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OVERLAP FUNCTION ANALYSIS OF $\pi^{\pm}p$ ELASTIC AND CHARGE EXCHANGE
SCATTERING AT HIGH ENERGIES

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A B S T R A C T

In terms of overlap functions, a new kind of analysis of πN elastic and charge exchange scattering data at high energies and small momentum transfers is given. The data analyzed cover a range $6 \text{ GeV/c} \lesssim q_L = \text{pion laboratory momentum} \lesssim 18 \text{ GeV/c}$ and $0 \leq -t = (\text{centre-of-mass momentum transfer})^2 \lesssim 1 (\text{GeV/c})^2$. The phenomenological expressions of the scattering amplitudes, including spin-flip, as they are obtained from experiment, are traced back to simple forms for the $\pi^{\pm}p$ overlap functions. The latter are interpreted in terms of different classes of inelastic $\pi^{\pm}p$ collisions, each contributing with a characteristic power of $s = (\text{centre-of-mass energy})^2$ to the overlap functions. The dominant deviations from the genuine asymptotic situation (where π^+p and π^-p elastic scatterings are expected to be identical) are attributed to the presence of "charge annihilation channels" among π^-p inelastic transitions which have no counterpart among the π^+p inelastic transitions. In this context the concepts of absorptive corrections and of a common interaction volume for both elastic and inelastic collisions are encountered within the overlap function framework. The actual simple forms we have adopted for the overlap functions imply that the charge exchange peak must remain somewhat broader than the elastic ones, so that in the case of constancy of the latter the observed shrinkage of the former is predicted to slow down and stop, or alternatively but less plausibly, elastic peaks have to start to shrink, the change occurring at an energy somewhat higher than the ones here studied and which is estimated to be $q_L \simeq 26 \text{ GeV/c}$. Another implication of our overlap functions is that the reggeized ρ exchange form of the charge exchange amplitude is supplemented by small corrections. As will be shown in a succeeding paper, these account in a natural manner for the measured polarization in $\pi^-p \rightarrow \pi^0n$. Also the $\pi^{\pm}p \rightarrow \pi^{\pm}p$ polarizations will be shown to be satisfactorily described in the present formalism.

I. INTRODUCTION

This paper gives an analysis of present high energy data at low momentum transfers on πp elastic and charge exchange scattering in terms of overlap functions ¹⁾. The data considered cover a region of $6 \text{ GeV/c} \lesssim q_L \lesssim 18 \text{ GeV/c}$ and $0 \leq -t \lesssim +1 (\text{GeV/c})^2$, where q_L is the pion laboratory momentum and $-t$ the momentum transfer squared between the pion and the nucleon in the centre-of-mass system.

Before proceeding to the detailed analysis, we recall the situation for elastic scattering and indicate the generalizations needed to include charge exchange scattering.

I.1) Elastic scattering

The notion of overlap function has been introduced by Van Hove ¹⁾ as providing an appropriate mathematical formulation to the idea of high energy diffraction or shadow scattering. We recall briefly the main ideas of this approach and the status of its applications to quantitative data analysis. The consideration is based on the unitarity relation for the s channel of the process in question (s = square of centre-of-mass energy). For instance for the partial wave amplitude f_ℓ of $\pi N \rightarrow \pi N$, neglecting nucleon spin for the moment, this reads

$$\text{Im } f_\ell(s) = \langle \phi_\ell | S^{(\ell)}(P-p) | \phi_\ell \rangle ; \quad (\text{I.1})$$

where P is the operator of total four-momentum and $p = (p^0 = \sqrt{s}, \vec{p} = 0)$ its eigenvalue for the incoming πN system. The states $|\phi_\ell\rangle$ are defined by the equation

$$T|\pi N; p, \ell\rangle = S^{(\ell)}(P-p) |\phi_\ell\rangle, \quad (\text{I.2})$$

2.

where $|\pi N; p, \ell\rangle$ is an incoming πN state with total momentum p and relative angular momentum ℓ and T is the transition operator related to the S matrix according to $S = 1 + 2iT$. Equation (I.2) corresponds to the introduction of energy shell transition matrix elements where the δ -function for energy momentum conservation has been split off.

If we split the outgoing wave, i.e., the state $|\phi_e\rangle$, into its elastic and inelastic components

$$|\phi_e\rangle = |\phi_e; \pi N\rangle + |\psi_e\rangle, \quad (I.3)$$

with

$$|\phi_e; \pi N\rangle = \int d^4p f_e(s) |\pi N; p, \ell\rangle, \quad f_e(s) = \langle \pi N; p, \ell | \phi_e \rangle,$$

Eq. (I.1) becomes

$$\text{Im } f_e(s) = \frac{1}{4} g_e(s) + |f_e(s)|^2, \quad (I.4)$$

where

$$\frac{1}{4} g_e(s) = \langle \psi_e | \delta^{(4)}(P-p) | \psi_e \rangle. \quad (I.5)$$

$g_e(s)$ is the ℓ component of the overlap function ¹⁾. If for some reason the phase of $f_e(s)$ is known, Eq. (I.4) leads to a simple algebraic determination of the amplitude in terms of $g_e(s)$. Since $g_e(s)$ describes in the elastic scattering channel the effects due to the presence of inelastic transitions, Eq. (I.4) provides a quantitative formulation of the concept of diffraction scattering.

Within our present knowledge, the observed diffraction peaks are most simply interpreted as predominantly due to the non-spin-flip amplitude. Furthermore, rather general hypotheses on the asymptotic s dependence of the scattering amplitudes (dominance of $C = +1$ exchange, C = charge conjugation quantum number), analyticity properties in this variable and the experimental fact that the diffraction phenomenon seems

to be independent of energy imply that the non-spin-flip amplitude is dominated by its imaginary part at sufficiently high energies,

$$f_e(s) \approx i \operatorname{Im} f_e(s), \quad s \rightarrow \infty. \quad (\text{I.6})$$

From (I.4) then follows

$$\operatorname{Im} f_e(s) = \frac{1}{2} \left(1 \pm \sqrt{1 - g_e(s)} \right), \quad (\text{I.7})$$

the positive sign being excluded by demanding continuity and the condition $\operatorname{Im} f_e(s) \rightarrow 0$ if $g_e(s) \rightarrow 0$. In this way it was possible to trace back the observed asymptotic shape of the elastic differential cross-section,

$$\frac{d\sigma_{ee}}{dt} = \left(\frac{d\sigma_{ee}}{dt} \right)_{t=0} e^{at+bt^2}, \quad 0 < \frac{b}{a^2} \ll 1, \quad (\text{I.8})$$

to an overlap function of purely Gaussian form [see Eq. (II.15)] in the impact parameter $b = (\ell + \frac{1}{2})/q$, where q is the magnitude of the centre-of-mass momentum, i.e., of purely exponential form in t . This form, in turn, can be understood in terms of certain bulk properties of inelastic collisions at high energies, i.e., dominance of many-particle production with weak correlation (beyond the correlation due to energy momentum conservation) between the particles in the final state.

In summary, it can be said that a definite simplification in the phenomenological description of high energy scattering in the diffraction region takes place at the overlap function level. Furthermore, the specific data reproducing form of this function is directly susceptible to physical interpretation. It should be noted, however, that the purely exponential form of the overlap function in t is only expected to describe the genuine asymptotic situation. To analyze the diffraction pattern at non-asymptotic energies, an appropriately modified overlap function and the real part of the elastic amplitude have to be introduced. Quantitative analyses of elastic scattering along Van Hove's lines have been carried out by Cottingham and Peierls²⁾ for πp and pp scattering and by Kokkedee³⁾ for $\bar{p}p$ scattering.

I.2) Generalization to include charge exchange scattering

As is well known, closer examination of elastic $\pi^\pm p$ cross-section data ⁴⁾ reveals small systematic deviations from the genuine asymptotic situation where $\pi^+ p$ and $\pi^- p$ scattering are expected to be identical. The total cross-sections ⁵⁾ differ from their common asymptotic value and their observed energy variation supports the interpretation that $\pi^+ p$ scattering is more asymptotic than $\pi^- p$ scattering in the energy region of interest. While the difference of $\pi^- p$ and $\pi^+ p$ differential cross-sections is positive for $t=0$, it is negative for $t=-0.3 \text{ (GeV/c)}^2$, which results in a cross-over of the differential cross-sections $d\sigma/dt$ around the value $t=-0.15 \text{ (GeV/c)}^2$ which seems to be energy independent ⁶⁾.

On the other hand, due to isospin invariance, the amplitudes $f^{(0)}$ and $f^{(1)}$ for elastic $\pi^\pm p$ scattering are connected with the amplitude $f^{(2)}$ for charge exchange by $f^{(2)} = 1/\sqrt{2}(f^{(0)} - f^{(1)})$. As a consequence the deviations of the elastic scatterings from their purely asymptotic values also manifest themselves directly in the charge exchange process for which recent experiments have given very detailed information for the s and t values we are concentrating on ^{10), 11), 12), 13)}.

The aim of the present investigation is to analyze these deviations from the true asymptotic situation with the help of the overlap function concept, generalized to allow for the detailed study of a special inelastic channel as charge exchange scattering. This simply means that we consider simultaneously the s channel unitarity relations for the three processes $\pi^\pm p \rightarrow \pi^\pm p$ and $\pi^- p \rightarrow \pi^0 n$ and define in each case, in analogy to (I.4), (I.5), the overlap function as originating from the components of the outgoing waves from the $\pi^+ p$, $\pi^- p$ and $\pi^0 n$ system orthogonal to the πN states. In addition, because of the observed importance of the spin-flip amplitude in charge exchange scattering, we shall have to take the spin degrees of freedom into account in full generality.

