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POLARIZATIONS IN THE OVERLAP FUNCTION ANALYSIS OF $\pi^{\pm}p$
ELASTIC AND CHARGE EXCHANGE SCATTERING AT HIGH ENERGIES

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A B S T R A C T

This paper presents the results on polarizations in $\pi^{\pm}p + \pi^{\pm}p$ and $\pi^{-}p \rightarrow \pi^{0}n$ high energy scattering which follow from our recent overlap function analysis combined with dispersion relations. The calculated $\pi^{-}p \rightarrow \pi^{0}n$ polarization is in agreement with present experimental results at 6 and 11 GeV/c. $\pi^{\pm}p \rightarrow \pi^{\pm}p$ polarizations at 6, 8, 10 and 12 GeV/c are fitted by adjusting an unknown, energy independent function of t . For higher energies (up to about 25 GeV/c) the $\pi^{-}p \rightarrow \pi^{0}n$ polarization is expected not to change qualitatively, while for $\pi^{\pm}p \rightarrow \pi^{\pm}p$ an increasingly asymmetric behaviour is found. Predictions for the Wolfenstein parameters A and R at present and higher energies are given. In the present scheme the $\pi^{-}p \rightarrow \pi^{0}n$ polarization results from exchange of a φ meson Regge pole $\alpha = \alpha_{\varphi}(t)$ supplemented by well-determined corrections of Regge cut type. The part of the amplitude even under crossing is found to be composed of a fixed Pommeranchuk pole term $\alpha = 1$, a P' pole term with cut corrections which is determined by the φ term and has the same trajectory, and a Pommeranchuk daughter term with $\alpha = 0$ operating as ghost compensator for the P' pole.

I. INTRODUCTION

This paper presents some consequences of our recent analysis of high energy $\pi^{\pm}p$ elastic and charge exchange scattering in the diffraction region in terms of overlap functions ¹⁾ [henceforth this reference will be denoted by I]. Our main concern will be the discussion of the available polarization measurements at 6 and 12 GeV/c for $\pi^{-}p \rightarrow \pi^{0}n$ ²⁾ and from 6 to 12 GeV/c for $\pi^{\pm}p \rightarrow \pi^{\pm}p$ ³⁾ within the framework proposed in I. As was already stressed there, these polarizations can be described within the scheme of I, and it will be shown how the values calculated from this model reproduce the experimental findings. In view of the forthcoming measurements of the Wolfenstein parameters in $\pi^{\pm}p \rightarrow \pi^{\pm}p$ ⁴⁾, we also list briefly the consequences for these quantities.

In I we mainly concentrated on finding data reproducing forms of the overlap functions and on giving them a physical interpretation. Now we proceed to the determination of the various scattering amplitudes from the overlap functions, a programme already outlined in I. The overlap functions fix at each energy the imaginary parts of the various amplitudes via unitarity. The imaginary parts in turn determine the real parts of the amplitudes via dispersion relations at fixed $t = (\text{centre-of-mass momentum transfer})^2$. Finally, polarizations and Wolfenstein parameters are found from the scattering amplitudes by standard formulae.

Before proceeding to the detailed calculations, we briefly recall some points of the overlap function description given in I and mention what is the source of polarization within this framework. The overlap functions $G^{(2)}$ and $\bar{G}^{(2)}$ for the spin-non-flip and spin-flip $\pi^{-}p \rightarrow \pi^{0}n$ scattering, respectively, are found in the form

$$G^{(2)}(b,s) = G_{\rho}^{(2)}(b,s) + G^{(2)'}(b,s), \quad (\text{I.1})$$

$$\bar{G}^{(2)}(b,s) = \bar{G}_{\rho}^{(2)}(b,s) + \bar{G}^{(2)'}(b,s), \quad (\text{I.2})$$

where $s = (\text{centre-of-mass energy})^2$ and b is the impact parameter. Their largest terms $G_{\rho}^{(2)}$, $\bar{G}_{\rho}^{(2)}$ correspond exactly to reggeized ρ exchange^{5),6)}, while the smaller additional terms $G^{(2)'}_i$, $\bar{G}^{(2)'}_i$ are such as to give to the total functions $G^{(2)}$, $\bar{G}^{(2)}$ a simple form which is amenable to a reasonable physical interpretation in terms of inelastic collisions. $G^{(2)}(b,s)$ and $\bar{G}^{(2)}(b,s)$ contain, beside the parameters characterizing the genuine asymptotic diffraction pattern of $\pi^{\pm}p \rightarrow \pi^{\pm}p$, only the parameters corresponding to reggeized ρ exchange, i.e., trajectory and residues of the ρ Regge pole.

Thus, the overlap function description given in I presents a well-determined way of calculating specific corrections to reggeized ρ exchange in πN scattering. In particular, the polarization in $\pi^-p \rightarrow \pi^0n$ originates from these correction terms since it vanishes for pure reggeized ρ exchange because of the phase equality of the corresponding spin-non-flip and spin-flip amplitudes. The calculation shows that, in the scattering amplitudes $f^{(2)}$ and $\bar{f}^{(2)}$ for $\pi^-p \rightarrow \pi^0n$, which, from (I.1) and (I.2), take a form

$$f^{(2)}(s,t) = f_{\rho}^{(2)}(s,t) + f^{(2)'}(s,t), \quad (\text{I.3})$$

$$\bar{f}^{(2)}(s,t) = \bar{f}_{\rho}^{(2)}(s,t) + \bar{f}^{(2)'}(s,t), \quad (\text{I.4})$$

the terms $f^{(2)'}_i$ and $\bar{f}^{(2)'}_i$, originating from $G^{(2)'}_i$ and $\bar{G}^{(2)'}_i$, explain the observed polarization.

The polarizations in $\pi^{\pm}p \rightarrow \pi^{\pm}p$ originate from the interference of the asymptotic with the leading non-asymptotic terms in the corresponding amplitudes. In the present framework, the latter are of the order of the charge exchange amplitudes and partially determined by $G^{(2)}$ and $\bar{G}^{(2)}$. All quantities measured up to now at high energies in $\pi^{\pm}p \rightarrow \pi^{\pm}p$ show negligible contributions from the asymptotic elastic spin-flip amplitudes.

The Wolfenstein parameters on the other hand will be found to be more sensitive to this amplitude which, in the present framework, is characterized by the single parameter $\bar{\mathcal{R}}'/M$ (we divide by M to get a dimensionless quantity) introduced in I; $\bar{\mathcal{R}}'/M$ was argued to be independent of s and t . Thus we are led to predictions for the Wolfenstein parameters. They depend on a single unknown constant, to be determined from the data.

To compare briefly the present results with other calculations of polarization parameters in high energy πp elastic and charge exchange scattering, we refer first of all to the survey given at the 1967 Heidelberg conference ⁷⁾. We limit ourselves to the remark that for charge exchange the amplitudes (I.3) and (I.4) may be thought of as originating from a pole and an effective branch point singularity in the Regge plane. Our specific overlap function model fixes the cut parameters entirely in terms of the pole parameters. Neglecting the cut contributions in the discussion of elastic scattering, we arrive at a situation similar to usual Regge analysis. However, the present theory differing substantially from the latter in the description of the genuine asymptotic part of elastic scattering (i.e., Pomeranchuk exchange), we are led to different behaviours for certain observable quantities, in particular for the Wolfenstein parameters which are highly sensitive to such dissimilarities.

As a further particularity of the theory, we shall encounter a P' Regge pole, whose parameters are determined by the parameters of the ρ pole and where a new type of ghost killing mechanism operates. We also recall the expected modification in the energy behaviour of either the $\pi^- p \rightarrow \pi^0 n$ or the $\pi^\pm p \rightarrow \pi^\pm p$ forward peaks (slowdown of shrinkage or setting in of shrinkage, respectively) at higher energies, which has been found in I from our specific forms of the overlap functions combined with general bounds from unitarity.

In Section II, we recall some results from I, and collect the input information for the subsequent calculations. Section III is concerned with charge exchange scattering and gives the determination of the imaginary parts of spin-non-flip and spin-flip amplitudes in terms of overlap functions from unitarity. This is followed by the calculation of the real parts from dispersion relations. Then polarization for $\pi^- p \rightarrow \pi^0 n$ will be discussed. Section IV gives the analogous programme for $\pi^\pm p \rightarrow \pi^\pm p$. An additional paragraph deals with the Wolfenstein parameters in elastic scattering. Finally, Section V presents some concluding remarks.

II. SUMMARY OF RESULTS FROM I

In this Section we summarize some formulae, statements and results from I, as far as they are needed for the aims of the present study.

The three reactions, $\pi^+p \rightarrow \pi^+p$, $\pi^-p \rightarrow \pi^-p$ and $\pi^-p \rightarrow \pi^0n$ are characterized by the indices 0, 1, 2 respectively. We work with the s channel spin-non-flip and spin-flip amplitudes $f^{(i)}$ and $\bar{f}^{(i)}$, $i = 0, 1, 2$, which are related to the scattering matrix F between Pauli spinors describing the spin state of the nucleons in their respective rest systems by

$$F^{(i)} = f^{(i)}(s, t) + i(\vec{\sigma} \cdot \vec{n}) \bar{f}^{(i)}(s, t), \quad (\text{II.1})$$

$\vec{n} = (\vec{q}_1 \times \vec{q}_2) / |\vec{q}_1 \times \vec{q}_2|$, $\vec{q}_1$ and $\vec{q}_2$ are the momenta of the incoming and outgoing pion in the centre-of-mass system. The normalization is chosen so that

$$\frac{d\sigma}{dt} = \frac{\pi}{q^2 s} (|f|^2 + |\bar{f}|^2), \quad (\text{II.2})$$

with $q = |\vec{q}_1| = |\vec{q}_2|$. The polarization $\vec{P}^{(i)}$ of the outgoing nucleon in the centre-of-mass system resulting from scattering of pions on unpolarized nucleons is given by

$$\vec{P}^{(i)} = 2 \frac{\text{Im}(f^{(i)} \bar{f}^{(i)*})}{|f^{(i)}|^2 + |\bar{f}^{(i)}|^2} \vec{n}. \quad (\text{II.3})$$

In the impact parameter description, the applicability of which was discussed in I, $f^{(i)}$ and $\bar{f}^{(i)}$ are given by

$$f^{(i)}(s, t) = 2q\sqrt{s} \int_0^\infty db b h^{(i)}(b, s) J_0(b\sqrt{-t}), \quad (\text{II.4})$$

$$\bar{f}^{(i)}(s, t) = 2q\sqrt{s} \int_0^\infty db b \bar{h}^{(i)}(b, s) J_1(b\sqrt{-t}), \quad (\text{II.5})$$

where b is the impact parameter, $h^{(i)}$ and $\bar{h}^{(i)}$ the corresponding weight functions and J_0 and J_1 are the Bessel functions of order 0 and 1.

