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MULTIPLE SCATTERING AND ISOBAR PRODUCTION

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At incident lab. momenta of 10 GeV/c or greater, there is experimental evidence ¹⁾⁻⁴⁾ on the following reactions for exciting nucleons to their isobaric levels :

If we write the cross-section for each reaction at small momentum transfers t in the form $d\sigma/dt = a e^{bt}$, a and b are found to have the following properties :

- i) : for those reactions which can proceed by Pomeron exchange, A is roughly independent of energy ; we consider only these cases, and interpret the evidence as indicating that Pomeron exchange is indeed dominant ;
- ii) : at small $|t|$ ($\lesssim 1 \text{ GeV}^2/c^2$), the parameter b/GeV^2 is about 9 for $pp \rightarrow pp$, 14 to 18 for $pp \rightarrow pN^*(1400)$, 4 or 5 for $pp \rightarrow pN^*(1520, 1690 \text{ or } 2190)$, and similar (slightly smaller) for the corresponding pion-initiated reactions ;
- iii) : at large $|t|$ ($\gtrsim 2 \text{ GeV}^2/c^2$), the reactions (a) where they are known have similar slopes, roughly the same as for pp elastic scattering.

In the present letter we apply a multiple scattering formalism to isobar production. This formalism is familiar in the theory of particle production in nuclei ⁵⁾, and is a natural extension of the method applied to elastic hadron scattering in a previous paper ⁶⁾. Our treatment will supply an explanation of experimental property iii), and an interesting connection between property ii) and the number of "inelastic steps" involved in the multiple scattering. It is important to note that although

we shall employ a Regge pole model, our main results on properties ii) and iii) will not depend on Regge behaviour. The key feature of our treatment is the absorptive corrections supplied by the multiple scattering formalism.

Our assumptions are as follows :

- i) : We neglect all spin dependence ⁶⁾.
- ii) : In the Glauber multiple scattering picture ^{7),5)}, the amplitude for a reaction $a + c \rightarrow a' + c$ can be written

$$A^{ij} = \sum_{r=1}^{\infty} A_r^{ij} \quad (1)$$

where $i \rightarrow j$ via one inelastic step in A_1 , two inelastic steps ($i \rightarrow m \rightarrow j$) in A_2 , and so forth. At high energies and small angles one can use the eikonal approximation and write

$$A_1^{ij} = s \int \frac{d^2b}{2\pi} \delta_{ij}(b) e^{2i\delta(b)} e^{i\vec{b} \cdot \vec{q}} \quad (2)$$

$$A_2^{ij} = -is \int \frac{d^2b}{2\pi} \left(\sum_m \delta_{im}(b) \delta_{mj}(b) \right) e^{2i\delta(b)} e^{i\vec{b} \cdot \vec{q}} \quad (3)$$

etc. Here $\vec{q}$ is the three-momentum transfer, b is an impact parameter, $\delta(b)$ is the eikonal phase for elastic scattering (here assumed the same for ac and $a'c$) at this impact parameter, and $\delta_{ij}(b)$ is related to the strength of the inelastic transition at impact parameter b . We shall adopt Eqs. (1)-(3) for our hadron problem, confining the discussion to the first two steps A_1 and A_2 for simplicity. The factor $\exp[2i\delta(b)]$, which is responsible for absorption of the beam on its way in and out, will play an important role in our results.

- iii) : The remaining problem is to specify $\delta(b)$ and $\delta_{ij}(b)$. In practice, we shall make the choice of Ref. ⁶⁾, identifying the "Born approximation" to the eikonal formalism with the Regge pole contribution A_{pole} :

$$2i\delta(b) = \int \frac{d\vec{q}_\perp}{2\pi} \left[\frac{A_{\text{pole}}^{ii}(s, -\vec{q}_\perp^2)}{-is/2} \right] e^{-i\vec{b} \cdot \vec{q}_\perp} \quad (4)$$

and similarly for $\delta_{ij}(b)$. It should be kept in mind, however, that any reasonable phenomenological parametrization of $\delta(b)$ would give absorptive corrections qualitatively similar to those we shall obtain, so that the main conclusions of the present paper are not tied to the identification of $\delta(b)$ with A_{pole} .

- iv) : For A_{pole} , we assume a straight line Pomeron trajectory

$$\alpha_p(t) = 1 + t\alpha' \quad (5)$$

and use the value $\alpha' = 0.82 \text{ GeV}/c^{-2}$ given by the fit to elastic scattering in a previous paper ⁶⁾. The coupling is assumed such that the single Pomeron exchange contribution to A_{ij}^{ij} is ⁶⁾

$$A_{\text{pole}}^{ij}(s, t) = c_{ij} \left(\frac{s}{s_0} e^{-i\pi/2} \right)^{\alpha_p(t)} \quad (6)$$

For simplicity, we take $s_0 = 1 \text{ GeV}^2$ independent of the reaction, and we take the elastic strengths c_{ii} for $NN^* \rightarrow NN^*$ to be the same as that determined from experiment ⁶⁾ for $NN \rightarrow NN$. Similarly, we take $c(\pi N^* \rightarrow \pi N^*)$ equal to the experimentally determined ⁶⁾ $c(\pi N \rightarrow \pi N)$. Given these assumptions, the s and t dependence of all A_{pole}^{ij} has the same form

$$A_{\text{pole}}^{ij}(s, t) = -i c_{ij} \left(\frac{s}{s_0} \right) e^{t\alpha'\mu} \quad (7)$$

where $\mu = \ln(s/s_0) - i\pi/2$, and only one strength parameter for each inelastic reaction remains undetermined⁸⁾.