The experimental fact that at high energy charge exchange is small compared to elastic scattering will be used, in addition to the properties of the elastic scattering amplitudes indicated before, to neglect small terms in the unitarity equations. Again, it becomes possible to express explicitly the imaginary parts of the amplitudes in terms of overlap functions. Once the imaginary parts are known, the real parts can be determined from crossing and analyticity properties (i.e., from dispersion relations) ¹⁴⁾.

The overlap functions (for non-spin-flip and spin-flip transitions) for $\pi^-p \rightarrow \pi^0 n$ will be seen to be related to the overlap functions for $\pi^\pm p \rightarrow \pi^\pm p$ by the same isospin relations as the scattering amplitudes. As a consequence, charge exchange scattering is related to the difference between the π^+p and π^-p elastic overlap functions.

The present study is based on the results of the phenomenological analysis of asymptotic elastic scattering in Ref. ²⁾ and uses the phenomenological information on charge exchange scattering in the form of a parametrization given by Höhler et al. ^{15),16)} in terms of reggeized ρ exchange. From this, the data reproducing forms of the asymptotic and the leading non-asymptotic contribution to the overlap functions are determined. Again it will be observed that the phenomenological description of data simplifies decisively at the overlap function level and that the data reproducing forms can be given an interesting physical interpretation. It is based on the phenomenological conception that among the inelastic transitions from the π^-p system there is, besides those which have a counterpart in π^+p transitions, an additional class missing in π^+p . These correspond to what might be called "charge annihilation channels", in analogy with the baryon annihilation channels in $\bar{p}p$ collisions. While in our model π^+p type transitions are responsible for asymptotic diffraction scattering, the presence of the additional class in the π^-p case produces the leading deviations from the asymptotic behaviour and is directly related to charge exchange scattering. In this context we shall encounter within the overlap function framework the concepts of absorption corrections and of a well-defined interaction volume common to elastic and inelastic processes, as have been proposed in the absorption model ¹⁷⁾ and coherent droplet model ¹⁸⁾ for two-body inelastic collisions.

In this way, rather complicated energy dependent properties can be accounted for in simple terms and the fact that $\pi^\pm p \rightarrow \pi^\pm p$ and $\pi^-p \rightarrow \pi^0 n$ peaks have approximately the same width is naturally understood. In fact, the actual form we have adopted for the overlap functions implies that the charge exchange peak must remain somewhat broader than

the elastic ones, so that in the case of constant elastic peaks the observed shrinkage of the former is predicted to slow down and to stop at an energy somewhat higher than the ones studied here. Another, less plausible alternative is the setting in of a shrinkage of the elastic peaks from the same energies onward. We estimate that the change in behaviour of the peaks should occur around 26 GeV laboratory energy.

Another interesting aspect of our considerations is that the Regge form of the charge exchange amplitude is supplemented by small corrections resulting from the specific form of the overlap functions. It will be shown in a succeeding paper ¹⁹⁾ that these corrections account in a natural manner for the measured polarization in $\pi^- p \rightarrow \pi^0 n$ ¹³⁾, which is known not to be explained by a pure reggeized ρ exchange mechanism. Also the $\pi^\pm p \rightarrow \pi^\pm p$ polarizations will be seen to be satisfactorily described in the present framework.

In Section II we collect some general formulae and summarize the phenomenological information. Section III is concerned with the general unitarity equations, their solution in the high energy approximation and the phenomenological determination of the overlap functions. In Section IV the physical interpretation of the corresponding expressions is given. Section V presents some conclusions.

II. GENERAL FORMULAE AND THE PHENOMENOLOGICAL INFORMATION

II.1) General formulae

We characterize the three reactions $\pi^+ p \rightarrow \pi^+ p$, $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^0 n$ by the indices 0,1,2, respectively. We first list some formulae for the scattering of a spin zero particle on a spin $\frac{1}{2}$ target, which are analogous in all three cases and omit for the moment this index. We shall work with the s channel spin-non-flip and spin-flip amplitudes f and $\bar{f}$, which are related to the scattering matrix F between Pauli spinors by

$$F = f(s,t) + i [\vec{\sigma}(\hat{q}_1 \times \hat{q}_2)] \bar{f}(s,t); \quad (\text{II.1})$$

$\hat{q}_1$ and $\hat{q}_2$ are unit vectors in the direction of the momenta $\vec{q}_1$ and $\vec{q}_2$ of the incoming and outgoing pion in the centre-of-mass system. The normalization will be chosen such that

$$\frac{d\sigma}{dt} = \frac{\pi}{q^2 s} (|f|^2 + |\bar{f}|^2). \quad (\text{II.2})$$

Between the amplitudes $(f^{(i)}, \bar{f}^{(i)})$, $i=0,1,2$, there exist, due to isospin invariance, the relations

$$\left. \begin{aligned} f^{(2)} &= \frac{1}{\sqrt{2}} (f^{(0)} - f^{(1)}), \\ \bar{f}^{(2)} &= \frac{1}{\sqrt{2}} (\bar{f}^{(0)} - \bar{f}^{(1)}). \end{aligned} \right\} \quad (\text{II.3})$$

The partial wave expansions of the amplitudes f and $\bar{f}$ are given by

$$f(s,t) = 2 \frac{\sqrt{s}}{q} \sum_{\ell=0}^{\infty} (\ell + \frac{1}{2}) \hat{f}_{\ell}(s) P_{\ell}(z), \quad (\text{II.4})$$

$$\bar{f}(s,t) = 2 \frac{\sqrt{s}}{q} \sum_{\ell=0}^{\infty} \bar{f}_{\ell}(s) P_{\ell}^{\prime}(z), \quad (\text{II.5})$$

where

$$\hat{f}_\ell = f_\ell + \frac{1}{2} \frac{1}{\ell + 1/2} \bar{f}_\ell, \quad (\text{II.6})$$

$$f_\ell = \frac{1}{2} (f_{\ell+} + f_{\ell-}), \quad (\text{II.7})$$

$$\bar{f}_\ell = \frac{1}{2} (f_{\ell+} - f_{\ell-}), \quad (\text{II.8})$$

and $f_{\ell\pm}$ are transition amplitudes between states of definite total angular momentum $J = \ell \pm \frac{1}{2}$ and parity $(-1)(-1)^\ell$; $z = \cos\theta = 1 + t/2q^2$.

At high energies and for small t these partial wave series are most conveniently evaluated by introducing the description of scattering with help of the impact parameter $b = (\ell + \frac{1}{2})/q$, which formally is obtained by substituting

$$\sum_{\ell=0}^{\infty} \rightarrow \int_0^{\infty} d(qb)$$

$$(\ell + \frac{1}{2}) P_\ell(z) \rightarrow qb J_0(b\sqrt{-t})$$

$$P'_\ell(z) = \sin\theta \cdot P'_\ell(z) \rightarrow qb J_1(b\sqrt{-t})$$

$$f_\ell(s) \rightarrow h(b,s) = f(\ell = bq - \frac{1}{2}, s), \quad (\text{II.9})$$

$$\bar{f}_\ell(s) \rightarrow \bar{h}(b,s) = \bar{f}(\ell = bq - \frac{1}{2}, s), \quad (\text{II.10})$$

where $f(\ell, s)$ and $\bar{f}(\ell, s)$ are defined in terms of the even ℓ and odd ℓ Carlson interpolations $f^{(\pm)}(\ell, s)$ and $\bar{f}^{(\pm)}(\ell, s)$ (which at high energies become equal to the extent that backward scattering is negligibly small compared to forward scattering) of the partial wave amplitudes $f_\ell(s)$ and $\bar{f}_\ell(s)$ ²⁰⁾,

$$f(\ell, s) = \frac{1}{2} (f^{(+)}(\ell, s) + f^{(-)}(\ell, s)),$$

$$\bar{f}(\ell, s) = \frac{1}{2} (\bar{f}^{(+)}(\ell, s) + \bar{f}^{(-)}(\ell, s)).$$

J_0 and J_1 are the Bessel functions of order 0 and 1. This results in

$$f(s, t) = 2q\sqrt{s} \int_0^\infty db b \hat{h}(b, s) J_0(b\sqrt{-t}), \quad (\text{II.11})$$

$$\bar{f}(s, t) = 2q\sqrt{s} \int_0^\infty db b \bar{h}(b, s) J_1(b\sqrt{-t}). \quad (\text{II.12})$$

As has been shown in Ref. ²⁰⁾, these expressions give f and $\bar{f}$ up to terms which are at most of order $o(s^\varepsilon)$, $\varepsilon > 0$ and arbitrarily small, for $s \rightarrow \infty$. Since phenomenologically $f^{(0)}$ and $f^{(1)}$, as well as possibly also $\bar{f}^{(0)}$ and $\bar{f}^{(1)}$, are $o(s^1)$, the errors introduced by using (II.11) and (II.12) are practically at most $o(s^{-1}f^{(0,1)})$ and $o(s^{-1}\bar{f}^{(0,1)})$ for elastic scattering. For charge exchange, $f^{(2)}$ and $\bar{f}^{(2)}$ are approximately $o(s^{\frac{1}{2}})$, thus the error is at most $o(s^{-\frac{1}{2}}f^{(2)})$ and $o(s^{-\frac{1}{2}}\bar{f}^{(2)})$. In the present context, only effects of order $o(s^1)$ and $o(s^{\frac{1}{2}})$ are investigated explicitly; as a consequence the errors introduced by the impact parameter description will be neglected.