The phenomenologically well supported assumptions

$$- \quad |\bar{h}^{(0)}| \ll |h^{(0)}|, \quad |\operatorname{Re} h^{(0)}| \ll \operatorname{Im} h^{(0)}, \quad (\text{II.6})$$

$$- \quad |h^{(2)}|, |\bar{h}^{(1)}| \ll \operatorname{Im} h^{(0)}, \quad (\text{II.7})$$

which were discussed in detail in I, together with the isospin relations

$$h^{(1)} = h^{(0)} - \sqrt{2} h^{(2)}, \quad (\text{II.8})$$

$$\bar{h}^{(1)} = \bar{h}^{(0)} - \sqrt{2} \bar{h}^{(2)}, \quad (\text{II.9})$$

allow to determine from unitarity the imaginary parts of the amplitudes $h^{(i)}$ and $\bar{h}^{(i)}$ in terms of the impact parameter components $G^{(i)}(b,s)$ and $\bar{G}^{(i)}(b,s)$ of the overlap functions (for brevity, we call $G^{(i)}$, $\bar{G}^{(i)}$ themselves overlap functions). For $\pi^+p \rightarrow \pi^+p$ and $\pi^-p \rightarrow \pi^0n$ these relationships read

$$\operatorname{Im} h^{(0)} = \frac{1}{2} \left(1 - \sqrt{1 - G^{(0)}} \right), \quad (\text{II.10})$$

$$\operatorname{Im} \bar{h}^{(0)} = \frac{1}{4} \frac{\bar{G}^{(0)}}{\sqrt{1 - G^{(0)}}}, \quad (\text{II.11})$$

$$\operatorname{Im} h^{(2)} = \frac{1}{4} \frac{G^{(2)}}{\sqrt{1 - G^{(0)}}}, \quad (\text{II.12})$$

$$\operatorname{Im} \bar{h}^{(2)} = \frac{1}{4} \frac{\bar{G}^{(2)}}{\sqrt{1 - G^{(0)}}}, \quad (\text{II.13})$$

while $\text{Im } h^{(1)}$ and $\text{Im } \bar{h}^{(1)}$ follow from (II.8) and (II.9), and $G^{(1)}$ and $\bar{G}^{(1)}$ verify

$$G^{(1)} = G^{(0)} - \sqrt{2} G^{(2)}, \quad (\text{II.14})$$

$$\bar{G}^{(1)} = \bar{G}^{(0)} - \sqrt{2} \bar{G}^{(2)}. \quad (\text{II.15})$$

At this point we recall that our study proposes to give a description of the asymptotic and the leading non-asymptotic terms of the scattering amplitudes. For the amplitudes $h^{(i)}$ and $\bar{h}^{(i)}$, they are found phenomenologically to be of the order s^0 and $s^{-\frac{1}{2}}$, respectively.

Our study neglects terms of order s^{-1} and smaller. To this degree of accuracy, the data reproducing overlap functions $G^{(0)}$, $\bar{G}^{(0)}$ and $G^{(2)}$, $\bar{G}^{(2)}$ found in I are given by

$$G^{(0)}(b,s) = G(b) = P e^{-\frac{1}{4B} b^2}, \quad (\text{II.16})$$

$$\bar{G}^{(0)}(b,s) = \bar{G}(b) = \bar{\alpha}' b P e^{-\frac{1}{4B} b^2}, \quad (\text{II.17})$$

$$G^{(2)}(b,s) = \frac{1}{\sqrt{2}} \alpha(s) \sqrt{1-G(b)} \left\{ P'(s) e^{-\frac{1}{4B'(s)} b^2} - P e^{-\frac{1}{4B} b^2} \right\}, \quad (\text{II.18})$$

$$\bar{G}^{(2)}(b,s) = \frac{1}{\sqrt{2}} \bar{\alpha}(s) \sqrt{1-G(b)} \left\{ \bar{P}'(s) e^{-\frac{1}{4B'(s)} b^2} - P e^{-\frac{1}{4B} b^2} \right\} \cdot b, \quad (\text{II.19})$$

where B and P characterize the genuine asymptotic diffraction pattern and $\bar{\alpha}'$ determines the strength of asymptotic elastic spin-flip which was argued to be small. These parameters are independent of s . $B'(s)$ determines the slope of the exponential factor in charge exchange and is given by

$$\beta'(s) = \alpha_1 \log\left(\frac{s}{M^2}\right) + \varphi_1 \quad (\text{II.20})$$

α_1 is the slope of the ξ trajectory, φ_1 determines a contribution to the exponential t dependence coming from kinematical and Γ function factors, and M = nucleon mass. The parameters $\mathcal{X}$, $\overline{\mathcal{X}}$, P' and $\overline{P}'$ are known functions of s which are given in I.

The forms (II.16) and (II.17) for the $\pi^+p \rightarrow \pi^+p$ overlap functions imply that in the energy region considered, and up to corrections of order s^{-1} , $G^{(0)}$ and $\overline{G}^{(0)}$ are well approximated by their asymptotic values. In I, this was justified on the basis of total π^+p cross-section data, and further interpreted in terms of the difference between inelastic π^+p and π^-p collisions. Svensson ⁸⁾ has added the following interesting observation which supports this hypothesis. The experimental values of the total inelastic π^+p cross-section, $\sigma_{\text{inel}}^{(0)} = \sigma^{(0)} - \sigma_{\text{el}}^{(0)}$, where $\sigma^{(0)}$ and $\sigma_{\text{el}}^{(0)}$ are the total and the total elastic π^+p cross-section, respectively, remain constant within the errors for $7 \text{ GeV/c} \lesssim q_{\text{L}} = \text{pion laboratory momentum} \lesssim 19 \text{ GeV/c}$. This is in contrast to the behaviour of the total inelastic π^-p cross-section, $\sigma_{\text{inel}}^{(1)} = \sigma^{(1)} - \sigma_{\text{el}}^{(1)}$, which decreases by about 2 mb in this region. This observation is based on the elastic differential π^+p cross-section measurements by Foley et al. ⁹⁾ and the latest total π^+p cross-section measurements by the same group ¹⁰⁾. The inelastic cross-sections are more directly related to the overlap functions $G^{(0)}$ and $G^{(1)}$ than σ or σ_{el} , since

$$\sigma_{\text{inel}}^{(0,1)} = 2\pi \int_0^\infty db \, b \, G^{(0,1)}(b, s).$$

III. CHARGE EXCHANGE SCATTERING : $\pi^- p \rightarrow \pi^0 n$

III.1 The imaginary parts

We concentrate first on the imaginary part of the spin-non-flip amplitude $f^{(2)}(s,t)$. If we introduce into the impact parameter integral (II.4) for the imaginary part of the weight function $h^{(2)}(b,s)$ the expression (II.12) in terms of the overlap function $g^{(2)}(b,s)$ as given by (II.18), we find

$$\text{Im } f^{(2)}(s,t) = -2q\sqrt{s} \frac{1}{4\sqrt{2}} \mathcal{A} \left\{ 2BPe^{Bt} - 2B'P'e^{B't} \right\}, \quad (\text{III.1})$$

where the relation

$$2Be^{Bt} = \int_0^\infty db b e^{-\frac{1}{4B}b^2} J_0(b\sqrt{-t})$$

has been applied. From Section II.3 of I, we take now the explicit forms of $\mathcal{A}(s)$ and $P'(s)$ in terms of B , of P and of the parameters characterizing reggeized φ exchange and we find

$$\text{Im } f^{(2)}(s,t) = -\frac{1}{\sqrt{2\pi}} \varphi(t) \left(\frac{s}{M^2}\right)^{\alpha(t)} \Gamma_J(s,t), \quad (\text{III.2})$$

$$\Gamma_J(s,t) = f_0^J + f_1^J \Delta B \frac{1}{B'^2} + f_1^J \left(\frac{B}{B'}\right)^2 \frac{1}{\Delta B} (e^{\Delta B t} - 1). \quad (\text{III.3})$$

Here, as in I, $\alpha(t) = \alpha_0 + \alpha_1 t$ is the φ trajectory, $f^J(t) = f_0^J + f_1^J t$ describes the t dependence of the reggeized φ contribution coming from the residue functions, $\varphi(t)$ collects some kinematical factors depending on t and $\alpha(t)$, and

$$\Delta B(s) = B - B'(s). \quad (\text{III.4})$$

To separate in (III.3) the term corresponding to reggeized ρ exchange explicitly from the correction term and also to simplify the subsequent calculations, we expand $B^{-2} = B^{-2}(1 - \Delta B/B)^{-2}$ and $\exp(\Delta B t)$ in powers of ΔB , and write Γ_J as a series

$$\Gamma_J(s, t) = \sum_{n=0}^{\infty} f^{(n)}(t) \left(\frac{\Delta B}{B} \right)^n. \quad (\text{III.5})$$

The $f^{(n)}(t)$ are explicitly known functions of t , which for $n=0, 1$ are

$$f^{(0)}(t) = f(t) = f_0 + f_1 t \quad (\text{III.6})$$

$$f^{(1)}(t) = f_1 \left(\frac{1}{B} + 2t + \frac{1}{2} B t^2 \right), \quad (\text{III.7})$$

The term $n=0$ gives the ρ contribution and $n=1$ the leading term in $\Delta B/B$ to $\Gamma_J(s, t)$. Our further analysis will be based on these two terms since they permit to deduce the essential consequences of the present model, while higher orders bring only minor numerical changes. For illustration, the two next higher orders are given in Appendix A.

The procedure for the spin-flip amplitude $\bar{f}^{(2)}(s, t)$ is analogous and gives

$$\text{Im } \bar{f}^{(2)}(s, t) = -2g \sqrt{s} \frac{1}{4\sqrt{2}} \bar{x} 2M \sqrt{\frac{-t}{4M^2}} \left\{ 4B^2 P e^{Bt} - 4B'^2 \bar{P}' e^{B't} \right\}, \quad (\text{III.8})$$

where

$$2M \sqrt{\frac{-t}{4M^2}} 4B^2 e^{Bt} = \int_0^{\infty} db b^2 e^{-\frac{b^2}{4B}} J_1(b\sqrt{-t})$$

has been used. The formulae analogous to (III.2) and (III.3) are

$$\text{Im } \bar{f}^{(n)}(s, t) = -\frac{1}{\sqrt{2\pi}} \varphi(t) \left(\frac{s}{M^2}\right)^{\alpha(t)} \sqrt{\frac{-t}{4M^2}} \bar{\Gamma}_J(s, t), \quad (\text{III.9})$$

$$\begin{aligned} \bar{\Gamma}_J(s, t) = & \bar{f}_0^e + \bar{f}_1^e 3B\Delta B \left(1 - \frac{2}{3} \frac{\Delta B}{B}\right) \frac{1}{B'^3} + \\ & + \bar{f}_1^e \left(\frac{B}{B'}\right)^3 \frac{1}{\Delta B} (e^{\Delta B t} - 1); \end{aligned} \quad (\text{III.10})$$

here again $\bar{f}^e(t) = \bar{f}_0^e + \bar{f}_1^e t$ describes the t dependence of the reggeized $\mathcal{Q}$ contribution coming from the residue functions. The same expansion as before gives for $\bar{\Gamma}_J(s, t)$

$$\bar{\Gamma}_J(s, t) = \sum_{n=0}^{\infty} \bar{f}^{(n)}(t) \left(\frac{\Delta B}{B}\right)^n, \quad (\text{III.11})$$

where the $\bar{f}^{(n)}$ are known functions of t . For $n=0, 1$, we have

$$\bar{f}^{(0)}(t) = \bar{f}^e(t) = \bar{f}_0^e + \bar{f}_1^e t, \quad (\text{III.12})$$

$$\bar{f}^{(1)}(t) = \bar{f}_1^e \left(\frac{3}{B} + 3t + \frac{1}{2} B t^2 \right). \quad (\text{III.13})$$

Again $n=0$ gives the $\mathcal{Q}$ contribution and $n=1$ the leading term in $\Delta B/B$ to $\bar{\Gamma}_J(s, t)$. The two next higher orders are given in Appendix A.

