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GENERAL PROPERTIES OF REGGE FAMILIES IN THE WICK-CUTKOSKY MODEL

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A B S T R A C T

General properties of Regge families as the coupling goes to zero or as the square of the energy goes to minus infinity are studied in the Bethe-Salpeter equation with zero mass exchange. Cutkosky-Deo mixings are shown to occur.

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Recently numerical calculations of the highest trajectories have been reported for the ladder Bethe-Salpeter (B-S) equation ^{1),2)}; perturbation treatments of some lower trajectories have been given either by Mellin transform techniques ^{3),4)} or ⁵⁾ by analytic continuation ⁶⁾ of the Fredholm method ⁷⁾, or, for the case of zero mass exchange, by power series expansion near zero energy ⁸⁾. These results indicate that for finite energy, as λ (square of the coupling constant) goes to zero, at first order in λ , the leading trajectory ⁷⁾ tends to $\ell = -1$ while three trajectories tend to -2 and five ⁴⁾ to -3 . Concerning the limit of infinite energy, it is known ⁹⁾ that the Regge poles of a non-relativistic Yukawa potential go to the negative integers or to infinity. A similar behaviour is not shown by the perturbative expressions of the B-S trajectories, e.g., the first daughter stays constant at $\ell = -2 + \lambda$ ³⁾⁻⁵⁾.

In this note we show for the equal mass Wick-Cutkosky model ¹⁰⁾ that :

- i) as $\lambda \rightarrow 0$ N Regge poles with even time parity and $N-1$ Regge poles with odd time parity go to the negative integer $-N$ in the complex ℓ plane, any other finite end point being excluded:
- ii) as $t = 4m^2 \rightarrow -\infty$ no Regge pole remains inside any bounded domain of the half-plane $\text{Re} \ell < -1-n'$, n' being the index of the Toller family ¹¹⁾ (see later).

In addition, our method can be used to derive power series expansions of all Regge trajectories.

We recall that due to the $O(4)$ symmetry of the model considered ¹⁰⁾ the bound state levels can be classified by a quantum number n , connected to the angular momentum ℓ by $n = \ell + n'$, ($n' = 1, 2, \dots$). The energy of the bound state depends only on n through the Wick-Cutkosky eigenvalue equation

$$(1-z^2)g_n''(z) + 2(n-1)zg_n'(z) - n(n-1)g_n(z) + \frac{\lambda}{1-\eta^2+\eta^2z^2}g_n(z) = 0 \quad (1)$$

$$\frac{\lambda \pi (-4\eta^2)^{-(n+1)}}{\eta \sin \pi(n+1)} \quad (7)$$

in any bounded domain of the half-plane $\text{Re } n < 0$.

We prove this by exactly evaluating the terms which need analytic continuation in the iterative solution of Eq. (3) and by majorizing the remaining integrals. From b) it follows that as $\lambda \rightarrow 0$ the solutions of Eq. (4.b) can go either to infinity in the n plane or to the negative integers, while those of Eq. (4.a) can in addition also go to zero. By using Rouché's theorem¹²⁾, one can prove the more detailed result that as $\lambda \rightarrow 0$ we have one and only one solution of Eq. (4.a) (even time parity) which tends to each non-positive integer and one and only one solution of (4.b) (odd time parity) which tends to each negative integer. From c) it follows that as $\eta^2 \rightarrow -\infty$ there are no solutions of Eqs. (4) inside a given domain of the half-plane $\text{Re } n < -1$. In particular, as $\eta^2 \rightarrow -\infty$ the Regge pole goes either to infinity or, through complex values, to the region $\text{Re } n > -1$. An explicit discussion can be made for the first order solutions of Eqs. (4) as $\eta^2 \rightarrow -\infty$ by inserting (7) into Eqs. (4). For odd time parity, one solution tends to $n = -1$ along the real axis [it corresponds to the first daughters of the Toller families $\kappa_+(n')$, defined below] with the behaviour

$$\left(\frac{\ell + n' + 1}{\lambda} \right) - \frac{1}{\ell + n' + 1} = -4\eta^2$$

On the left of $\text{Re } n = -1$, for small $\eta^2 < 0$, pairs of solutions start from the real neighbourhoods of consecutive negative integers $(-2, -3)$ $(-4, -5)$, ..., move one against the other along the real axis, collide and go into the complex plane orthogonally to the real axis.

As $-\eta^2 \rightarrow +\infty$ they turn to the right, finally accumulating at $n=-1$. For even time parity trajectories, the first order solution of Eq. (4.a) gives the well-known result of Lee and Sawyer ⁷⁾ for the leading trajectory which goes to $n=0$ as $-\eta^2 \rightarrow \infty$. For $\text{Re } n < -1$, pairs of solutions start from the real neighbourhoods of consecutive negative integers $(-1,-2)$ $(-3,-4)$, ..., move one against the other along the real axis, collide and go into the complex plane orthogonally to the real axis, turn to the right and finally accumulate at $n=-1$. Examples of such collision phenomena among trajectories have already been given by Cutkosky and Deo ²⁾ for $\ell \geq -\frac{3}{2}$.

The zero energy intercepts are ¹⁰⁾ :

$$\ell(0) = -\kappa - \frac{1}{2} + \sqrt{\frac{1}{4} + \lambda} - n' \quad (8.a)$$

$$\ell(0) = -\kappa - \frac{1}{2} - \sqrt{\frac{1}{4} + \lambda} - n' \quad (8.b)$$

where κ is the nodal quantum number ($\kappa = 0, 1, 2, \dots$) whose parity coincides with the time parity of φ [Eq. (2)]. Accordingly, the Toller families can be denoted by $\mathcal{K}_+(n')$ and $\mathcal{K}_-(n')$ for fixed n' ($n' = 1, 2, \dots$) and variable κ ($\kappa = 0, 1, 2, \dots$), the plus and minus corresponding to the intercepts (8.a) and (8.b) respectively.

A simple way to see this is to remark that according to Domokos' formula ¹³⁾, for general λ , two trajectories belonging to the same family cannot equal derivative at zero energy. Equations (3) and (4) can also be used to derive power series expansions in the coupling strength and in the energy which, however, as shown by the preceding results, can only be valid for small η^2 . The first order expressions for the zero energy intercepts and derivatives with respect to η^2 are summarized in the Table.

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6.

Regge trajectory	Intercept	Derivative
$\alpha_+(n')$	$-\alpha - n' + \lambda$	$\frac{1}{3}\lambda(2-\lambda) - \frac{1}{3} \frac{\lambda(1+\alpha-\lambda)(\alpha-\lambda)}{1-\lambda}$
$\alpha_-(n')$	$-\alpha - 1 - n' - \lambda$	$-\lambda(1+\lambda) + \frac{\lambda(2+\alpha+\lambda)(1+\alpha+\lambda)}{2+\lambda}$

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