For the amplitude $\hat{h}(b, s)$ follows from (II.6)

$$\hat{h}(b, s) = h(b, s) + \frac{1}{2qb} \bar{h}(b, s).$$

For charge exchange, the second term can safely be neglected because it gives a contribution $O(s^{-\frac{1}{2}} f^{(2)})$; for the elastic scatterings it can be neglected in the calculation of observable quantities to the degree that $\bar{h}(b,s)$ is small compared to $h(b,s)$. Thus we put

$$\hat{h}(b,s) = h(b,s). \quad (\text{II.13})$$

II.2) Phenomenological information on asymptotic $\pi^{\pm}p \rightarrow \pi^{\pm}p$ scattering

As mentioned before, we use the phenomenological information on asymptotic elastic scattering in the parametrization of Cottingham and Peierls²⁾. The description is based on the equation

$$\text{Im } h^{(0,1)}(b,s) = \frac{1}{4} G(b) + \left(\text{Im } h^{(0,1)}(b,s) \right)^2, \quad (\text{II.14})$$

for $h^{(0,1)}(b,s)$, which follows from an equation of type (I.6) for the partial wave amplitudes $f_{\ell}^{(0,1)}$ if the transition (II.9) is carried out, where elastic spin-flip as well as the real parts are neglected. $G(b,s)$ is, similarly to $h^{(0,1)}(b,s)$, defined in terms of Carlson interpolations of the partial wave components $g_{\ell}(s)$ of the overlap function. Since asymptotically $h^{(0)}$ and $h^{(1)}$ are equal, they are given in terms of the single function $G(b,s)$.

Asymptotic scattering for $0 \leq -t \lesssim 1 \text{ (GeV/c)}^2$ is well fitted with an energy independent function $G(b)$ of purely Gaussian form

$$G(b) = P \cdot e^{-\frac{1}{4B} b^2}; \quad (\text{II.15})$$

$$P = 0.91, \quad B = 4.86 \text{ (GeV/c)}^{-2}. \quad (\text{II.16})$$

II.3) Regge description of $\pi^- p \rightarrow \pi^0 n$

The formulae for exchange of a ρ Regge pole in $\pi^- p \rightarrow \pi^0 n$ have been given by Singh²¹⁾. Höhler et al.¹⁶⁾ have analyzed the experimental information in terms of the free parameters of this Regge pole model. They are the trajectory and residue functions $\alpha(t)$ and $\beta_+(t)$, $\beta_-(t)$ corresponding to the ρ Regge pole in the t channel, where

$$\beta_+(t) = 2\pi \lim_{J \rightarrow \alpha(t)} \left[M^{2J-1} f_+^{(-)}(J, t) (\alpha(t) - J) \right],$$

$$\beta_-(t) = 2\pi \lim_{J \rightarrow \alpha(t)} \left[M^{2J} \frac{f_-^{(-)}(J, t)}{\sqrt{J(J+1)}} (\alpha(t) - J) \right],$$

$f_{\pm}^{(-)}(J, t)$ are the Carlson interpolations of the helicity non-flip and flip amplitudes in the t channel $\pi^- \pi^0 \rightarrow \bar{p} n$ and $M = \text{nucleon mass}$.

For the s channel amplitudes $f^{(2)}$ and $\bar{f}^{(2)}$ the form resulting from reggeized ρ exchange, derived in Appendix A, reads

$$f^{(2)} = -\frac{1}{\sqrt{2\pi}} \frac{1 - e^{-i\pi\alpha(t)}}{\sin[\pi\alpha(t)]} \varphi(t) f(t) \left(\frac{2q_L}{M} \right)^{\alpha(t)},$$

$$\bar{f}^{(2)} = -\frac{1}{\sqrt{2\pi}} \frac{1 - e^{-i\pi\alpha(t)}}{\sin[\pi\alpha(t)]} \varphi(t) \sqrt{\frac{-t}{4M^2}} \bar{f}(t) \left(\frac{2q_L}{M} \right)^{\alpha(t)},$$

$$\varphi(t) = \frac{1}{1 - t/4M^2} \frac{\Gamma(\alpha(t) + \frac{3}{2})}{\Gamma(\alpha(t) + 1)},$$

$$f(t) = \beta_+(t) - \frac{t}{4M^2} \alpha(t) \beta_-(t),$$

$$\bar{f}(t) = \beta_+(t) - \alpha(t) \beta_-(t).$$

$q_L \approx s/2M$ is the incident pion laboratory momentum.

According to the analysis by Hhler et al. $\alpha(t) = \alpha_0 + \alpha_1 t$ for $0 \leq -t \leq 0.8 \text{ (GeV/c)}^2$, where $\alpha_0 = 0.57$, $\alpha_1 = 0.96 \text{ (GeV/c)}^{-2}$. As is shown in Fig. 1, $\varphi(t)$ can be fitted by an exponential

$$\varphi(t) = \varphi_0 e^{\varphi_1 t},$$

where $\varphi_0 = 1.16$, $\varphi_1 = 0.74 \text{ (GeV/c)}^{-2}$. Figure 2 shows the functions $\gamma(t)$ and $\bar{\gamma}(t)$ and reveals the remarkable phenomenological fact that both are linear in t , while following Hhler et al. neither $\beta_+(t)$ nor $\beta_-(t)$ show any simple t dependence. We put

$$\gamma(t) = \gamma_0 + \gamma_1 t,$$

$$\bar{\gamma}(t) = \bar{\gamma}_0 + \bar{\gamma}_1 t,$$

with $\gamma_0 = 0.95$, $\gamma_1 = 3.55 \text{ (GeV/c)}^{-2}$, $\bar{\gamma}_0 = -9.1$ and $\bar{\gamma}_1 = -13.8 \text{ (GeV/c)}^{-2}$.

With the formulae in Appendix B and $(1 - e^{-i\pi\alpha})/\sin(\pi\alpha) = (1 - \cos(\pi\alpha))/\sin(\pi\alpha) + i$ these results yield for the imaginary parts of the impact parameter weight functions $h^{(2)}(b,s)$ and $\bar{h}^{(2)}(b,s)$

$$\text{Im } h^{(2)}(b,s) = -\frac{1}{4\sqrt{2}} \alpha e \left\{ \Delta P + P \frac{\Delta B}{48B'} b^2 \right\} e^{-\frac{1}{48B'} b^2}, \quad (\text{II.17})$$

$$\text{Im } \bar{h}^{(2)}(b,s) = -\frac{1}{4\sqrt{2}} \bar{\alpha} e \left\{ \bar{\Delta P} + P \frac{\Delta B}{48B'} b^2 \right\} b e^{-\frac{1}{48B'} b^2}, \quad (\text{II.18})$$

$$B' = B'(s) = \alpha_1 \log\left(\frac{2q_L(s)}{M}\right) + \varphi_1, \quad (\text{II.19})$$

$$\Delta B = \Delta B(s) = B - B'(s), \quad (\text{II.20})$$

$$\mathcal{H} = \mathcal{H}(s) = \frac{1}{M^2} \frac{2}{\sqrt{\pi}} \varphi_0 \frac{B}{P} \left(\frac{2q_L(s)}{M} \right)^{\alpha_0-1} \frac{1}{B'^2(s) \Delta B(s)} \mathcal{H}_1^e \quad (\text{II.21})$$

$$\Delta P = \Delta P(s) = \frac{P}{B} \left(\frac{B'(s) \mathcal{H}_0^e}{\mathcal{H}_1^e} - 1 \right) \Delta B(s), \quad (\text{II.22})$$

$$\overline{\mathcal{H}} = \overline{\mathcal{H}}(s) = \frac{1}{M^2} \frac{2}{\sqrt{\pi}} \varphi_0 \frac{B}{P} \left(\frac{2q_L(s)}{M} \right)^{\alpha_0-1} \frac{1}{B'^2(s) \Delta B(s)} \frac{1}{4MB'(s)} \overline{\mathcal{H}}_1^e, \quad (\text{II.23})$$

$$\overline{\Delta P} = \overline{\Delta P}(s) = \frac{P}{B} \left(\frac{B'(s) \overline{\mathcal{H}}_0^e}{\overline{\mathcal{H}}_1^e} - 2 \right) \Delta B(s), \quad (\text{II.24})$$

P and B are the parameters characterizing asymptotic scattering introduced in subsection II.2). They are independent of s. As will appear in Section III, the rather complicated form adopted for (II.17) and (II.18) will lead to simple expressions for the overlap functions, see Eqs. (III.32) and (III.33). They will also be suitable for the physical interpretation described in Section IV. $B'(s)$ determines the slope of the exponential part of the t dependence of the charge exchange amplitudes and contains in particular the typical Regge pole contribution $\propto_1 \cdot \log(2q_L/M)$ which gives rise to the shrinking effect.

In the table the values of the parameters (II.19)-(II.23) are given for $q_L = 6 \text{ GeV}/c$ and $18.2 \text{ GeV}/c$ in order to illustrate their behaviour in the energy region considered.

III. THE UNITARITY EQUATIONS

III.1) The general structure of the unitarity equations

We define states for any charge configuration of the πN system

$$|\pi N; p, \ell\rangle = \frac{1}{\sqrt{2}} (|\pi N; p, \ell^+\rangle + |\pi N; p, \ell^-\rangle), \quad (\text{III.1})$$

$$|\bar{\pi} N; p, \ell\rangle = \frac{1}{\sqrt{2}} (|\pi N; p, \ell^+\rangle - |\pi N; p, \ell^-\rangle), \quad (\text{III.2})$$

where $|\pi N; p, \ell^\pm\rangle$ are incoming πN states with total four-momentum p , relative angular momentum $J = \ell \pm \frac{1}{2}$ and parity $(-1)(-1)^\ell$. Analogously to (I.3), we introduce

$$\delta^{(i)}(P-p) |\phi_\ell^{(i)}\rangle = T |\pi N^{(i)}; p, \ell\rangle, \quad (\text{III.3})$$

$$\delta^{(i)}(P-p) |\bar{\phi}_\ell^{(i)}\rangle = T |\bar{\pi} N^{(i)}; p, \ell\rangle, \quad (\text{III.4})$$

where $i=0,1,2$, and these indices characterize π^+p , π^-p and π^0n states, respectively.