In Figs. 1 and 2 we show the functions $(\Delta B/B) \bar{f}^{(1)}(t)$ and $(\Delta B/B) \bar{f}^{(1)}(t)$, together with $\bar{f}^e(t)$ and $\bar{f}^e(t)$, respectively, at the energies 6, 11 and 18 GeV/c which span the energy region investigated.

We underline at this point that our corrections $\text{Im} f^{(2)}$, and $\text{Im} \bar{f}^{(2)}$, to reggeized φ exchange, which are defined by the terms with $n > 0$ in (III.5) and (III.11), contain, beside B , only the same parameters as $\text{Im} f^{(2)}$ and $\text{Im} \bar{f}^{(2)}$, which are given by $n=0$. Except for the usual power $s^\alpha(t)$, $\text{Im} f^{(2)}$, and $\text{Im} \bar{f}^{(2)}$, have an additional s dependence entering via $\Delta B(s)$ in the series (III.5) and (III.11); it is logarithmic. It is precisely this additional s dependence which causes the phase difference between the total amplitudes $f^{(2)}$ and $\bar{f}^{(2)}$ and therefore induces a non-vanishing polarization. We also note that the logarithmic s dependence of $\Delta B(s)$ indicates that the corrections $f^{(2)}$, and $\bar{f}^{(2)}$, may be thought of as originating from an effective cut contribution in the Regge plane. According to the preceding remarks, the parameters of this cut term are entirely fixed in terms of the parameters of the ρ pole in the present framework, as has already been indicated in the Introduction.

III.2 The real parts : principle of the method

In I it has been shown that at high energy $f^{(2)}$ and $\bar{f}^{(2)}$ are related to the invariant amplitudes $A^{(2)}(s,t,u)$ and $B^{(2)}(s,t,u)$ by

$$f^{(2)}(s,t) = \frac{2M}{8\pi} \left\{ A^{(2)}(s,t,u) + \frac{s}{2M} B^{(2)}(s,t,u) \right\}, \quad (\text{III.14})$$

$$\bar{f}^{(2)}(s,t) = \frac{2M}{8\pi} \sqrt{\frac{-t}{4M^2}} A^{(2)}(s,t,u), \quad (\text{III.15})$$

where

$$u = 2(M^2 + \mu^2) - t - s,$$

μ = pion mass, M = nucleon mass.

The right-hand sides of Eqs. (III.14) and (III.15) are the leading terms of $f^{(2)}$ and $\bar{f}^{(2)}$ for $s \rightarrow \infty$: terms of order $\mathcal{O}(s^{-\frac{1}{2}} A^{(2)})$ and $\mathcal{O}(s^{-\frac{1}{2}} (s/2M) B^{(2)})$ and lower have been neglected.

The functions on the right-hand sides of (III.14) and (III.15) fulfil once-subtracted dispersion relations at fixed $t \leq 0$: this follows from the analyticity properties of $A^{(2)}$ and $B^{(2)}$ and their asymptotic behaviour. Considering first the spin-non-flip amplitude $f^{(2)}$, we can write as a consequence for the leading term of $\text{Re } f^{(2)}$, neglecting terms of order s^0

$$\text{Re } f^{(2)}(s, t) = \frac{1}{\pi} s P \int_{s_0}^{\infty} ds' \frac{\text{Im } \tilde{f}^{(2)}(s', t)}{(s' - s) s'} + \frac{1}{\pi} u \int_{s_0}^{\infty} du' \frac{\text{Im } \tilde{f}^{(2)}(s[u'], t)}{(u' - u) u'} \quad (\text{III.16})$$

$$s \rightarrow \infty, \quad s'[u'] = 2(M^2 + \mu^2) - t - u', \quad s_0 = (M + \mu)^2,$$

where P denotes the principal value, and

$$\tilde{f}^{(2)}(s, t) \equiv \frac{2M}{8\pi} \left\{ A^{(2)}(s, t, u) + \frac{s}{2M} B^{(2)}(s, t, u) \right\} \quad (\text{III.17})$$

is the leading term of $f^{(2)}$ considered for all values of s . Since (III.17) does not account exactly for the crossing properties of $f^{(2)}$, terms of order s^0 occur in (III.16) and the following equations. These can be eliminated by proper reformulation of the theory (see Appendix D) and are neglected as not being significant.

Now, we apply the crossing properties of $A^{(2)}$ and $B^{(2)}$,

$$A^{(2)}(s'[u'], t, u') = A_c^{(2)}(u', t, s'[u']) ,$$

$$B^{(2)}(s'[u'], t, u') = - B_c^{(2)}(u', t, s'[u']) ,$$

where the index c denotes u channel quantities. They show that up to negligible orders in u' one has

$$\tilde{f}^{(2)}(s'[u'], t) = \tilde{f}_c^{(2)}(u', t) \quad , \quad (\text{III.18})$$

where $\tilde{f}_c^{(2)}$ is defined by

$$\tilde{f}_c^{(2)}(u', t) = \frac{2M}{8\pi} \left\{ A_c^{(1)}(u', t, s'[u']) + \frac{u'}{2M} B_c^{(2)}(u', t, s'[u']) \right\}. \quad (\text{III.19})$$

Indeed, (III.18) follows from

$$\tilde{f}^{(2)}(s'[u'], t) = \frac{2M}{8\pi} \left\{ A^{(2)}(s'[u'], t, u') + \frac{2(M^2 + u'^2) - t - u'}{2M} B^{(2)}(s'[u'], t, u') \right\}$$

if one uses the crossing properties of $A^{(2)}$, $B^{(2)}$ and keeps only the leading order in u' .

The crossed u channel is $\pi^0 p \rightarrow \pi^+ n$, which can be rotated by isospin and time reversal to $\pi^- p \rightarrow \pi^0 n$: the corresponding relation being

$$\tilde{f}_c^{(2)}(u, t) = - \tilde{f}^{(2)}(u, t). \quad (\text{III.20})$$

With the results (III.18) and (III.20) and putting $u \simeq -s$, (III.16) gives now for the leading term of $\text{Re } f^{(2)}$

$$\text{Re } f^{(2)}(s, t) = \frac{1}{\pi} \mathcal{P} \int_{s_0}^{\infty} ds' \frac{\text{Im } \tilde{f}^{(2)}(s', t)}{(s' - s) s'} + \frac{1}{\pi} \mathcal{P} \int_{s_0}^{\infty} du' \frac{\text{Im } \tilde{f}^{(2)}(u', t)}{(u' + s) u'} \quad , \quad (\text{III.21})$$

$$s \rightarrow \infty$$

Equation (III.21) determines the leading term of $\text{Re } f^{(2)}$ in terms of the imaginary part of $\tilde{f}^{(2)}$.

Finally, we observe that above a certain energy $s_1 = u_1$ $\text{Im } \tilde{f}^{(2)}$ in (III.21) is given by the high energy expression (III.2)-(III.5); phenomenologically, dominance of the high energy mechanism seems to set in around $s_1 \approx 8 \text{ (GeV/c)}^2$ [which corresponds to $q_L = \text{pion laboratory momentum} \approx 4 \text{ (GeV/c)}$]¹¹⁾. We replace now $\text{Im } \tilde{f}^{(2)}$ in the whole region of integration in (III.21), i.e., also between threshold and s_1 , by its high energy expression $\text{Im } f_{HE}^{(2)}$ defined by (III.2)-(III.5). It is easily seen that this does not influence the leading term of $\text{Re } f^{(2)}$. We thus find, to leading order,

$$\begin{aligned} \text{Re } f^{(2)}(s, t) = & \frac{1}{\pi} s P \int_{s_0}^{\infty} ds' \frac{\text{Im } f_{HE}^{(2)}(s', t)}{(s' - s) s'} + \\ & + \frac{1}{\pi} s \int_{s_0}^{\infty} du' \frac{\text{Im } f_{HE}^{(2)}(u', t)}{(u' + s) u'} , \end{aligned} \quad (III.22)$$

$s \rightarrow \infty$.

As a side remark, it is interesting to note that the statement that the leading order of the amplitude dominates above the value s_1 implies a consistency condition for the amplitude in the low energy region, i.e., for $s_0 \leq s \leq s_1$. This means that the value of the integral

$$\frac{2}{\pi} s P \int_{s_0}^{s_1} ds' \frac{\text{Im } \tilde{f}^{(2)}(s', t) - \text{Im } f_{HE}^{(2)}(s', t)}{s'^2 - s^2} \quad (III.23)$$

which contributes to the deviations from the leading order, has to be consistent with the smallness of the latter. Such a kind of condition is of course intimately related to superconvergence relations¹²⁾. In this context we only mention that the existence of such relations also requires terms in addition to reggeized ρ exchange for charge exchange scattering.

It can be seen that the corrections defined by the present model qualitatively behave like those required to satisfy the conditions implied by superconvergence relations. A more quantitative study of this question could be carried out as part of a best fit analysis of all data in terms of the free parameters of our model, which has not yet been done.

For the spin-flip amplitude $\bar{f}^{(2)}$, the same considerations hold as for $f^{(2)}$ with an equation for $\text{Re } \bar{f}^{(2)}$ similar to (III.22), where $\text{Im } f_{\text{HE}}^{(2)}$ is replaced by $\text{Im } \bar{f}_{\text{HE}}^{(2)}$. $\text{Im } \bar{f}_{\text{HE}}^{(2)}$ is given by the results in Section II. The spin-flip case is even somewhat simpler; since $\bar{f}^{(2)}$ is proportional to $A^{(2)}$ the crossing relation analogous to (III.18) is immediate.

III.3 The real parts : explicit form

We give now the explicit forms of $\text{Re } f^{(2)}$ and $\text{Re } \bar{f}^{(2)}$ as they follow from the equations of type (III.22) through insertion of the results from Section III.1.