The model defined by these assumptions is easy to solve. Putting the A_{pole} of Eq. (7) into (4), one obtains as in Ref. 6)

$$2i\delta(b) = -\frac{\xi}{\mu} e^{-b^2/(4\alpha'\mu)} \quad (8)$$

with the strength parameter $\xi = -c/(\alpha's_0)$. Similarly for the inelastic reactions, one obtains

$$\delta_{ij}(b) = -\frac{\xi_{ij}}{2i\mu} e^{-b^2/(4\alpha'\mu)} \quad (9)$$

where $\xi_{ij} = -c_{ij}/(\alpha's_0)$. One then substitutes (8) and (9) into Eqs. (2) and (3), expands $e^{2i\delta}$ into a power series in $(2i\delta)^n$, and performs the resulting integrals over Gaussian integrands, obtaining

$$A_1 = i s \alpha' \xi_{ij} \sum_{n=1}^{\infty} \frac{1}{n(n-1)!} \left(-\frac{\xi}{\mu}\right)^{n-1} e^{t\alpha'\mu/n} \quad (10)$$

$$A_2 = \frac{i s \alpha'}{2\mu} \left(\sum_m \xi_{im} \xi_{mj} \right) \sum_{n=2}^{\infty} \frac{1}{n(n-2)!} \left(-\frac{\xi}{\mu}\right)^{n-2} e^{t\alpha'\mu/n} \quad (11)$$

The n^{th} term in these expressions can be identified with n^{th} order scattering. For comparison, the elastic amplitude derived in Ref. 6) from closely related assumptions was

$$A_{el} = i s \alpha' \xi \sum_{n=1}^{\infty} \frac{1}{n(n-1)!} \left(-\frac{\xi}{\mu}\right)^{n-1} e^{t\alpha'\mu/n} \quad (12)$$

In both elastic and inelastic cases, the connection to the differential cross-section is given by

$$\frac{d\sigma}{dt} = \frac{\pi}{k_i^2 s} |A|^2 \quad (13)$$

where k_i is the incoming centre-of-mass momentum.

Among the coefficients in these equations, $s_0 = 1 \text{ GeV}^2$, $\alpha' = 0.82 \text{ GeV}/c^{-2}$ and $\xi = 7$ have already been determined from the elastic scattering⁶⁾, but the inelastic strengths ξ_{ij} are undetermined, so we have the shape but not the overall normalization of each A_r . This means we cannot predict the absolute rate for production of any isobar, or how many inelastic steps dominate in each case, but we can plot the shape for each A_r and attempt to identify phenomenologically how many steps dominate for each production reaction.

The t dependence of $|A_1|^2$ and $|A_2|^2$ at 20 GeV/c incident lab. momentum is plotted in Fig. 1. The t dependence of our model⁶⁾ for pp elastic scattering at the same incident momentum is also shown for comparison. At small t , the forward peak in $|A_1|^2$ is considerably sharper than the elastic peak, while the peak in $|A_2|^2$ is about twice as broad as $|A_{el}|^2$. These differences are brought out more clearly on the smaller scale in Fig. 2, where we see that in the small t expression

$$d\sigma/dt = a e^{bt}, \quad b \text{ is } 8 (\text{GeV}/c)^{-2} \text{ for } |A_{el}|^2, \\ 12 \text{ for } |A_1|^2, \text{ and } 5 \text{ for } |A_2|^2.$$

This leads us to identify production of the 1400 MeV isobar as a dominantly one-step inelastic process, and production of the 1520, 1690 and 2190 MeV isobars as dominantly two-step (or perhaps multiple-step) inelastic processes. In line with this identification the curves in Fig. 1 have been normalized at $t = 0$ so that the one-step process corresponds to $d\sigma/dt$ (pp $\rightarrow$ p.N* 1400) and the two-step process to

We conclude by listing some further possible applications of our method for estimating multiple scattering corrections.

- i) : For reactions in which a single Pomeranchon can be exchanged, such as $\pi N \rightarrow \pi^* N$, $\pi N \rightarrow \pi^* N^*$ and $\gamma N \rightarrow \bar{V} N$, the method of the present paper can be taken over with relatively minor modifications.
- ii) : For reactions in which a single Pomeranchon cannot be exchanged, first order scattering involves other exchanges which will generally require a more complicated parametrization. The absorption factor $e^{2i\delta}$, introducing higher order corrections, will have the same simple form as in the present paper, however. Many estimates of absorptive corrections have already been made for small t ¹⁰⁾, of course, and introduce such interesting qualitative changes as converting the forward dip for single pion exchange in $np \rightarrow pn$ into a narrow forward peak ¹¹⁾.
- iii) : Multiple scattering involving more than one non-Pomeranchon exchange also occurs, and is probably important for reactions such as double strangeness exchange ¹²⁾. As a correction to processes which go in first order, however, it is normally smaller at high energies than the simpler absorptive corrections associated with Pomeranchon exchanges.

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(North-Holland Public., Amsterdam, 1967).
- 8) In connection with our earlier comment that the Regge features are not
crucial for our purposes, note that if we replaced $\alpha' \mu(s)$
in Eq. (7) by a real constant we would obtain a non-Regge model
which would reproduce all our results except the shrinking at
large t , and the details of how dips are filled in.
- 9) In particular, even though A_2 is weaker than A_1 for forward 1400
production, the two-step amplitude may considerably modify the
1400 production cross-section at larger t values.
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FIGURE CAPTIONS

- Figure 1 : Proton-proton elastic scattering and "one-step" and "two-step" inelastic processes, calculated using formulae (10)-(13).
- Figure 2 : The small momentum transfer behaviour for pp elastic scattering and one-step and two-step inelastic processes at 20 GeV/c. The solid curves are theoretical, and the dotted straight lines correspond to experimental mean values. The list of slopes is experimental data from Ref. 3), in GeV/c^{-2} .

FIG.1