The unitarity equations for the amplitudes $f_\ell^{(i)}$ and $\bar{f}_\ell^{(i)}$, $i=0,1,2$, can now be written as

$$\begin{aligned} \text{Im } f_\ell^{(0,1)} &= \langle \phi_\ell^{(0,1)} | \delta^{(4)}(P-p) | \phi_\ell^{(0,1)} \rangle, \\ \text{Im } \bar{f}_\ell^{(0,1)} &= \langle \bar{\phi}_\ell^{(0,1)} | \delta^{(4)}(P-p) | \phi_\ell^{(0,1)} \rangle, \\ \text{Im } f_\ell^{(2)} &= \langle \phi_\ell^{(2)} | \delta^{(4)}(P-p) | \phi_\ell^{(1)} \rangle, \\ \text{Im } \bar{f}_\ell^{(2)} &= \langle \bar{\phi}_\ell^{(2)} | \delta^{(4)}(P-p) | \phi_\ell^{(1)} \rangle. \end{aligned}$$

We split $|\phi_e^{(i)}\rangle$ and $|\bar{\phi}_e^{(i)}\rangle$ into components in the πN system and components $|\psi_e^{(i)}\rangle$, $|\bar{\psi}_e^{(i)}\rangle$ orthogonal to it (i.e., coming from more than two particles)

$$|\phi_e^{(0)}\rangle = |\phi_e^{(0)}; \pi^+ p\rangle + |\psi_e^{(0)}\rangle, \quad (\text{III.5})$$

$$|\phi_e^{(1)}\rangle = |\phi_e^{(1)}; \pi^- p\rangle + |\phi_e^{(1)}; \pi^0 n\rangle + |\psi_e^{(1)}\rangle, \quad (\text{III.6})$$

$$|\phi_e^{(2)}\rangle = |\phi_e^{(2)}; \pi^0 n\rangle + |\phi_e^{(2)}; \pi^- p\rangle + |\psi_e^{(2)}\rangle, \quad (\text{III.7})$$

with similar equations for $|\bar{\phi}_e^{(i)}\rangle$, and we define overlap functions

$$\frac{1}{4} g_e^{(0,1)} = \langle \psi_e^{(0,1)} | \delta^{(4)}(P-p) | \psi_e^{(0,1)} \rangle, \quad (\text{III.8})$$

$$\frac{1}{4} g_e^{(2)} = \langle \psi_e^{(2)} | \delta^{(4)}(P-p) | \psi_e^{(1)} \rangle, \quad (\text{III.9})$$

$$\frac{1}{4} \bar{g}_e^{(0,1)} = \langle \bar{\psi}_e^{(0,1)} | \delta^{(4)}(P-p) | \psi_e^{(0,1)} \rangle, \quad (\text{III.10})$$

$$\frac{1}{4} \bar{g}_e^{(2)} = \langle \bar{\psi}_e^{(2)} | \delta^{(4)}(P-p) | \psi_e^{(1)} \rangle. \quad (\text{III.11})$$

The above unitarity equations now become

$$\text{Im } f_e^{(0)} = \frac{1}{4} g_e^{(0)} + |f_e^{(0)}|^2 + |\bar{f}_e^{(0)}|^2, \quad (\text{III.12})$$

$$\text{Im } \bar{f}_e^{(0)} = \frac{1}{4} \bar{g}_e^{(0)} + 2 \text{Re}(f_e^{(0)*} \bar{f}_e^{(0)}), \quad (\text{III.13})$$

$$\text{Im } f_e^{(1)} = \frac{1}{4} g_e^{(1)} + |\bar{f}_e^{(2)}|^2 + |f_e^{(2)}|^2 + |\bar{f}_e^{(1)}|^2 + |f_e^{(1)}|^2, \quad (\text{III.14})$$

$$\text{Im } \bar{f}_e^{(1)} = \frac{1}{4} \bar{g}_e^{(1)} + 2 \text{Re} (f_e^{(2)*} \bar{f}_e^{(2)}) + 2 \text{Re} (f_e^{(1)*} \bar{f}_e^{(1)}), \quad (\text{III.15})$$

$$\text{Im } f_e^{(2)} = \frac{1}{4} g_e^{(2)} + f_e^{(2)*} \bar{f}_e^{(2)} + \bar{f}_e^{(1)} f_e^{(2)*} + f_e^{(3)*} f_e^{(2)} + f_e^{(1)} f_e^{(2)*}, \quad (\text{III.16})$$

$$\text{Im } \bar{f}_e^{(2)} = \frac{1}{4} \bar{g}_e^{(2)} + \bar{f}_e^{(3)} f_e^{(2)} + \bar{f}_e^{(1)} f_e^{(2)*} + f_e^{(3)*} \bar{f}_e^{(2)} + f_e^{(1)} \bar{f}_e^{(2)*}, \quad (\text{III.17})$$

where $f_e^{(3)}$ and $\bar{f}_e^{(3)}$ describe $\pi^0 n \rightarrow \pi^0 n$ scattering.

If isospin invariance is introduced $f_e^{(1)}$, $\bar{f}_e^{(1)}$ as well as $f_e^{(3)}$, $\bar{f}_e^{(3)}$ can be eliminated from these equations by using relation (II.3) between reactions 0, 1 and 2, relation $f_e^{(3)} = f_e^{(0)} - 1/\sqrt{2} f_e^{(2)}$ and the same relation for $\bar{f}_e$. Then, Eqs. (III.16) and (III.17) become

$$\text{Im } f_e^{(2)} = \frac{1}{4} g_e^{(2)} - \frac{3}{\sqrt{2}} (|f_e^{(2)}|^2 + |f_e^{(2)}|^2) + 2 \text{Re} (f_e^{(0)*} \bar{f}_e^{(2)}) + 2 \text{Re} (f_e^{(0)*} f_e^{(2)}), \quad (\text{III.18})$$

$$\text{Im } \bar{f}_e^{(2)} = \frac{1}{4} \bar{g}_e^{(2)} - 3\sqrt{2} \text{Re} (f_e^{(2)*} \bar{f}_e^{(2)}) + 2 \text{Re} (f_e^{(0)*} \bar{f}_e^{(2)}) + 2 \text{Re} (f_e^{(0)*} f_e^{(2)}), \quad (\text{III.19})$$

and (III.14) and (III.15) reduce to the relations

$$g_e^{(2)} = \frac{1}{\sqrt{2}} (g_e^{(0)} - g_e^{(1)}), \quad (\text{III.20})$$

$$\bar{g}_e^{(2)} = \frac{1}{\sqrt{2}} (\bar{g}_e^{(0)} - \bar{g}_e^{(1)}), \quad (\text{III.21})$$

between the overlap functions. It is on the basis of these last equations that we shall relate in the next Section the experimental information on $\pi^- p \rightarrow \pi^0 n$ to properties of the overlap functions $g_e^{(0,1)}$ for elastic $\pi^\pm p$ scattering.

The general structure of the unitarity relations implies the following bounds for the overlap functions

$$0 \leq g_e^{(0,1)} \leq +1, \quad -\frac{1}{2} \leq \bar{g}_e^{(0,1)} \leq +\frac{1}{2}, \quad (\text{III.22})$$

$$-\frac{1}{\sqrt{2}} \leq g_e^{(2)}, \bar{g}_e^{(2)} \leq +\frac{1}{\sqrt{2}}. \quad (\text{III.23})$$

These bounds are derived in Appendix C.

III.2) Approximations at high energies

We introduce now our assumptions on the high energy amplitudes, as already discussed in the Introduction,

$$- \quad |f_e^{(0)}| \ll |f_e^{(1)}|, \quad |\text{Re } f_e^{(0)}| \ll \text{Im } f_e^{(0)}, \quad (\text{III.24})$$

$$- \quad |f_e^{(2)}|, |\bar{f}_e^{(2)}| \ll \text{Im } f_e^{(0)}, \quad (\text{III.25})$$

the relations

$$\begin{aligned} f_e^{(1)} &= f_e^{(0)} - \sqrt{2} f_e^{(2)}, \\ \bar{f}_e^{(1)} &= \bar{f}_e^{(0)} - \sqrt{2} \bar{f}_e^{(2)}, \end{aligned}$$

implying similar inequalities for $f_e^{(1)}, \bar{f}_e^{(1)}$.

It should be noted that beside hypotheses (III.24) and (III.25), no further assumptions are introduced. In particular, no assumptions on the relative magnitudes of $\text{Re } f_e^{(0)}, \bar{f}_e^{(0)}$ and $f_e^{(2)}, \bar{f}_e^{(2)}$ have to be made. Thus $\bar{f}_e^{(0)}$ could be $o(s^0)$ like $f_e^{(0)}$ for $s \rightarrow \infty$, and $f_e^{(2)}, \bar{f}_e^{(2)}$ can be $o(s^{\alpha_0-1})$ as suggested by the Regge pole model for $\pi^- p \rightarrow \pi^0 n$. But we need not specify such properties at this stage.

Introducing inequalities (III.24) and (III.25) into Eqs. (III.12), (III.13) and (III.16), (III.17), and linearizing the latter systematically in all small quantities, one reduces them to equations determining the imaginary parts of the amplitudes in terms of the overlap functions. The solutions of these equations are found to be

$$\text{Im } f_e^{(0)} = \frac{1}{2} \left(1 \pm \sqrt{1 - g_e^{(0)}} \right), \quad (\text{III.26})$$

$$\text{Im } \bar{f}_e^{(0)} = \frac{1}{4} \frac{\bar{g}_e^{(0)}}{\sqrt{1 - g_e^{(0)}}}, \quad (\text{III.27})$$

$$\left. \begin{aligned} \text{Im } f_e^{(2)} &= \frac{1}{4} \frac{g_e^{(2)}}{\sqrt{1 - g_e^{(0)}}} \\ \text{Im } \bar{f}_e^{(2)} &= \frac{1}{4} \frac{\bar{g}_e^{(2)}}{\sqrt{1 - g_e^{(0)}}} \end{aligned} \right\} \quad (\text{III.28})$$

These results now permit easily to find the form of the overlap functions $g_e^{(2)}$ and $\bar{g}_e^{(2)}$, reproducing the charge exchange data. This is done by comparison with equations (III.17) and (III.18) for $\text{Im } h^{(2)}$ and $\text{Im } \bar{h}^{(2)}$. Some additional considerations will lead to the forms of $g_e^{(0)}$ and $\bar{g}_e^{(0)}$.