Again, we consider first the spin-non-flip case and define

$$\text{Re } f^{(2)}(s, t) = -\frac{1}{\sqrt{2\pi}} \varphi(t) \left(\frac{s}{M^2}\right)^{\alpha(t)} \Gamma_R(s, t), \quad (\text{III.24})$$

$$\Gamma_R(s, t) = \sum_{n=0}^{\infty} g^{(n)}(t) \frac{C^{(n)}(s, t)}{B^n}, \quad (\text{III.25})$$

where $C^{(n)}(s, t)$ follows from (III.22) and (III.5) as

$$\begin{aligned} C^{(n)}(s, t) = & \frac{1}{\pi} P \int_{s_0/s}^{\infty} dx \frac{(\Delta B(s) - \alpha_1 \log x)^n x^{\alpha(t)-1}}{x-1} + \frac{1}{\pi} \int_{s_0/s}^{\infty} dx \frac{(\Delta B(s) - \alpha_1 \log x)^n x^{\alpha(t)-1}}{x+1} \\ & - \frac{2}{\pi} \int_{s_2/s}^{\infty} dx \frac{(\Delta B(s) - \alpha_1 \log x)^n x^{\alpha(t)}}{x^2-1} + \frac{2}{\pi} (\Delta B_2)^n \left(\frac{s_2}{s}\right)^{\alpha_1 t} \int_{s_2/s}^{\infty} dx \frac{x^{\alpha_0}}{x^2-1}. \end{aligned} \quad (\text{III.26})$$

To arrive at this equation, the integration variables in the dispersion integrals were changed into

$$\frac{s'}{s} = x, \quad \frac{u'}{s} = x. \quad (\text{III.27})$$

Furthermore, the relation

$$\Delta B(s') = \Delta B(s) + \alpha_1 \log \frac{s'}{s} \quad (\text{III.28})$$

has been applied. Finally, the modification implied by our model for the energy dependence of the charge exchange or of the elastic forward peak (as described in detail in I) has been taken into account and is represented by the last two terms in (III.26). It consists simply in subtracting the contribution from the unmodified behaviour, and replacing it by the contribution of the modified behaviour. According to I and our remarks at the end of this Section such modifications should occur around

$$s_2 \approx 70 (\text{GeV}/c)^2. \quad (\text{III.29})$$

The quantities B_2 and ΔB_2 in (III.26) are defined by

$$B_2 = B'(s_2), \quad \Delta B_2 = \Delta B(s_2). \quad (\text{III.30})$$

The integrations in (III.26) can be carried out with the help of the formulae in Appendix B and give for the two lowest orders $n=0, 1$, to which we restrict the discussion in the present paper,

$$C^{(0)}(s, t) = \frac{\sin \pi \alpha(t)}{1 + \cos \pi \alpha(t)} + \frac{2}{\pi} (-\alpha_1 t) \left(\frac{s}{s_2} \right)^{1-\alpha(t)} \sum_{k=0}^{\infty} \frac{1}{1-\alpha_0+2k} \frac{1}{1-\alpha(t)+2k} \left(\frac{s}{s_2} \right)^{2k}, \quad (\text{III.31})$$

$$C^{(1)}(s,t) = \Delta B(s) \frac{\sin \pi \alpha(t)}{1 + \cos \pi \alpha(t)} - \pi \alpha_1 \frac{1}{1 + \cos \pi \alpha(t)} +$$

$$+ \frac{2}{\pi} \left(\frac{s}{s_2} \right)^{1-\alpha(t)} \sum_{k=0}^{\infty} \left[\frac{-\alpha_1 t}{1-\alpha_0+2k} \Delta B_2 + \frac{\alpha_1}{1-\alpha(t)+2k} \right] \frac{1}{1-\alpha(t)+2k} \left(\frac{s}{s_2} \right)^{2k} \quad (\text{III.32})$$

The higher terms, $C^{(n)}(s,t)$, $n \geq 2$, are easily determined by the method described in Appendix B.

For the spin-flip amplitude, the procedure is completely analogous and gives with the definition

$$\text{Re } f^{(2)}(s,t) = - \frac{1}{\sqrt{2\pi}} \varphi(t) \sqrt{\frac{-t}{4M^2}} \left(\frac{s}{M^2} \right)^{\alpha(t)} \overline{f}_R(s,t), \quad (\text{III.33})$$

for the function $\overline{f}_R$ a form analogous to (III.25),

$$\overline{f}_R(s,t) = \sum_{n=0}^{\infty} \overline{f}^{(n)}(t) \frac{C^{(n)}(s,t)}{B^n} \quad (\text{III.34})$$

where the $\overline{f}^{(n)}(t)$ are given by (III.11).

To discuss briefly some aspects of the functions (III.31) and (III.32), we note that the first term in (III.31) gives the reggeized ρ exchange contribution to the real part of $f^{(2)}$ or $\bar{f}^{(2)}$; the second term is due to the modification in shrinkage. All terms in (III.32) are corrections to reggeized ρ exchange. While the third term in (III.32) is due to the modification in shrinkage, the dominant modification to reggeized ρ exchange near $q_L \approx 6 \text{ GeV}/c$ comes from the first two terms in (III.32), the further corrections becoming more important with increasing energy.

Before proceeding to the calculation of polarization, we briefly discuss some aspects of the problem of fitting our complete amplitudes $f^{(2)} = f_{\rho}^{(2)} + f^{(2)}$, and $\bar{f}^{(2)} = \bar{f}_{\rho}^{(2)} + \bar{f}^{(2)}$, to the charge exchange angular distribution ¹³⁾. This will result in modifications of the phenomenological values of the ρ Regge pole parameters, as compared to a fit with a pole term only. These modifications are expected to be small in general since $f_{\rho}^{(2)}$ and $\bar{f}_{\rho}^{(2)}$ dominate over $f^{(2)}$ and $\bar{f}^{(2)}$, respectively. There is, however, one exception for γ_1 , which is recognized as follows. The dip-bump structure of course is still accounted for by the spin-flip Regge pole term which dominates except in the forward direction and near the second dip, i.e., $t \approx -0.6 \text{ (GeV/c)}^2$. In the single pole model, the cross-section in the second dip has to come entirely from $\text{Im} f_{\rho}^{(2)}$. In the present model, closer examination using the explicit form of $\gamma^{(0)}$, $\gamma^{(1)}$, $\bar{\gamma}^{(0)}$ and $\bar{\gamma}^{(1)}$, as shown in Figs. 1 and 2, indicates that there is a substantial contribution from $\bar{f}^{(2)}$, [more specifically from $\text{Re } \bar{f}^{(2)}$] at this place. Since $\bar{f}^{(2)}$ is numerically fixed once $\bar{f}_{\rho}^{(2)}$ is specified, we necessarily have to modify $f_{\rho}^{(2)}$, as compared to the single pole model, in order to diminish sufficiently its contribution in the second dip. As a consequence, the zero of the spin-non-flip residue function $\gamma^{(0)} = \gamma(t)$ will have to occur more to the right as compared to the situation described in I [see Fig. 2 in I]: a reasonable choice is $t \approx -0.5 \text{ (GeV/c)}^2$, which corresponds roughly to half of the value of the constant γ_1 given in I. In Fig. 1 of the present paper, the functions $\gamma(t)$ and $(\Delta B/B) \gamma^{(1)}(t)$ are given with this value of γ_1 .

It is important that with the procedure just described, the calculated angular distribution is not significantly modified because of the dominance of $\bar{f}_{\rho}^{(2)}$. Also the compatibility with the results from finite energy dispersion sum rules ¹²⁾ is preserved. It is also interesting to see that with this procedure we approach the situation where the ρ trajectory $\alpha(t)$ chooses nonsense ⁶⁾ at $\alpha(t) = 0$, i.e., where both ρ residue functions vanish at this place. As to the cross-over phenomenon ^{5),6),14)}, it results in the present case from the combined action of the spin-non-flip

residue function and the corresponding correction terms presumably together with some effect from spin-flip [see also end of Section IV.5]. Furthermore we remark that the above conclusions concerning the spin-non-flip residue are in agreement with indications from recent more direct phenomenological information on these quantities ¹⁵⁾.

Finally, the change $y_1^* \rightarrow y_1^*/2$ increases the energy around which, as has been seen in I, the proposed modifications in the behaviour of the forward peaks should occur. This energy becomes ~ 35 GeV/c.

III.4 The polarization

With the knowledge of the amplitudes $f^{(2)}$ and $\bar{f}^{(2)}$, it is now possible to calculate the polarization vector $\vec{P}^{(2)}$ given by (II.3). With the definitions (III.2), (III.24), (III.9) and (III.33), its length $P^{(2)}$ is given by :

$$P^{(2)} = 2 \frac{-\bar{\Gamma}_J \Gamma_R + \bar{\Gamma}_R \Gamma_J}{\Gamma_R^2 + \Gamma_J^2 + \frac{-t}{4M^2} (\bar{\Gamma}_R^2 + \bar{\Gamma}_J^2)} \sqrt{\frac{-t}{4M^2}} \quad (\text{III.35})$$

In our calculation up to first order in n we get with (III.5), (III.25), (III.11) and (III.34) for the numerator of (III.35) :

$$-\bar{\Gamma}_J \Gamma_R + \bar{\Gamma}_R \Gamma_J = \left\{ -\bar{f}^{(1)} f^{(10)} + \bar{f}^{(10)} f^{(1)} \right\} \frac{\alpha_1}{B}.$$

$$\left\{ \frac{\pi}{1 + \cos \pi \alpha(t)} - \frac{2}{\pi} \left(\frac{s}{s_2} \right)^{1 - \alpha(t)} \sum_{k=0}^{\infty} \frac{1}{1 - \alpha(t) + 2k} \right\} \quad (\text{III.36})$$

$$\left\{ \frac{1}{1 - \alpha(t) + 2k} - \frac{-\alpha_1 t}{1 - \alpha_0 + 2k} \log \left(\frac{s_0}{s} \right) \right\} \left(\frac{s}{s_2} \right)^{2k} \Bigg\}.$$

These formulae describe in the present theory the main features of polarization, i.e., sign and approximate magnitude for $0 \leq -t \lesssim 0.2-0.3 (\text{GeV}/c)^2$ and for energies in the region we are considering. For larger values of $-t$ higher orders, i.e., $n=2$ and 3 , in the correction terms have to be taken into account in order to get a sufficiently accurate expression for the amplitudes; these terms also increase the magnitude of polarization by a few per cent for the lower values of $-t$. We have calculated for $q_{\perp} = 6$ and $11 \text{ GeV}/c$ the polarization resulting from (III.35), (III.36), putting in the parameter values given in I with the modification of β_1 described at the end of the preceding Section. The result is shown in Figs. 3a and 3b, and compared with the experimental data of the Orsay-Saclay-Pisa collaboration ²⁾; the agreement is good.

To discuss briefly the expression (III.36), which determines the main features of polarization, we note that the first factor is positive and is determined by the residue type functions $\beta^{(0)}$, $\beta^{(1)}$, $\bar{\beta}^{(0)}$ and $\bar{\beta}^{(1)}$. The second factor, in which the first term dominates for the s, t values considered, also is positive and is determined via the dispersion integrals by the type of energy dependence of the imaginary part. Note that the positivity of this factor follows from the fact that the second term in (III.32) and the pure Regge type term in (III.31) have opposite signs. It is interesting to realize that such a change of sign can only come from the lower energy part of the dispersion integrals, i.e., $x < 1$, as may be seen from (III.26). In (III.26) this occurs for $n > 0$ because of the higher weighting of the low energy part as compared to the higher energy part, since $\Delta B(s) \sim \alpha_1 \log x$ decreases with increasing x for $x \gg s_0/s$. The second term in the second factor determines the energy dependence.

Finally, we stress that the calculated polarization is entirely given in terms of the parameters of the reggeized ϕ exchange approximation, which themselves are determined phenomenologically by the angular distribution. There are in our model no further parameters which could be adjusted to fit the observed polarization.

