken $\sqrt{\langle r^2 \rangle} = 1.68 \text{ f}$ and $\frac{d\sigma}{dt}(\gamma p \rightarrow \rho^0 p)|_{t=0} = 100 \mu\text{b}/(\text{GeV}/c)^2$ and have used simple Gaussian wave function.
- Figure 3 : The reaction $p + {}^{16}\text{O} \rightarrow p + {}^{16}\text{O}$ at 1.67 GeV/c. This shows the difference between Eq. (3.1) and Eq. (3.2). We have taken $\sqrt{\langle r^2 \rangle} = 2.61 \text{ f}$, $\sigma_{pN} = 43.8 \text{ mb}$ and $\alpha_{pN} = -0.36$. The experimental points are from Ref. 3).
- Figure 4 : The reaction $\gamma + {}^{16}\text{O} \rightarrow \rho^0 + {}^{16}\text{O}$ at 6 GeV/c. We have taken $\sqrt{\langle r^2 \rangle} = 2.61 \text{ f}$ and $\frac{d\sigma}{dt}(\gamma p \rightarrow \rho^0 p)|_{t=0} = 100 \mu\text{b}/(\text{GeV}/c)^2$ and have used the harmonic oscillator wave function.
- Figure 5 : Results for energy resolution problem, showing cross-sections for transitions to the ground state and single excited state and the total differential cross-section for the two combined.
- Figure 6 : Differential cross-section for different parameters in $f_{\rho N}$ when one excited state is included in the problem.

(1a)

(1b)

(1c)

FIG.1

FIG 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6