III.3) The data reproducing form of $g_e^{(2)}$ and $\bar{g}_e^{(2)}$

All considerations concerning unitarity are also valid for the impact parameter weight functions $h^{(i)}$ and $\bar{h}^{(i)}$ if the relations (II.9) and (II.10) are taken into account and overlap functions $G^{(i)}(b,s)$ and $\bar{G}^{(i)}(b,s)$ depending on the impact parameter are introduced. These are defined in analogy to $h^{(i)}$ and $\bar{h}^{(i)}$ in terms of the even ℓ and odd ℓ Carlson interpolations of the partial wave components $g_\ell^{(i)}(s)$ and $\bar{g}_\ell^{(i)}(s)$. From (II.17) and (II.18), and (III.28), follows

$$G^{(2)}(b,s) = -\frac{1}{\sqrt{2}} \mathcal{E} \sqrt{1-G^{(0)}(b,s)} \left\{ \Delta P + P \frac{\Delta B}{4BB'} b^2 \right\} e^{-\frac{1}{4B'} b^2}, \quad (\text{III.29})$$

$$\bar{G}^{(2)}(b,s) = -\frac{1}{\sqrt{2}} \bar{\mathcal{E}} \sqrt{1-G^{(0)}(b,s)} \left\{ \bar{\Delta P} + P \frac{\Delta B}{4BB'} b^2 \right\} b e^{-\frac{1}{4B'} b^2}, \quad (\text{III.30})$$

where all symbols were defined in II.2) and II.3). Defining new parameters $P'(s)$ and $\bar{P}'(s)$ by

$$P'(s) = P - \Delta P(s), \quad \bar{P}'(s) = \bar{P} - \Delta \bar{P}(s), \quad (\text{III.31})$$

and applying $\exp[(\Delta B/4BB')b^2] = 1 + (\Delta B/4BB')b^2 + \dots$, these expressions can be written in the simple form

$$G^{(2)}(b,s) = \frac{1}{\sqrt{2}} \mathcal{E} \sqrt{1-G^{(0)}(b,s)} \left\{ P'(s) e^{-\frac{1}{4B'} b^2} - P e^{-\frac{1}{4B} b^2} \right\}, \quad (\text{III.32})$$

$$\bar{G}^{(2)}(b,s) = \frac{1}{\sqrt{2}} \bar{\mathcal{E}} \sqrt{1-G^{(0)}(b,s)} \left\{ \bar{P}'(s) e^{-\frac{1}{4B'} b^2} - \bar{P} e^{-\frac{1}{4B} b^2} \right\} b. \quad (\text{III.33})$$

The expressions (III.29), (III.30) and (III.32), (III.33) agree up to terms proportional to $[(\Delta B/4BB')b^2]^2$. This is a small quantity for the significant values of b for the energies considered. From the discussion in the next Section, which is concerned with the physical meaning of these expressions, we will conclude that (III.32), (III.33) should be considered as the appropriate expressions for $G^{(2)}$ and $\bar{G}^{(2)}$. As a consequence, the description of $\pi^- p \rightarrow \pi^0 n$ in terms of overlap functions of form (III.32), (III.33) which contains the reggeized ρ exchange mechanism equivalent to (III.29), (III.30) as a lowest order approximation yields in addition well defined corrections to it.

As is now well known ¹⁶⁾, pure reggeized ρ exchange describes $\pi^- p \rightarrow \pi^0 n$ only approximately since it yields vanishing polarization contrary to experiment ¹³⁾. In a subsequent publication ¹⁸⁾, it will be shown that a description of charge exchange scattering based on the forms (III.32) and (III.33) for the overlap functions $G^{(2)}$ and $\bar{G}^{(2)}$ accounts for all observed quantities, in particular for the polarization. According to the preceding remarks, this polarization is predicted to decrease logarithmically with q_L when q_L increases from 6 to 18 GeV/c. Also the polarizations in $\pi^\pm p \rightarrow \pi^\pm p$ can be calculated and are shown to agree with the observed values ¹⁹⁾. For these calculations, as well as for determining differential cross-sections, the real parts of the amplitudes have to be known; they can be calculated from the imaginary parts by the dispersion relations.

In the next Section the forms (III.32) and (III.33) of $G^{(2)}$ and $\bar{G}^{(2)}$ will be traced back to the presence of inelastic collisions of the $\pi^- p$ system which have no counterpart in those of the $\pi^+ p$ system. To this end we shall use the relations (III.20) and (III.21) determining $G^{(2)}$, $\bar{G}^{(2)}$ in terms of $G^{(i)}$, $\bar{G}^{(i)}$, $i=0,1$, and the definitions (III.8) of the latter quantities in terms of the inelastic components $|\psi_\ell^{(i)}\rangle$ of the outgoing waves from the $\pi^\pm p$ system.

IV. PHYSICAL INTERPRETATION OF THE PHENOMENOLOGICAL EXPRESSIONS OF THE OVERLAP FUNCTIONS

IV.1) The two classes of inelastic π^-p transitions and the splitting of $G^{(1)}$

We first concentrate on the non-spin-flip amplitudes since the arguments for spin-flip are largely analogous. Our aim is to trace back the phenomenological form of the overlap functions to a specific assumption on the high energy behaviour of the possible inelastic transitions $|\pi^+p\rangle \rightarrow |\psi^{(0)}\rangle$ and $|\pi^-p\rangle \rightarrow |\psi^{(1)}\rangle$, where the notation in (III.1) and (III.5) is used and non-essential indices are omitted. As will be explained later in more detail, the main feature assumed for this behaviour is the splitting of the π^-p inelastic collisions into a class of transitions which are a counterpart of π^+p transitions (type A) and transitions of essentially new character (type B). The type B transitions are regarded as the cause of the dominant deviations from the purely asymptotic scattering. This hypothesis means that in our approximation the quantities $G^{(0)}(b,s)$ and $\text{Im} h^{(0)}(b,s)$ for π^+p scattering are already asymptotic at the energies considered. Such a picture is supported by a fit by Lindenbaum⁵⁾ of recent total $\pi^\pm p$ cross-section data. The total cross-sections $\sigma^{(0)}$ for π^+p and $\sigma^{(1)}$ for π^-p are fitted by $\sigma^{(0,1)} = a + b^{(0,1)} / s^{n^{(0,1)}}$ with $n^{(0)} \approx 1$ and $n^{(1)} \approx 0.6$.

We accordingly assume that $G^{(0)}(b,s)$ can be written as

$$G^{(0)}(b,s) = G(b) + \sigma(s^{\delta-1}), \quad \delta \approx 0; \quad (\text{IV.1})$$

$\sigma(s^{\delta-1})$ is an unknown function of b of order $s^{\delta-1} \approx s^{-1}$ which in our approximation can be neglected. $G(b)$ is the asymptotic overlap function given by (II.16). From Eq. (III.27), it follows that in our approximation

$$\text{Im} h^{(0)}(b,s) = \frac{1}{2} \left(1 - \sqrt{1 - G(b)} \right).$$

Since it will be convenient in the following to visualize our considerations diagrammatically, we symbolize the inelastic transitions from the π^+p system,

$$|\pi^+p\rangle \longrightarrow |\psi^{(0)}\rangle, \quad (\text{IV.2})$$

by the graph in Fig. 3a; Fig. 3b visualizes the relationship between overlap function and $|\psi^{(0)}\rangle$ according to (III.8).

It should be noted that the asymptotic situation for $\text{Re} h^{(0)}$ is different from that for $\text{Im} h^{(0)}$. $\text{Re} h^{(0)}$ is determined through a dispersion relation from $\text{Im} h^{(0)}$ and $\text{Im} h^{(1)}$. As indicated before, $G^{(1)}$ and therefore $\text{Im} h^{(1)}$ will contain the leading non-asymptotic corrections which are $o(s^{\alpha_0-1}, \alpha_0 \approx 0.5)$; this induces in $\text{Re} h^{(0)}$ a term of order s^{α_0-1} as leading asymptotic contribution.