IV. ELASTIC SCATTERING : $\pi^{\pm}_p \rightarrow \pi^{\pm}_p$

We now discuss briefly elastic scattering along the same lines as in Section III.

IV.1 The imaginary parts

We have only to consider the spin-non-flip and the spin-flip amplitudes $f^{(0)}$ and $\bar{f}^{(0)}$ of $\pi^+_p \rightarrow \pi^+_p$ scattering since $\pi^-_p \rightarrow \pi^-_p$ is given by the isospin relations corresponding to (II.8) and (II.9).

According to Van Hove ¹⁶⁾, Eqs. (II.4), (II.10) and (II.16) imply

$$\text{Im } f^{(0)}(s,t) = M^2 B \mathcal{G}(Bt) \frac{s}{M^2} \quad (\text{IV.1})$$

where the s independent function $\mathcal{G}(Bt)$ describes the asymptotic elastic diffraction peak and is completely determined by the overlap function $G(b)$ as given by (II.16); we briefly recall Van Hove's formulae and procedure in Appendix C. The only s dependence in (IV.1) comes from the factor s/M^2 .

As to the imaginary part of the spin-flip amplitude $\text{Im } \bar{f}^{(0)}(s,t)$, we find from (II.5), (II.11) and (II.17)

$$\text{Im } \bar{f}^{(0)}(s,t) = 4M^4 B^2 \frac{\bar{\mathcal{X}}'}{M} \sqrt{\frac{-t}{4M^2}} \mathcal{G}(Bt) \frac{s}{M^2} \quad (\text{IV.2})$$

The derivation, which is analogous to Van Hove's calculation of $\text{Im } f^{(0)}(s,t)$, is also indicated briefly in Appendix C. It should be noted that (IV.1) and (IV.2) contain the same function $\mathcal{G}(Bt)$. $\bar{\mathcal{X}}'/M$ is the dimensionless strength factor of asymptotic elastic spin-flip. Here we underline the fact that the present model makes definite predictions on the t dependence of the elastic spin-flip amplitude. They will have to be compared with experimental findings, provided $\bar{\mathcal{X}}'/M$ is not found experimentally to be too small.

IV.2 The real parts : principle of the method

Up to Eq. (III.18), the considerations of Section III.2 apply also to $\pi^{\pm} p \rightarrow \pi^{\pm} p$ scattering. However, the crossing properties now are different, i.e.,

$$\tilde{f}_c^{(0)}(u, t) = \tilde{f}^{(1)}(u, t), \quad \tilde{f}_c^{(1)}(u, t) = \tilde{f}^{(0)}(u, t). \quad (\text{IV.3})$$

If we introduce with

$$f^{(\pm)}(s, t) = \frac{1}{2} \left(f^{(0)}(s, t) \pm f^{(1)}(s, t) \right) \quad (\text{IV.4})$$

amplitudes which are even (+) and odd (-) under crossing, we find for them, again after the substitution $\text{Im} \tilde{f}^{(\pm)}(s' \text{ or } u', t) \rightarrow \text{Im} f_{\text{HE}}^{(\pm)}(s' \text{ or } u', t)$, the dispersion relation corresponding to (III.22). Because of isospin invariance

$$f^{(-)}(s, t) = \frac{1}{\sqrt{2}} f^{(1)}(s, t), \quad (\text{IV.5})$$

and the corresponding dispersion relation has been investigated in Section III. We now consider the case of $f^{(+)}$, for which one has for the leading term of $\text{Re} f^{(+)}$

$$\begin{aligned} \text{Re} f^{(+)}(s, t) = & \frac{1}{\pi} s \mathcal{P} \int_{s_0}^{\infty} ds' \frac{\text{Im} f_{\text{HE}}^{(+)}(s', t)}{(s' - s) s'} - \frac{1}{\pi} s \int_{s_0}^{\infty} du' \frac{\text{Im} f_{\text{HE}}^{(+)}(u', t)}{(u' + s) u'} + \\ & + \text{terms of order } \log s \text{ and } s^0 \text{ for } s \rightarrow \infty, \end{aligned} \quad (\text{IV.6})$$

the $\log s$ terms come from the terms of order s^0 neglected in $\text{Im} \tilde{f}^{(+)}$ (up to this point we have not discussed such terms except in Section IV.1 of I), while the contributions of order s^0 include the subtraction term, the pole term, a low energy integral of type (III.23) as well as corrections to the approximation $u = 2M^2 + 2\mu^2 - t - s \approx -s$.

The input into the integrals of (III.6) will be

$$\begin{aligned} \text{Im } f_{HE}^{(+)}(s,t) &= \frac{1}{2} \left(\text{Im } f_{HE}^{(0)}(s,t) + \text{Im } f_{HE}^{(1)}(s,t) \right) \\ &= \text{Im } f_{HE}^{(0)}(s,t) - \frac{1}{\sqrt{2}} \text{Im } f_{HE}^{(2)}(s,t), \quad (\text{IV.7}) \end{aligned}$$

the latter equality resulting from isospin invariance.

$\text{Im } f_{HE}^{(0)}$ and $\text{Im } f_{HE}^{(2)}$ have to be taken from equations (IV.1) and (III.2)-(III.5), respectively. As will be seen explicitly in the next subsection, $f^{(+)}$ being even under crossing, the integrals in (IV.6) give terms of order $s^{\alpha(t)}$ and s^0 and smaller. We can then write

$$f^{(+)}(s,t) = f_P(s,t) + f_{P'}(s,t) + f_{P''}(s,t) \quad (\text{IV.8})$$

with the convention

$$\left. \begin{aligned} f_P(s,t) &= \frac{s}{M^2} \phi_P(t) \\ f_{P'}(s,t) &= \left(\frac{s}{M^2} \right)^{\alpha(t)} \phi_{P'}(t) \\ f_{P''}(s,t) &\text{ of orders } \log s, s^0 \text{ and lower} \end{aligned} \right\} \quad (\text{IV.9})$$

and we conclude that f_P is purely imaginary.

A similar treatment applies to the spin-flip amplitude which takes the form

$$\bar{f}^{(-)}(s,t) = \bar{f}_P(s,t) + \bar{f}_{P'}(s,t) + \bar{f}_{P''}(s,t), \quad (\text{IV.10})$$

the orders in s being the same as in (IV.9), and $\bar{f}_P$ being again purely imaginary.

It is obvious that the contributions f_P , $f_{P'}$, and $\bar{f}_P$, $\bar{f}_{P'}$ in the present framework have to be identified with the Pomeron and P' terms in Regge pole theory. It follows that the Pomeron pole in our model has trajectory $\alpha_P(t) = 1$, while according to (IV.1) and (IV.2) the residues are essentially determined by the function $\zeta(Bt)$ as given in Appendix C. Our P' pole has the same trajectory $\alpha_{P'}(t) = \alpha_\rho(t) = \alpha(t)$ and the same residue functions $\gamma_{P'}^l(t) = \gamma_\rho^l(t) = \gamma^l(t)$, $\bar{\gamma}_{P'}^l(t) = \bar{\gamma}_\rho^l(t) = \bar{\gamma}^l(t)$ as the ρ Regge pole; it differs only in the signature properties from the latter. It is interesting to compare this with the results of the latest fits of the Regge pole model by Rarita et al.¹⁷⁾; their favoured solution, which is solution (3)¹⁸⁾, gives indeed values of certain P' parameters, especially for $\alpha_{P'}(0)$, which are strikingly similar to the corresponding ρ parameters. The P' contribution in our model is also complemented by correction terms originating from $n > 0$ in (III.5) and (III.11).

The terms $f_{P''}$ and $\bar{f}_{P''}$ defined in (IV.8)-(IV.10) will be important for killing the P' ghost in the present framework.

In the following discussion, which will be concerned with the explicit form of the real parts and the polarization parameters for $\pi^\pm p \rightarrow \pi^\pm p$, we limit ourselves to the pure Regge pole contributions of our model, i.e., the $n=0$ terms. This approximation is sufficient for the investigation of the dominant properties of these quantities. We also neglect the contributions coming from the modification in shrinkage, which are small in the energy region investigated.

IV.3 The real parts : explicit form

Similarly to the procedure in Section III.3 we obtain now from (IV.6), (IV.7), together with (IV.1) and (III.2)-(III.5) (with $n=0$) for $s \rightarrow \infty$

$$\begin{aligned}
\operatorname{Re} f^{(*)}(s, t) &= M^2 B \zeta(Bt) \frac{s}{M^2} C_P(s, t) + \\
&+ \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \bar{f}(t) \left(\frac{s}{M^2}\right)^{\alpha(t)} C_{P'}(s, t) + \quad (\text{IV.11}) \\
&+ \text{terms of order } \log s \text{ and } s^0,
\end{aligned}$$

where

$$C_j(s, t) = \frac{1}{\pi} \mathcal{P} \int_{s_0/s}^{\infty} dx \frac{x^{\alpha_j-1}}{x-1} - \frac{1}{\pi} \int_{s_0/s}^{\infty} dx \frac{x^{\alpha_j-1}}{x+1}, \quad (\text{IV.12})$$

with $j = P, P'$; $\alpha_P = 1$, $\alpha_{P'} = \alpha(t)$. For spin-flip we get analogously with (IV.2) and (III.9)-(III.11) (with $n=0$)

$$\begin{aligned}
\operatorname{Re} \bar{f}^{(*)}(s, t) &= 4M^4 B^2 \frac{\bar{\varphi}'(t)}{M} \sqrt{\frac{-t}{4M^2}} \zeta(Bt) \frac{s}{M^2} C_P(s, t) + \\
&+ \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \sqrt{\frac{-t}{4M^2}} \bar{f}(t) \left(\frac{s}{M^2}\right)^{\alpha(t)} C_{P'}(s, t) + \quad (\text{IV.13}) \\
&+ \text{terms of order } \log s \text{ and } s^0.
\end{aligned}$$

From the detailed considerations in Appendix B, we find for (IV.12) with (B.4)

$$C_P = \frac{2}{\pi} \frac{s_0}{s} + \frac{2}{\pi} \frac{s_0}{s} \sum_{k=1}^{\infty} \frac{1}{2k+1} \left(\frac{s_0}{s}\right)^{2k}, \quad (\text{IV.14})$$

$$\begin{aligned}
C_{P'} &= - \frac{\cos \pi \alpha(t) + 1}{\sin \pi \alpha(t)} + \frac{2}{\pi \alpha(t)} \left(\frac{s_0}{s}\right)^{\alpha(t)} + \\
&+ \frac{2}{\pi} \left(\frac{s_0}{s}\right)^{\alpha(t)} \sum_{k=1}^{\infty} \frac{1}{2k + \alpha(t)} \left(\frac{s_0}{s}\right)^{2k}. \quad (\text{IV.15})
\end{aligned}$$

The second term in (IV.14) and the third term in (IV.15) will be neglected; they give rise to terms of order s^{-2} in the part $f_{P''}$ of $f^{(+)}$. The first term of (IV.15) contributes to $f_{P'}$, the second term gives in $f_{P''}$ a contribution which cancels the ghost pole occurring in the first term at $\alpha(t) = 0$. The first term of (IV.14) contributes again to $f_{P''}$. We note that our model automatically provides a ghost compensating mechanism for the t value where the P' trajectory goes through zero (this point is discussed somewhat further in Appendix B).