Having adopted the value (IV.1) for $G^{(0)}$, we apply relation (III.20) between $G^{(i)}(b,s)$, $i=0,1,2$, as well as (III.32) and (III.33), and we obtain the following phenomenological expression for $G^{(1)}(b,s)$

$$G^{(1)}(b,s) = G^{(1A)}(b,s) + G^{(1B)}(b,s), \quad (\text{IV.3})$$

$$G^{(1A)}(b,s) = G(b) - \alpha(s) \sqrt{1-G(b)} P'(s) e^{-\frac{b^2}{4B(s)}}, \quad (\text{IV.4})$$

$$G^{(1B)}(b,s) = \alpha(s) \sqrt{1-G(b)} G(b), \quad (\text{IV.5})$$

where terms of order s^{-1} or lower have been neglected [in this approximation the above formulae could also be written with $G^{(0)}(b,s)$ replacing $G(b)$, thereby taking into account an eventual term $o(s^{\delta-1}, 0 \leq \delta \leq \frac{1}{2})$ in $G^{(0)}(b,s)$]. The physical interpretation of this form of $G^{(1)}(b,s)$ is based on the splitting mentioned earlier for the inelastic transitions $|\pi^-p\rangle \rightarrow |\psi^{(1)}\rangle$ from the π^-p system into the classes A and B, which results for $|\psi^{(1)}\rangle$ in

$$|\psi^{(1)}\rangle = |\psi^{(1A)}\rangle + |\psi^{(1B)}\rangle, \quad (\text{IV.6})$$

where $|\psi^{(1A)}\rangle$ and $|\psi^{(1B)}\rangle$ are supposed orthogonal to each other. Introducing this into the definition (III.8) of $G^{(1)}$ in terms of $|\psi^{(1)}\rangle$ reproduces the splitting (IV.3) of $G^{(1)}$ with

$$G^{(1A,B)} = \langle \psi^{(1A,B)} | \delta^{(4)}(P-p) | \psi^{(1A,B)} \rangle. \quad (\text{IV.7})$$

Before discussing the detailed algebraic form of the expressions (IV.3)-(IV.5), we describe more explicitly the physical justification for our assumed splitting of the inelastic transitions $|\pi^- p\rangle \rightarrow |\psi^{(1)}\rangle$ into classes A and B. For both $\pi^\pm p$ inelastic transitions, the following phenomenological properties seem to emerge at high energies. The final states of appreciable cross-section show jet structure, i.e., the outgoing particles can be approximately subdivided into two groups or "jets" going forward and backward in the centre-of-mass system. Each group has predominantly the same internal quantum numbers as the corresponding incoming particle¹⁴⁾. This suggests an analysis of many-particle transitions, analogous to two-body transitions, in terms of exchanged quantum numbers, whereby exchange of the quantum numbers of the vacuum gives the dominant contribution to the transition amplitude at high energies. The same consideration of exchanged quantum numbers applies also to the case where the final state consists of more than two groups of particles, as is the case in the multiperipheral model with exchange of Regge poles.

These considerations allow to define for both $\pi^\pm p$ inelastic transitions a structure $|\psi^{(0,1)}\rangle = |\psi^{(0,1A)}\rangle + |\psi^{(0,1B)}\rangle$, where A corresponds to the exchange of the vacuum quantum numbers and B to those different from the vacuum. This induces a splitting of both

$G^{(0,1)}$ into two terms $G^{(0,1A)}$ and $G^{(0,1B)}$. From the preceding discussion it follows that the leading term for $s \rightarrow \infty$ in $G^{(0,1A)}$ is $G(b)$, the asymptotic Gaussian. The dominating non-asymptotic contributions are expected to be caused by the presence of $G^{(0,1B)}$. However, as will be seen immediately, it is reasonable to expect $G^{(0B)}$ to be smaller than $G^{(1B)}$ and to neglect it in our approximation.

We show this for the case of two groups or jets of particles in the final state. Let $\pi^\pm$ and p' be the outgoing jets corresponding to the incoming $\pi^\pm$ and p , respectively. Predominantly, and especially in class A, we have $(\pi^\pm)^\pm$ and $(p')^\pm$, where upper (lower) signs go together and where the index outside the bracket indicates the total charge of the jet. Neglecting exchange of hypercharge 2, 4, ..., it is easily seen that beside the dominant configuration $(\pi^+)^\pm, (p')^\pm$ there are no inelastic $\pi^+ p$ transitions possible where π^+ has isospin $I=1$ and p' has $I=\frac{1}{2}$. On the other hand, among the inelastic $\pi^- p$ transitions, we have the possibilities $(\pi^-)^\pm$ and $(p')^\pm$ with $I=1$ and $I=\frac{1}{2}$, respectively. It is reasonable to expect that those inelastic transitions which lead to minimal deviations from the dominant configurations determine the dominant corrections to the genuine asymptotic situation. In our case these are the particular final states $(\pi^-)^\pm (p')^\pm$ contained in $|\psi^{(1B)}\rangle$; we shall call them the charge annihilation channels. All other channels contributing to $|\psi^{(0,1B)}\rangle$ involve $\pi^\pm$ with isospin $I \geq 2$ or non-vanishing strangeness S or non-vanishing baryon number B , and p' with $I \geq \frac{3}{2}$ or $S \neq 0$ or $B \neq 1$; our basic assumption is that their contributions are mainly controlled by powers of s lower than the one involved in the charge annihilation channels of $|\psi^{(1B)}\rangle$. It should be noted that these considerations are to a certain extent analogous to Kokkedee's investigation ³⁾ of $\bar{p}p$ scattering, where the presence of the $\bar{p}p$ annihilation channels causes the main contributions to the non-asymptotic difference between $\bar{p}p$ and pp scattering. Our charge annihilation channels are in a certain sense the analogues of the $\bar{p}p$ annihilation channels in Kokkedee's work, although it must be realized that the definition of the former is model dependent, whereas the latter are of course not.

IV.2) The algebraic form of $G^{(1A)}$ and $G^{(1B)}$

Now we proceed to discuss the detailed form of the contributions $G^{(1A)}$ and $G^{(1B)}$ to $G^{(1)}$ given in Eqs. (IV.3)-(IV.5).

The type A transitions $|\pi^- p\rangle \rightarrow |\psi^{(1A)}\rangle$ must proceed asymptotically in the same way as $|\pi^+ p\rangle \rightarrow |\psi^{(0)}\rangle$ ($\approx |\psi^{(0A)}\rangle$ in our approximation) in order to produce the necessary equality

$$\lim_{s \rightarrow \infty} G^{(1)}(b, s) = \lim_{s \rightarrow \infty} G^{(1A)}(b, s) = \lim_{s \rightarrow \infty} G^{(0A)}(b, s) = G(b).$$

Clearly, (IV.4) has this property. At non-asymptotic energies, $s \neq \infty$, the presence of the type B transitions $|\pi^- p\rangle \rightarrow |\psi^{(1B)}\rangle$ has to be taken into account and we have assumed it to dominate the deviations from the genuine asymptotic situation. This presence manifests itself at two places in the state vector $|\psi^{(1)}\rangle$. Firstly, of course, $|\psi^{(1B)}\rangle \neq 0$, for $s \neq \infty$; secondly, there will be a reaction felt in $|\psi^{(1A)}\rangle$ due to the presence of $|\psi^{(1B)}\rangle$ for $s \neq \infty$. For the overlap function $G^{(1)} = G^{(1A)} + G^{(1B)}$ this implies $G^{(1B)} \neq 0$ for $s \neq \infty$ and a corresponding deviation of $G^{(1A)}$ from its asymptotic value $G(b)$ which is of the same order as $G^{(1B)}$. The expressions (IV.4) and (IV.5) for $G^{(1A, B)}$ have the property that $G^{(1B)}$ and the reaction effect in $G^{(1A)}$ vanish with the typical power law $s^{-\alpha_0-1}$, $0 < \alpha_0 \approx 0.5 < 1$.

The phenomenological expression (IV.5) for $G^{(1B)}(b, s)$ suggests the formulation of a general phenomenological principle for the description of new kinds of transitions, in our context type B, determining dominant deviations from the genuine asymptotic transitions :

at high energies the impact parameter distribution of every new type of transition is the same as for the asymptotic transition $|\pi^+ p\rangle \rightarrow |\psi^{(0)}\rangle$ giving rise to a Gaussian overlap function, except for the occurrence of absorption corrections in the sense of Jackson¹⁷⁾.

This principle is graphically illustrated in Fig. 4a: Fig. 4b symbolizes the corresponding overlap function $G^{(1B)}(b,s)$. In the phenomenological expression (IV.5) for $G^{(1B)}(b,s)$ the absorption effect is contained in the factor $\mathcal{R}(s)\sqrt{1-G(b)}$. Since $\mathcal{R}$ is independent of b , the spatial distribution of absorption is the same as in the usual distorted wave Born approximation described for instance in Ref. 17). This follows from $\sqrt{1-G(b)} = e^{2i\delta}$, where δ is the $\pi^+p \rightarrow \pi^+p$ phase shift. The factor $G(b)$ in the expression (IV.5) for $G^{(1B)}$ is symbolized by the central part of diagram 4b, i.e., it originates from the residual part of the type B transitions which, after the absorption effect is taken out, proceeds in the same way as the standard transition $|\pi^+p\rangle \rightarrow |\psi^{(0)}\rangle$. This corresponds to a well-determined volume of interaction between the particles at high energies, the same for the type B transitions as for the asymptotic transitions of type A; this idea is closely related to the basic concept of coherent droplet model as proposed by Byers and Yang 18). The volume of interaction is "measured" by the genuine asymptotic diffraction pattern and is quantitatively described by $G(b)$, the "radius R " of this volume being given by

$$R = 2\sqrt{B}$$

As to $\mathcal{R}(s)$, it expresses the strength of the B transitions and has the power dependence s^{α_0-1} characteristic of the leading non-asymptotic effects.

Next, we consider the reaction term $\mathcal{R}(s)\sqrt{1-G(b)}P'(s)\exp.(-b^2/4B'(s))$ of order s^{α_0-1} in $G^{(1A)}(b,s)$, which has to be understood as the dominant non-asymptotic correction to the main term $G(b)$ in $G^{(1A)}$. Except for the strength factor $\mathcal{R}(s)$ and the absorption factor $\sqrt{1-G(b)}$ also occurring in $G^{(1B)}$, the reaction term consists in a Gaussian differing from the fundamental Gaussian $G(b)$ by the small modifications $P \rightarrow P'(s)$ and $B \rightarrow B'(s)$. That these modifications are small is natural to expect because we saw that after separating off the absorption effect, the B transitions have the same spatial distribution as the asymptotic ones.