From the preceding results we find now the explicit forms of the quantities defined in (IV.8)-(IV.10), i.e.,

$$\left. \begin{aligned} f_P(s,t) &= i M^2 B \zeta(Bt) \frac{s}{M^2} \\ \bar{f}_P(s,t) &= i 4 M^4 B^2 \frac{\bar{\zeta}'}{M} \sqrt{\frac{-t}{4M^2}} \zeta(Bt) \frac{s}{M^2} \end{aligned} \right\} \quad (\text{IV.16})$$

$$\left. \begin{aligned} f_{P'}(s,t) &= \left(-\frac{\cos \pi \alpha(t) + 1}{\sin \pi \alpha(t)} + i \right) \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \bar{\gamma}(t) \left(\frac{s}{M^2} \right)^{\alpha(t)} \\ \bar{f}_{P'}(s,t) &= \left(-\frac{\cos \pi \alpha(t) + 1}{\sin \pi \alpha(t)} + i \right) \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \sqrt{\frac{-t}{4M^2}} \bar{\gamma}(t) \left(\frac{s}{M^2} \right)^{\alpha(t)} \end{aligned} \right\} \quad (\text{IV.17})$$

$$\left. \begin{aligned} f_{P''}(s,t) &= \frac{2}{\pi \alpha(t)} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \bar{\gamma}(t) \left(\frac{s_0}{M^2} \right)^{\alpha(t)} + \frac{2}{\pi} M^2 \zeta(Bt) \frac{s_0}{M^2} + \\ &\quad + \text{other terms of order } s^0 \text{ and } \log s \\ \bar{f}_{P''}(s,t) &= \frac{2}{\pi \alpha(t)} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \sqrt{\frac{-t}{4M^2}} \bar{\gamma}(t) \left(\frac{s_0}{M^2} \right)^{\alpha(t)} + \\ &\quad + \frac{2}{\pi} 4 M^4 B^2 \frac{\bar{\zeta}'}{M} \sqrt{\frac{-t}{4M^2}} \zeta(Bt) \frac{s_0}{M^2} + \\ &\quad + \text{other terms of order } s^0 \text{ and } \log s \end{aligned} \right\} \quad (\text{IV.18})$$

We recall that the terms not explicitly given in (IV.18) are those defined in the context of the dispersion relations of type (IV.6) for $\text{Re } f^{(+)}$ and $\text{Re } \bar{f}^{(+)}$.

To describe with these expressions the entire interval of diffractive phenomena, i.e., $0 \leq -t \lesssim -1 \text{ (GeV/c)}^2$, we should take into account the terms f_{p}'' and $\bar{f}_{\text{p}}''$, which are needed for ghost killing, and being of order s^0 and $\log s$, become of the same order as f_{p} and $\bar{f}_{\text{p}}$, near $\alpha(t) = 0$ and dominate them for larger $|t|$. As will be seen in the next Section, they will also be needed to account for present data on $\pi^{\pm} p \rightarrow \pi^{\pm} p$ scattering in our framework.

To avoid unnecessary complications regarding these terms, we describe our scattering amplitudes in an approximation where the asymptotic expansion ¹⁹⁾ contains only powers of s . Beside explicitly known terms necessary in order to fulfill crossing symmetry exactly, no further terms of order s^0 can then occur in $\text{Im } f_{\text{HE}}^{(+)}$ or $\text{Im } \bar{f}_{\text{HE}}^{(+)}$ since such terms would lead to contributions of order $\log s$ in the real parts, as has been indicated in the context of (IV.6). In other words, we assume that eventual logarithmic terms would only give small corrections to the expansion containing pure power terms (some implications of this assumption are discussed in Appendix D). As a consequence, we can describe the unknown terms in (IV.18) up to their leading order s^0 by two real valued, regular functions $\phi(t)$ and $\bar{\phi}(t)$ for spin-non-flip and spin-flip, respectively. They have to be determined phenomenologically from the data.

IV.4 The polarizations in $\pi^{\pm} p \rightarrow \pi^{\pm} p$

We consider now the polarization for elastic scattering; our aim is to show how the present theory can be fitted to the observed polarizations in the interval $0 \leq -t \lesssim 0.6 \text{ (GeV/c)}^2$ and for $q_{\perp} = 6-12 \text{ GeV/c}$ as they have been found in a recent CERN experiment ³⁾. We intend to discuss only the bulk properties of the experimental findings, detailed quantitative description requiring a more elaborate study.

From (IV.4), (IV.5) and (IV.8) we get for $f^{(0)}$ and $f^{(1)}$

$$f^{(i)}(s,t) = f_p(s,t) + f_{p'}(s,t) + f_{p''}(s,t) \pm \frac{1}{\sqrt{2}} f^{(i)}(s,t) \quad (\text{IV.19})$$

where the upper (lower) index goes together with the upper (lower) sign. Similar equations hold for the spin-flip amplitudes $\bar{f}^{(0)}$ and $\bar{f}^{(1)}$.

From (II.3) we find now for the polarization $P^{(i)}$

$$P^{(i)} = 2 \frac{\text{Re } \bar{f}^{(i)}}{\text{Im } f_p} \quad (\text{IV.20})$$

where we have taken into account that f_p is purely imaginary and neglected terms which are bilinear in the small quantities $\bar{f}_p$, $f_{p'} + f_{p''}$, $\bar{f}_{p'} + \bar{f}_{p''}$, $f^{(2)}$ and $\bar{f}^{(2)}$, or smaller ones.

We observe first that the dominant contribution (IV.20) to polarization does not depend on $\bar{f}_p$ since this quantity is purely imaginary. This means that polarization will give very little information on Pomeron spin-flip: it will be seen in the next Section that the Wolfenstein parameters on the other hand are sensitive to this quantity.

Furthermore, experimentally, the polarizations for $\pi^+ p$ scattering are roughly of equal magnitude and opposite sign for the energies considered. This implies that for these energies

$$| \text{Re} (\bar{f}_{p'} + \bar{f}_{p''}) | \ll \frac{1}{\sqrt{2}} | \text{Re } \bar{f}^{(2)} |, \quad (\text{IV.21})$$

since $\text{Re}(\bar{f}_{p'} + \bar{f}_{p''})$ has the same sign for both reactions. This condition is not fulfilled if we keep only the term of leading order s^α in $\text{Re}(\bar{f}_{p'} + \bar{f}_{p''})$, which is the explicitly known term $\text{Re } \bar{f}_{p'}$. We therefore have to take this fact into account by keeping also $\text{Re } \bar{f}_{p''}$, which according to the considerations at the end of the last Section is taken to be of order s^0 . The condition (IV.21) can now be fulfilled by adjusting the unknown function $\bar{\phi}(t)$ contained in $\bar{f}_{p''}$, which was defined

at the end of the last Section. To fit present data, we choose $\bar{\rho}(t)$ such that $\text{Re}(\bar{f}_p + \bar{f}_{p''}) = 0$ for $q_L = 8 \text{ GeV/c}$, this choice being far from unique because of the large uncertainties in the data. It should be noted that no information concerning $\rho(t)$ is obtained from the present considerations. Figures 4a-4d show the corresponding values calculated from (IV.20) compared with the data at 6, 8, 10 and 12 GeV/c. This result shows that the dominant contribution (IV.20) to $P^{(0)}_1$ accounts for the sign and order of magnitude of the observed polarizations; quantitative agreement is improved by a more complete discussion of $P^{(0)}_1$.

We discuss now predictions of the present theory for higher energies. We restrict the consideration in the following to energies up to about 25 GeV/c, since for higher energies effects due to a possible modification in shrinkage of ρ and P' contributions are expected (cf., the corresponding discussion in I and in Section III; note that in the present framework, modifications in the ρ contribution automatically give rise to corresponding modifications in P'); the latter are neglected in the present discussion. In Fig. 5 we show the polarizations at 18 and 25 GeV/c calculated from (IV.20) with the choice of $\bar{\rho}(t)$ as described above. The change in sign of the $\pi^- p$ polarization $P^{(1)}$ between $t=0$ and $t=-0.6 (\text{GeV/c})^2$ and the corresponding zero moving with increasing energy towards larger values of t are due to the different energy dependences of $\bar{f}_p$ and $\bar{f}_{p''}$. With our choice of $\bar{\rho}(t)$, one has $\text{Re}(\bar{f}_p + \bar{f}_{p''}) \geq 0$ for $-0.6 (\text{GeV/c})^2 \leq t \leq 0$ and $q_L \geq 8 \text{ GeV/c}$; the interval where $P^{(1)} > 0$ is the region where the sign of $\pi^- p$ polarization is determined by $\text{Re}(\bar{f}_p + \bar{f}_{p''})$, while in the interval where $P^{(1)} < 0$, the sign is determined by $-1/\sqrt{2} \text{Re} \bar{f}^{(2)} \leq 0$. For $\pi^+ p \rightarrow \pi^+ p$ both terms are positive and $P^{(0)} > 0$ in the whole interval. Note that the zero of $P^{(0)}$ near $t = -0.5 (\text{GeV/c})^2$ in our results for $q_L = 8 \text{ GeV/c}$ is of similar origin. We realize that such a behaviour of polarizations with increasing energies is different from the usual Regge pole model ¹⁷⁾, where both polarizations stay of opposite sign in the t interval considered.

IV.5 The Wolfenstein parameters

In view of the forthcoming experiments by the Saclay group at CERN ⁴⁾ concerning the observation of polarization in the scattering plane for $\pi^{\pm}p$ elastic scattering, we indicate in this subsection briefly the implications of the overlap function analysis for the relevant parameters introduced by Wolfenstein ²⁰⁾. A detailed discussion of these quantities may be found, e.g., in Ref. ²¹⁾.