It is interesting to realize that in our interpretation the small modification $B \rightarrow B'(s)$ is the origin of the slope $B'(s)$ of the shrinking charge exchange peak usually described by reggeized φ exchange. We also arrive at a natural explanation for the observed similarity of the slopes B, B' in $[[p]$ elastic and charge exchange scattering, again in analogy with the coherent droplet model¹⁸⁾. Furthermore, in our phenomenological theory, the sign of $\Delta B = B - B'(s)$ is determined by unitarity. Because the phenomenological expression (IV.5) for $G^{(1B)}$ is traced back to an expression of type (IV.7), it has to be positive, i.e., $\mathcal{R} > 0$. Combining this information with the phenomenological expression (II.21) for $\mathcal{R}$ leads to $\Delta B > 0$ (since $\mathcal{R}_1 > 0$), i.e., $B' < B$ as it is observed. On the other hand, (II.21) shows that if the actually observed energy dependences of B and B' , i.e., constancy of the elastic peak and shrinking of the charge exchange peak, would continue to hold for higher energies, this would lead to $B' \rightarrow B$ and $\mathcal{R} \rightarrow \infty$ for some value of the energy. But (IV.7), together with the bounds (III.22) leads immediately to $0 \leq G^{(1A,B)} \leq 1$, which is seen to be fulfilled by the phenomenological expressions (IV.4) and (IV.5) in the energy region investigated here. This in turn leads to a restriction for the values of $\mathcal{R}(s)$ which is roughly

$$0 \leq \mathcal{R}(s) \lesssim +1. \quad (\text{IV.8})$$

Thus the phenomenological expression (II.21) gets into conflict with these bounds. In our framework, the simplest way out of this dilemma is that the energy dependences of the peak slopes observed so far will change for higher energies in such a way that $B - B'$ remains positive and does not become too small as required by inequality (IV.8). This means that either the charge exchange peak stops shrinking or alternatively, but less plausibly, the elastic peaks have to start shrinking at some energy above 18 GeV/c. This prediction presents a clear-cut experimental test of the ideas developed in the present work. Taking (IV.8) as allowed values for $\mathcal{R}(s)$, using (II.21) and making the assumption that the observed energy dependences for B and B' can be extrapolated to higher energies, one can give a rough estimate of the laboratory energy ω_L at which the proposed change should occur. One finds $\omega_L \sim 26$ GeV.

IV.3) Spin-flip amplitudes

We end with a discussion of spin-flip effects along the same lines as followed for the spin-non-flip terms. For the spin-flip overlap functions $\bar{G}^{(0,1)}(b,s)$ we write analogously to (IV.1)-(IV.5) :

$$\bar{G}^{(0)}(b,s) = \bar{\mathcal{E}}' b G(b) + \bar{\sigma} (s^{\delta-1}), \quad \delta \approx 0, \quad (\text{IV.9})$$

$$\bar{G}^{(1)}(b,s) = \bar{G}^{(1A)}(b,s) + \bar{G}^{(1B)}(b,s), \quad (\text{IV.10})$$

$$\bar{G}^{(1A)}(b,s) = \bar{\mathcal{E}}' b G(b) - \bar{\mathcal{E}}'' \sqrt{1-G(b)} \bar{\mathcal{E}}' \bar{P}'(s) b e^{-\frac{b^2}{4B'(s)}}, \quad (\text{IV.11})$$

$$\bar{G}^{(1B)}(b,s) = \bar{\mathcal{E}}'' \sqrt{1-G(b)} \bar{\mathcal{E}}' b G(b), \quad (\text{IV.12})$$

where we have introduced $\bar{\mathcal{E}}(s) \equiv \bar{\mathcal{E}}' \bar{\mathcal{E}}''$ for the quantity $\bar{\mathcal{E}}(s)$ given in (II.18) and (II.23). $\bar{\mathcal{E}}' b G(b)$ is the asymptotic overlap function for elastic spin-flip, which is related to the asymptotic elastic spin-flip amplitude $\bar{h}$ by an equation of type (III.27). $\bar{\mathcal{E}}'$ is a parameter determining the strength of elastic spin-flip at asymptotic energies. $\bar{\mathcal{E}}''$ plays the same role as $\mathcal{E}$ in the non-spin-flip case, i.e., it determines the over-all strength of the type B contribution $\bar{G}^{(1B)}$ for $\pi^- p \rightarrow \pi^- p$. Therefore it is natural to expect that it shows the same general characteristics as $\mathcal{E}(s)$, i.e., independence of b and same type of energy dependence. This in turn would imply for $\bar{\mathcal{E}}'$ independence of b and s as is seen from $\bar{\mathcal{E}} = \bar{\mathcal{E}}' \bar{\mathcal{E}}''$ and the phenomenological expressions (II.21) and (II.23) for $\mathcal{E}$ and $\bar{\mathcal{E}}$. Present experimental information is compatible with $\bar{\mathcal{E}}'$ small; more refined polarization measurements will be needed for obtaining more precise information on $\bar{G}^{(0,1)}(b,s)$.

We have included an additional factor b as only essential modification in the spin-flip overlap functions $\bar{G}^{(i)}$. It may be motivated by a potential model of the high energy interaction; it has been shown in Ref. 20) that at high energies it is always possible to find a potential reproducing the relativistic scattering process. If it is assumed that the total interaction potential V_{tot} is given by

$$V_{\text{tot}}(|\vec{x}|) = V(|\vec{x}|) + (\vec{\ell} \cdot \vec{\sigma}) \bar{V}(|\vec{x}|),$$

where V and $\bar{V}$ depend only on the relative distance $|\vec{x}|$ of the colliding particles and $\vec{\ell}$ is the relative orbital angular momentum operator, at high energies and sufficiently small momentum transfers the impact parameter weight functions $h(b,s)$ and $\bar{h}(b,s)$ are given by 22)

$$h(b,s) = \frac{1}{2i} \left(e^{i\chi(b)} \cos[bq\bar{\chi}(b)] - 1 \right),$$

$$\bar{h}(b,s) = \frac{1}{2} e^{i\chi(b)} \sin[bq\bar{\chi}(b)],$$

$$\chi(b) = -\frac{2m}{q} \int_0^\infty dz V(\sqrt{z^2 + b^2}),$$

$$\bar{\chi}(b) = -\frac{2m}{q} \int_0^\infty dz \bar{V}(\sqrt{z^2 + b^2});$$

m is the reduced mass of the particles. For a small and sufficiently regular spin-flip potential $\bar{V}$, this shows that $\bar{h}(b,s)$ becomes proportional to b , at small b , while the cosine in the first equation can be replaced by one. From the connection of type (III.27) between $\text{Im}\bar{h}(b,s)$ and $\bar{G}(b,s)$ it follows that this factor b has to come from $\bar{G}(b,s)$. The forms (IV.9)-(IV.12) of the $\bar{G}^{(i)}(B,s)$ represent in a certain sense the minimal modifications of the non-spin-flip overlap functions $G^{(i)}(b,s)$, taking into account spin dependent interactions. In this context we note that the situation is less satisfactory in the coherent droplet model analysis of $\pi^- p \rightarrow \pi^0 n$ scattering 18), where, for a range of $-t \lesssim 0.4 (\text{GeV}/c)^2$, quantitative agreement between theory and experiment can be reached only by having an additional factor b^2 rather than b in the spin-flip amplitude.

V. CONCLUSIONS AND OUTLOOK

The preceding Sections have shown that the overlap function analysis reveals several interesting aspects of high energy collisions in a considerable range of s and t . From the formal point of view, we recall the reduction of the general unitarity equations (III.13)-(III.19) to simple equations determining at high energy the imaginary parts of the amplitudes explicitly in terms of the overlap functions. This, taken together with the determination of the real parts via crossing and analyticity properties, represents a special case of the solution of the S matrix equations if the overlap functions are assumed to be known; a problem which has been shown by Martin ²³⁾ to have a unique answer in a more general context. From the phenomenological side we obtained simple data reproducing forms for the overlap functions, expressed in terms of Gaussians together with absorption corrections and a minimal modification for spin-flip. They hold in a range of s and t where the scattering amplitudes themselves show complicated variations. Finally, an interesting physical interpretation of these results has been possible by analyzing the overlap functions in terms of inelastic transitions; this has led to the concept of two classes of inelastic transitions determining the genuine asymptotic diffraction pattern, as well as the dominant non-asymptotic deviations. Our phenomenological description incorporates all observed particularities, including polarization, of pion-nucleon elastic and charge-exchange scattering in the considered ranges of s and t . It predicts a clear-cut change in the qualitative behaviour of the elastic and charge-exchange peaks for higher energies, the latter having to remain wider than the former even above the energy where extrapolation of present trends predicts the converse situation to set in. The generality of the theoretical concepts used in the present approach leads to the conjecture that they will be applicable to the analysis of more complicated reactions: the next case in complexity would be Kp scattering.

In the physical interpretation of the phenomenological expressions found for the overlap functions we have encountered the concepts of absorption corrections and of a well-defined interaction volume for various types of transitions. The former have been introduced by

Sopkovich²⁴⁾ and have been applied with limited success to peripheral interaction mechanisms of meson exchange type^{17),25)}. We encounter them here in a very different context, completely unrelated to single particle exchange models. The interaction volume concept occurred in the context of the coherent droplet model¹⁸⁾ which compared with the present considerations has similar aims but is of more restricted scope. The present investigation shows that both concepts represent essential aspects of high energy interactions within the framework of the overlap function method.