If the target nucleon has longitudinal polarization P_{initial} with respect to the incident beam, the transverse polarization P_{final} of the recoil nucleon in the scattering plane is

$$P_{\text{final}} = A_{\text{recoil}} \cdot P_{\text{initial}} \quad (\text{IV.22})$$

The directions of the polarization vectors are defined in Fig. 6. The same holds if the target nucleon has transverse polarization P_{initial} with respect to the incident beam and (see Fig. 6) :

$$P_{\text{final}} = R_{\text{recoil}} \cdot P_{\text{initial}} \quad (\text{IV.23})$$

A_{recoil} and R_{recoil} are the Wolfenstein parameters; they are defined in the laboratory system. A_{recoil} and R_{recoil} are expressed in terms of centre-of-mass quantities as follows ²¹⁾

$$R_{\text{recoil}} = \sin(\phi - \phi_L) \cdot A_{\text{CM}} + \cos(\phi - \phi_L) \cdot R_{\text{CM}} \quad (\text{IV.24})$$

$$A_{\text{recoil}} = -\cos(\phi - \phi_L) \cdot A_{\text{CM}} + \sin(\phi - \phi_L) \cdot R_{\text{CM}} \quad (\text{IV.25})$$

where ϕ = centre-of-mass recoil angle, ϕ_L = laboratory recoil angle, and

$$A_{\text{CM}} = 2 \frac{\text{Re}(f \bar{f}^*)}{|f|^2 + |\bar{f}|^2} \quad (\text{IV.26})$$

$$R_{\text{CM}} = \frac{|f|^2 - |\bar{f}|^2}{|f|^2 + |\bar{f}|^2} \quad (\text{IV.27})$$

From (IV.19) together with the corresponding expressions for spin-flip and taking into account that f_p and $\bar{f}_p$ are purely imaginary, we get for $\pi^\pm p$ scattering, respectively

$$A_{CM}^{(2)} = 2 \frac{1}{1 + \left(\frac{\text{Im} \bar{f}_p}{\text{Im} f_p} \right)^2} \frac{\text{Im} \bar{f}^{(2)}}{\text{Im} f_p} \quad (\text{IV.28})$$

$$R_{CM} = 1 - 2 \frac{1}{1 + \left(\frac{\text{Im} \bar{f}_p}{\text{Im} f_p} \right)^2} \left(\frac{\text{Im} \bar{f}^{(2)}}{\text{Im} f_p} \right)^2 \quad (\text{IV.29})$$

where we have kept only terms of order s^0 and $s^{\alpha-1}$. Terms of order $s^{2(\alpha-1)}$ and lower are neglected, as well as terms bilinear in the small quantities $(\text{Im} \bar{f}_p)^2$, $\text{Im}(f_p + f_{p''})$, $\text{Im}(\bar{f}_p + \bar{f}_{p''})$, $\text{Im} f^{(2)}$ and $\text{Im} \bar{f}^{(2)}$. We note that to leading order $A_{CM}^{(0,1)}$ and $R_{CM}^{(0,1)}$ and therefore $A_{\text{recoil}}^{(0,1)}$ and $R_{\text{recoil}}^{(0,1)}$ do not depend on the functions ϕ and $\bar{\phi}$.

We introduce now this result into (IV.24) and (IV.25) and keep at first only the terms which survive asymptotically and are seen to be identical for $\pi^\pm p$ scattering. Omitting the indices, this gives

$$R_{\text{recoil,as.}} = - \frac{\sqrt{-t}}{\sqrt{1 + \frac{-t}{4M^2}}} \left(1 - 2 \cdot 4M^2 B \frac{\bar{\mathcal{R}}'}{M} \frac{1 + 4M^2 B \frac{\bar{\mathcal{R}}' - t}{M} \frac{-t}{4M^2}}{1 + (4M^2 B \frac{\bar{\mathcal{R}}'}{M})^2 \frac{-t}{4M^2}} \right) \quad (\text{IV.30})$$

$$A_{\text{recoil,as.}} = \frac{1}{\sqrt{1 + \frac{-t}{4M^2}}} \left(1 + 2 \left[1 - 4M^2 B \frac{\bar{\mathcal{R}}'}{M} \right] \frac{4M^2 B \frac{\bar{\mathcal{R}}' - t}{M} \frac{-t}{4M^2}}{1 + (4M^2 B \frac{\bar{\mathcal{R}}'}{M})^2 \frac{-t}{4M^2}} \right) \quad (\text{IV.31})$$

For finite energies these asymptotic contributions are supplemented by corrections which are determined by (IV.24)-(IV.29). In Fig. 7 we show $R_{\text{recoil,as.}}$ and $A_{\text{recoil,as.}}$ for the values $\bar{\mathcal{R}}'/M = 0; \pm 0.02; \pm 0.04$. For $\bar{\mathcal{R}}'/M = \pm 0.1$ already the assumption of dominance of $\text{Im} f_p$ over $\text{Im} \bar{f}_p$ is no longer justified in the entire interval $0 \leq -t \lesssim 1 \text{ (GeV/c)}^2$. We see immediately that $R_{\text{recoil,as.}}$ and $A_{\text{recoil,as.}}$ are very sensitive to the parameter $\bar{\mathcal{R}}'/M$ and that especially the former quantity is very well suited for its experimental determination.

As a side remark, we mention that considerations in the context with the cross-over phenomenon [cf., end of Section III.3] favour positive values of $\bar{\alpha}'/M$ of order 0.05.

In Fig. 8, finally, we show $R_{\text{recoil}}^{(0,1)}$ and $A_{\text{recoil}}^{(0,1)}$ as they are given by (IV.24), (IV.25) for $q_L = 10, 18$ and 25 GeV/c; as an example, we have chosen $\bar{\alpha}'/M = 0.04$.

Note the relatively smooth variation of $R_{\text{recoil,as.}}$ and $A_{\text{recoil,as.}}$ in the interval of t considered, as compared to the situation in the usual Regge model ²¹⁾. This results from the difference in description of the Pommeranchuk part f_P and $\bar{f}_P$ of elastic scattering in the Regge model. In the latter considerable phase variations, together with important contributions from Pommeranchuk spin-flip, induce marked structure in these parameters. In the present model there are no phase variations in f_P and $\bar{f}_P$.

V. CONCLUDING REMARKS

It has been shown in the preceding Sections how the overlap function analysis of πp scattering (as it was developed in I) combined with dispersion relations leads to a description of polarization parameters accounting for present experimental findings. Furthermore, predictions for the Wolfenstein parameters A and R in elastic scattering have been found which are predominantly sensitive to the Pomeranchuk part of the elastic spin-non-flip and spin-flip amplitudes. These show marked differences with other models and should allow discrimination between different descriptions as soon as appropriate experimental information is available. As to predictions for the polarization P, we find an increasingly asymmetric behaviour in $\pi^{\pm}p \rightarrow \pi^{\pm}p$ with increasing energy. The behaviour of $\pi^{-}p \rightarrow \pi^{0}n$, as it is described by the formulae in Section III.4, will not be qualitatively different at higher energies from the one actually observed. It is important to note that with increasing energy the influence of an eventual modification of shrinkage in forward peaks, as predicted by our model, becomes more and more detectable; however, such changes being expected around 35 GeV/c (observe the remarks at the end of Section III.3), their influence on polarization is small up to about 25 GeV/c.

We recall that the basic assumptions of the present analysis of πp scattering data, beside general properties of relativistic S matrix theory, are the following :

- the $\pi^{+}p \rightarrow \pi^{+}p$ overlap functions are well approximated by their asymptotic values for energies $\gtrsim 6$ GeV/c;
- the $\pi^{-}p \rightarrow \pi^{-}p$ overlap functions have more energy variation above 6 GeV/c, but they have simple algebraic forms which can be interpreted in terms of different classes of inelastic transitions and absorptive corrections. They lead to a description of $\pi^{-}p \rightarrow \pi^{0}n$ in terms of φ Regge pole exchange supplemented by small corrections.

The overlap functions actually chosen in our analysis lead to cut-type corrections to reggeized ϱ exchange. For the even amplitude under crossing, our model leads to a description by the sum of a Pomeranchuk term, a P' term determined by the ϱ term and also supplemented by cut-type corrections, and a Pomeranchuk daughter term P'' operating as ghost compensator. While P and P'' correspond to fixed pole contributions for exchanged angular momenta $\alpha = 1$ and $\alpha = 0$, P' is of moving pole type with $\alpha = \alpha(t)$, where $\alpha(t)$ is the ϱ trajectory.

It has to be stressed that the presence or absence of increasingly asymmetric behaviour of the $\pi^+ p \rightarrow \pi^+ p$ polarization at higher energy will directly test the first of the above assumptions. The absence of such effects would indicate the existence of further sizeable contributions to the $\pi^+ p$ overlap function, for instance caused by classes of inelastic transitions with strangeness exchange between jets. Such a situation can easily be taken into account in the present overlap function framework and simply leads to an additional independent P' type term.

Finally, we mention that an interesting question still to be studied concerns the actual shapes of overlap functions which follow from concrete dynamical models of multiparticle production, in particular from the multi-Regge model^{22),23)}. Such shapes should be compared with the results of the present semi-phenomenological analysis and this comparison would undoubtedly lead to further tests of the multiparticle production mechanisms.

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A P P E N D I X A

We give the functions $j^{(2)}(t)$ and $j^{(3)}(t)$ determining the second and third order contribution in $\Delta B/B$ to $\Gamma_J(s,t)$:

$$j^{(2)}(t) = j_1 \left(\frac{2}{B} + 3t + Bt^2 + \frac{1}{6} B^2 t^3 \right) \quad (\text{A.1})$$

$$j^{(3)}(t) = j_1 \left(\frac{3}{B} + 4t + \frac{3}{2} Bt^2 + \frac{1}{3} B^2 t^3 + \frac{1}{24} B^3 t^4 \right) \quad (\text{A.2})$$

For the spin-flip quantity, $\overline{\Gamma}_J(s,t)$, the corresponding functions are

$$\overline{j}^{(1)}(t) = \overline{j}_1 \left(\frac{7}{B} + 6t + \frac{3}{2} Bt^2 + \frac{1}{6} B^2 t^3 \right) \quad (\text{A.3})$$

$$\overline{j}^{(3)}(t) = \overline{j}_1 \left(\frac{12}{B} + 10t + 3 Bt^2 + \frac{1}{2} B^2 t^3 + \frac{1}{24} B^3 t^4 \right) \quad (\text{A.4})$$

A P P E N D I X B

In this Appendix we first summarize the explicit calculation of $\text{Re } f^{(2)}$ and $\text{Re } \bar{f}^{(2)}$, i.e., of $C^{(n)}(s,t)$ as given by (III.26). Then we consider the real parts for elastic scattering, i.e., $C_P(s,t)$ and $C_{\bar{P}}(s,t)$ as given by (IV.14).

We calculate the first two integrals in (III.26) by putting the lower limit to zero and correct by subtracting the integrals from zero to s_0/s . With the residue calculus, one finds ²⁴⁾ in the case $n=0$,

$$\int_0^{\infty} dx \frac{x^{\alpha-1}}{x \pm 1} = \begin{cases} \pi \frac{1}{\sin \pi \alpha} & , \text{ for the } + \text{ sign} \\ -\pi \text{ctg } \pi \alpha & , \text{ for the } - \text{ sign} , \end{cases} \quad (\text{B.1})$$

$$0 < \text{Re } \alpha < 1 ,$$

and where in the case of the $-$ sign the principal value of the integral is understood. This gives the first term in the expression (III.31) for $C^{(0)}(s,t)$ ²⁵⁾. It has to be corrected by subtraction of

$$\frac{2}{\pi} \int_0^{s_0/s} dx \frac{x^{\alpha(t)}}{x^2 - 1} = -\frac{2}{\pi} \sum_{k=0}^{\infty} \frac{1}{\alpha(t) + 2k + 1} \left(\frac{s_0}{s}\right)^{2k} \cdot \left(\frac{s_0}{s}\right)^{\alpha(t) + 1} \quad (\text{B.2})$$

because of its smallness this correction has been neglected in (III.31).