Our overlap function analysis incorporates the observed Regge asymptotic behaviour of $\pi^-p \rightarrow \pi^0n$ and defines in principle a dynamical scheme to determine the parameters of the Regge pole model from bulk properties of inelastic transitions. It should be noted, however, that it is not equivalent to a pure Regge pole model, since additional singularities (cuts) are necessary in the crossed channel complex angular momentum plane in order to reproduce the overlap function expressions exactly. From the practical point of view, overlap function analysis and Regge theory seem at present to play rather complementary roles. Regge theory makes definite statements on the energy dependence of amplitudes by associating it to particles and their Regge recurrences in the crossed channel, but it is rather vague on the t dependence (residue functions). Just the opposite has been seen to be true for overlap function considerations. Here the t dependences have been traced back to an appropriate jet structure of different classes of inelastic transitions, while each class contributes with a typical power of s to the overlap functions: s^{α_P-1} for the leading terms from the type A transitions, where α_P is the Pomeranchuk trajectory; $s^{\alpha_\rho-1}$ for the leading terms from the type B transitions in $\pi^-p \rightarrow \pi^-p$ and the corresponding reaction corrections in the type A transitions. The vagueness of the overlap function approach lies in its inability to predict the values of α_P and α_ρ . It would be a very interesting development to correlate also the values of α_P , α_ρ , i.e., energy dependences, with bulk properties of high energy inelastic transitions. This might be attempted by describing the latter in terms of exchange processes as briefly discussed in Section IV.1. It is premature, however, to speculate on the results which could be obtained along such lines.

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q_L GeV/c	B' $(\text{GeV}/c)^{-2}$	$\Delta B = B - B'$ $B = 4.86$ $(\text{GeV}/c)^{-2}$	$\mathcal{H}$	ΔP	$\overline{\mathcal{H}}$ GeV/c	$\overline{\Delta P}$
6	3.25	1.61	0.56	$-3.9 \cdot 10^{-2}$	-0.18	$4.2 \cdot 10^{-2}$
18.2	4.25	0.61	0.50	$+1.5 \cdot 10^{-2}$	-0.12	$9 \cdot 10^{-2}$

The phenomenological values of the parameters in Eqs. (II.17) and (II.18) for $q_L = 6 \text{ GeV}/c$ and $18.2 \text{ GeV}/c$.

A P P E N D I X A

In this Appendix we indicate briefly the derivation of the expressions for $f^{(2)}$ and $\bar{f}^{(2)}$ resulting from reggeized φ exchange. We introduce the Lorentz invariant amplitudes ²⁶⁾ by writing the transition matrix T as (for the moment we omit the index 2)

$$T = -A + i \frac{1}{2} \left[\gamma^{\mu} (q_1 + q_2) \right] B ;$$

q_1 and q_2 are the four-momenta of the incoming and of the outgoing pion. The matrix F given by (II.1) and T are related by $\bar{\chi}_2 F \chi_1 = -M/4\pi \bar{u} T u$, where χ_1 and χ_2 are Pauli, u_1 and u_2 Dirac spinors corresponding to the incoming and outgoing nucleon, respectively. In addition we use the amplitude A' introduced by Singh ²¹⁾

$$A' = A + \frac{\omega_L + t/4M}{1 - t/4M^2} B \xrightarrow{s \rightarrow \infty} A + \frac{s/2M}{1 - t/4M^2} B ,$$

where $\omega_L = (2M)^{-1}(s - M^2 - \mu^2)$ is the laboratory energy of the incident pion (μ = pion mass). The right-hand side gives the leading term for $s \rightarrow \infty$ and small t .

$-(M/4\pi \sqrt{t})(p_t/M)A'$, $p_t = t$ channel centre-of-mass momentum of the nucleon-antinucleon pair, and $(M/4\pi \sqrt{t})(q_t \sqrt{t}/2M)\sin\theta_t B$, $q_t = t$ channel centre-of-mass momentum of the pion pair, θ_t = scattering angle in the t channel, are the t channel helicity non-flip and flip amplitudes ²⁷⁾. As a consequence for A' and B the Sommerfeld-Watson representation in the t channel can be written and Regge poles can be introduced ²¹⁾.

The connection between f , $\bar{f}$ and A, B reads

$$f = \frac{1}{8\pi} \left\{ ([1 - \cos \theta] E + [1 + \cos \theta] M) A + \right. \\ \left. ([1 - \cos \theta][W - E] M + [1 + \cos \theta][EW - M^2]) B \right\},$$

$$\xrightarrow{s \rightarrow \infty} \frac{2M}{8\pi} \left\{ A + \frac{s}{2M} B \right\} = \frac{2M}{8\pi} \left\{ A' - \frac{t/4M^2}{1 - t/4M^2} \frac{s}{2M} B \right\},$$

$$\bar{f} = -\sin \theta \frac{E - M}{8\pi} \left\{ -A + (W - M) B \right\},$$

$$\xrightarrow{s \rightarrow \infty} \frac{2M}{8\pi} \sqrt{\frac{-t}{4M^2}} A = \frac{2M}{8\pi} \sqrt{\frac{-t}{4M^2}} \left\{ A' - \frac{1}{1 - t/4M^2} \frac{s}{2M} B \right\},$$

where $E = \sqrt{M^2 + q^2}$ and $W = \sqrt{s}$. For the high energy limit $s \rightarrow \infty$ we have only kept orders $o(A)$ and $o(\frac{s}{2M}B)$: the highest neglected orders are $o(s^{-\frac{1}{2}}A)$ and $o(s^{-\frac{1}{2}}\frac{s}{2M}B)$. As discussed in II.1), we treat charge exchange only to its leading order so that this accuracy is sufficient. Introducing into these formulae the expressions for A' and B resulting from reggeized ρ exchange²¹⁾, one obtains the expressions given in II.3) for $f^{(2)}$ and $\bar{f}^{(2)}$.

A P P E N D I X B

For the derivation of (II.17) and (II.18) we use

$$A(1+Ct)e^{Bt} = 2q\sqrt{s} \int_0^{\infty} db b J_0(b\sqrt{-t}).$$

$$\cdot \frac{1}{M^2} \left(\frac{2q_L}{M} \right)^{-1} \frac{A}{2B} \left\{ 1 - \frac{C}{B} + \frac{C}{4B^2} b^2 \right\} e^{-\frac{b^2}{4B}},$$

$$A(1+C) \sqrt{\frac{-t}{4M^2}} e^{Bt} = 2q\sqrt{s} \int_0^{\infty} db b J_1(b\sqrt{-t}).$$

$$\cdot \frac{1}{M^2} \left(\frac{2q_L}{M} \right)^{-1} \frac{A}{8MB^2} \left\{ 1 - \frac{2C}{B} + \frac{C}{4B^2} b^2 \right\} b e^{-\frac{b^2}{4B}},$$

where $A, B (>0)$ and C are independent of t and $2q\sqrt{s} = 2Mq_L$ has been applied. These formulae are easily derived on the basis of results given in Ref. 28).

A P P E N D I X C

While the bounds for the non-spin-flip overlap functions, (III.22) and (III.23), follow immediately from (III.12), (III.14) and (III.20), those for the spin-flip overlap functions are easily derived by introducing overlap functions corresponding to transitions with definite total angular momentum and parity

$$g_{e^{\pm}}^{(0,1)} = \frac{1}{4} \langle \psi_{e^{\pm}}^{(0,1)} | \delta^{(4)}(P-P) | \psi_{e^{\pm}}^{(0,1)} \rangle,$$

where $|\psi_{e^{\pm}}^{(0,1)}\rangle$ are defined analogously to (III.5) and (III.6). These are related to $g_{\ell}^{(0,1)}$ and $\bar{g}_{\ell}^{(0,1)}$ by

$$g_{\ell}^{(0,1)} = \frac{1}{2} (g_{e^{+}}^{(0,1)} + g_{e^{-}}^{(0,1)}) , \quad \bar{g}_{\ell}^{(0,1)} = \frac{1}{2} (g_{e^{+}}^{(0,1)} - g_{e^{-}}^{(0,1)}) .$$

These overlap functions enter into the unitarity relations for $f_{\ell}^{(0,1)}$,

$$\text{Im } f_{\ell}^{(0)} = \frac{1}{4} g_{\ell}^{(0)} + |f_{\ell}^{(0)}|^2 ,$$

$$\text{Im } f_{\ell}^{(1)} = \frac{1}{4} g_{\ell}^{(1)} + |f_{\ell}^{(2)}|^2 + |f_{\ell}^{(1)}|^2 .$$

From this follows $0 \leq g_{\ell}^{(0,1)} \leq 1$. This, together with the relations (III.20) and (III.21), implies the bounds (III.22) and (III.23).

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FIGURE CAPTIONS

- Figure 1 : The function $\varphi(t)$ according to the values of $\alpha(t)$ from Ref. ¹⁶⁾.
- Figure 2 : The functions $f(t)$ and $\bar{f}(t)$ according to the values of $\alpha(t)$, $\beta_+(t)$ and $\beta_-(t)$ from Ref. ¹⁶⁾.
- Figure 3a : Diagram symbolizing the transition $|\pi^+p\rangle \rightarrow |\psi^{(0)}\rangle$ leading to a Gaussian overlap function $G^{(0)}(b,s)$ at high energies.
- Figure 3b : Diagram symbolizing the overlap function $G^{(0)}(b,s)$.
- Figure 4a : Diagram symbolizing the mechanism for the type B transition $|\pi^-p\rangle \rightarrow |\psi^{(1B)}\rangle$ from the initial π^-p state at high energies. First there is absorption due to the presence of the type A transitions $|\pi^-p\rangle \rightarrow |\psi^{(1A)}\rangle$, then the final states of class B are produced with the same impact parameter distribution as in $|\pi^+p\rangle \rightarrow |\psi^{(0)}\rangle$.
- Figure 4b : Diagram symbolizing the overlap function $G^{(1B)}(b,s)$ as determined by the mechanism indicated in Fig. 4a.

FIG.1

FIG.2

FIG. 3a

$$G^{(0)} = \langle \psi^{(0)} | \delta^{(4)}(P-p) | \psi^{(0)} \rangle$$

FIG. 3b

FIG. 4 a

$$G^{(1B)} = \langle \psi^{(1B)} | \delta^{(4)}(P-p) | \psi^{(1B)} \rangle$$

FIG. 4 b