For $n \geq 1$ the procedure is analogous. Beside the power $x^{\alpha-1}$ we have now in addition powers of $\log x$ with positive integer exponents. The corresponding integrals of type (B.1) can be calculated by partial differentiations of (B.1) with respect to α . For $n=1$ we obtain in this way the first two terms of (III.32). The correction to be subtracted is

$$\begin{aligned} & \frac{2}{\pi} \int_0^{s_0/s} dx \frac{(\Delta B(s) - \alpha_1 (\log x)) x^{\alpha(t)}}{x^2 - 1} = \\ & = -\frac{2}{\pi} \left(\frac{s_0}{s}\right)^{\alpha(t)+1} \sum_{k=0}^{\infty} \left(\Delta B_0 + \frac{\alpha_1}{\alpha(t)+2k+1} \right) \frac{1}{\alpha(t)+2k+1} \left(\frac{s_0}{s}\right)^{2k}, \quad (B.3) \end{aligned}$$

and has again been neglected (for higher n the corrections of this type become more important and have also to be taken into account for values of s which are not sufficiently high).

The last two integrals in (III.26) finally give for $n=0, 1$ rise to the second term in (III.31) and to the third term in (III.32). They are easily obtained by introducing again power series for the denominators x^2-1 . The same applies for higher n .

We now proceed in the same way for elastic scattering. Applying again formulae (B.1), we find for $C_j(s, t)$ as given by (IV.12)

$$C_j(s, t) = -\frac{\cos \pi \alpha_j + 1}{\sin \pi \alpha_j} + \frac{2}{\pi} \left(\frac{s_0}{s}\right)^{\alpha_j} \sum_{k=0}^{\infty} \frac{1}{\alpha_j + 2k} \left(\frac{s_0}{s}\right)^{2k}. \quad (B.4)$$

Again the first term originates from the integration from 0 to ∞ , the second is the correction from the interval 0 to s_0 . For $\alpha_j = \alpha_P = 1$, the first term vanishes and we have the result (IV.14). For $\alpha_j = \alpha_P = \alpha$, we find equation (IV.15).

As to the ghost killing mechanism, which the present theory provides to kill the P' ghost at $\alpha_{P'} = \alpha = 0$ and which was already mentioned in Section IV.3, we realize that the P' Regge pole chooses sense at $\alpha = 0$ and the ghost is killed by an additive compensating term (see also end of Appendix D). This is an alternate mechanism to what is usually taken for sense choosing trajectories, where one kills the ghost according to the Chew mechanism in a multiplicative way ²⁶⁾. For nonsense choosing trajectories, the Gell-Mann mechanism ²⁷⁾ also works with additive compensation, but the nature of the compensating term is different [also in this case we have with the no-compensation-mechanism a multiplicative version of ghost killing ²⁸⁾].

A P P E N D I X C

To find the function $\mathcal{G}(Bt)$ defined by (IV.1), one first writes the right-hand side of (II.10), where according to (II.16) $G^{(0)} \rightarrow G$, as a power series in G ,

$$\operatorname{Im} h^{(0)}(b) = \frac{1}{2} \sum_{m=1}^{\infty} C_m \left(\frac{G}{2}\right)^m, \quad (C.1)$$

$$C_m = \begin{cases} 1, & m=1 \\ \frac{1 \cdot 3 \cdot 5 \cdots (2m-3)}{m!}, & m \geq 2 \end{cases}$$

From (II.4), together with the high energy approximation $2q\sqrt{s} \approx s$ follows now

$$\mathcal{G}(Bt) = \sum_{m=1}^{\infty} \frac{C_m}{m} \left(\frac{P}{2}\right)^m e^{\frac{B}{m}t}, \quad (C.2)$$

where the relation following (III.1), with $B \rightarrow B/m$, has been applied. Defining

$$\mathcal{G}(Bt) = \mathcal{G}(0) e^{\chi(Bt)}, \quad (C.3)$$

one finds first

$$\mathcal{G}(0) = \sum_{m=1}^{\infty} \frac{C_m}{m} \left(\frac{P}{2}\right)^m = 2 \left\{ 1 - \sqrt{1-P} - \log \left(2 \frac{1 - \sqrt{1-P}}{P} \right) \right\}. \quad (C.4)$$

Writing $\chi(Bt)$ as a power series in Bt results in

$$\chi(Bt) = \sum_{n=1}^{\infty} a_n (Bt)^n, \quad (C.5)$$

where the coefficients are given by

$$a_n = \frac{1}{n!} \left. \frac{d^n (\log \mathcal{Y}(Bt))}{d(Bt)^n} \right|_{Bt=0}, \quad n \geq 1. \quad (C.6)$$

With this formula the coefficients a_n are determined explicitly in terms of the parameter P . Detailed investigation shows that a_n decreases rapidly with n . For our parameter values $P=0.91$ and $B=4.86 (\text{GeV}/c)^{-2}$ resulting from the Cottingham-Peierls analysis ²⁹⁾ of asymptotic diffraction scattering, one finds

$$\begin{aligned} a_1 &= 0.91, & a_1 B &= 4.42 (\text{GeV}/c)^{-2}, \\ a_2 &= 0.02, & a_2 B^2 &= 0.56 (\text{GeV}/c)^{-4}. \end{aligned}$$

The parameters P and B are also directly related to total cross-sections. From the optical theorem, $\sigma_T^{(0)} = (4\pi/q\sqrt{s}) \text{Im} f^{(0)}(s,0)$, follows for the total cross-section $\sigma_T^{(0)}$

$$\sigma_T^{(0)} = 8\pi B \mathcal{Y}(0), \quad (C.7)$$

where $\mathcal{Y}(0)$ is given by (C.4). The total inelastic cross-section $\sigma_{\text{inel}}^{(0)}$ is

$$\begin{aligned} \sigma_{\text{inel}}^{(0)} &= \frac{4\pi}{q\sqrt{s}} \cdot 2q\sqrt{s} \int_0^\infty db b \frac{1}{4} G(b) \mathcal{Y}(0) \\ &= 4\pi B P, \end{aligned} \quad (C.8)$$

which may be related by $\sigma_T^{(0)} = \sigma_{\text{el}}^{(0)} + \sigma_{\text{inel}}^{(0)}$ to the total elastic cross-section $\sigma_{\text{el}}^{(0)}$.

The procedure for $\text{Im} \bar{f}^{(0)}(s,t)$ is analogous. Developing (II.11) with $G^{(0)} \rightarrow G$, in a power series in G results together with (II.17) in

$$\text{Im} \bar{h}^{(0)}(b) = \bar{x}' b \frac{1}{2} \sum_{m=1}^{\infty} m C_m \left(\frac{G}{2}\right)^m. \quad (\text{C.9})$$

If this expression is introduced into (II.5) we find after application of the relation following (III.8), with $B \rightarrow B/m$, the result (IV.2). Note the cancellation of the extra factor m in (C.9) by the factor $(B/m)^2$ leading to the factor $\mathcal{Y}(Bt)$ also in the case of spin-flip.

A P P E N D I X D

In this Appendix we first give a formulation of our theory which accounts explicitly for crossing to all orders. Then we consider the crossing even amplitudes defined in Section IV.2, discuss briefly superconvergence type restrictions for the functions $\phi(t)$ and $\bar{\phi}(t)$ introduced at the end of Section IV.3, and study the consequences of the assumption that the asymptotic expansion contains only pure power terms. Finally, we add some remarks on the crossing odd amplitudes defined in Section IV.2, i.e., $f^{(-)} = (1/\sqrt{2})f^{(2)}$ and $\bar{f}^{(-)} = (1/\sqrt{2})\bar{f}^{(2)}$, where $f^{(2)}$, $\bar{f}^{(2)}$ are the amplitudes studied in Section III.

To account explicitly for crossing, it is convenient to introduce in place of s the variable

$$\nu = \frac{1}{2}(s-u) = S - M^2 - \mu^2 + \frac{t}{2} ; \quad (D.1)$$

$s \rightarrow \infty$ implies $\nu \rightarrow \infty$ in the diffraction region and the crossing operation $s \leftrightarrow u$ now means $\nu \leftrightarrow -\nu$. In the same way as before, it can be shown that for our amplitudes f , $\bar{f}$ for high energy, i.e., for $\nu \rightarrow \infty$,

$$f(\nu, t) = \frac{2M}{8\pi} \left\{ A(s, t, u) + \frac{\nu}{2M} B(s, t, u) \right\} , \quad (D.2)$$

$$\bar{f}(\nu, t) = \frac{2M}{8\pi} \sqrt{\frac{-t}{4M^2}} A(s, t, u) , \quad (D.3)$$

where A and B are the invariant amplitudes and further indices will be specified later. To study this leading term for $\nu \rightarrow \infty$ systematically we consider analogously to (III.17) the functions

$$\tilde{f}(\nu, t) \equiv \frac{2M}{8\pi} \left\{ A(s, t, u) + \frac{\nu}{2M} B(s, t, u) \right\} , \quad (D.4)$$

$$\tilde{f}(\nu, t) = \frac{2i\gamma}{8\pi} \sqrt{\frac{-t}{4m^2}} A(s, t, u), \quad (D.5)$$

which are defined in the whole ν plane. The crossing relations are similar to (III.18), i.e., $\tilde{f}(-\nu, t) = \tilde{f}_c(\nu, t)$ and $\tilde{f}(-\nu, t) = \tilde{f}_c(\nu, t)$, where the first relation is now exact and $\tilde{f}_c, \tilde{f}_c$ are again u channel amplitudes.

We consider now the crossing even amplitudes and concentrate on $\tilde{f}^{(+)}$, the discussion of $\tilde{f}^{(-)}$ being analogous. We define the high energy part $\tilde{f}_{HE}^{(+)}(\nu, t)$ we are interested in explicitly and which will later be assumed to be given by a sum of powers in ν by

$$\tilde{f}^{(+)}(\nu, t) = \tilde{f}_{HE}^{(+)}(\nu, t) + \tilde{f}^{(+)\prime}(\nu, t), \quad (D.6)$$

where, for $\nu \rightarrow \infty$

$$\tilde{f}^{(+)\prime}(\nu, t) = O(\nu^{-\varepsilon}; t), \quad \varepsilon > 0, \quad (D.7)$$

and $\tilde{f}^{(+)}$ is assumed to be negligible in the description of high energy phenomena in the diffraction region. If (D.6) is introduced into the (exact) dispersion relation for $\tilde{f}^{(+)}$,

$$\text{Re } \tilde{f}^{(+)}(\nu, t) = \tilde{f}^{(+)}(0, t) + \tilde{f}_{p.t.}^{(+)}(\nu, t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \tilde{f}^{(+)}(\nu', t)}{\nu'} \frac{1}{\nu'^2 - \nu^2}, \quad (D.8)$$

where $\tilde{f}_{p.t.}^{(+)}$ is the pole term, the decomposition corresponding to (D.6) is found to be

$$\text{Re } \tilde{f}^{(+)}(\nu, t) = \phi(t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \tilde{f}_{HE}^{(+)}(\nu', t)}{\nu'} \frac{1}{\nu'^2 - \nu^2} + O(\nu^{-\varepsilon}; t), \quad (D.9)$$

$$\phi(t) = \tilde{f}^{(+)}(0,t) + \tilde{f}_{p.t.}^{(+)}(t) - \frac{2}{\pi} \int_{\gamma_0}^{\infty} d\nu' \frac{\text{Im}(\tilde{f}^{(+)}(\nu',t) - \tilde{f}_{HE}^{(+)}(\nu',t))}{\nu'} \quad (D.10)$$

where $\phi(t)$ and the corresponding quantity $\bar{\phi}(t)$ for spin-flip are identical with the functions introduced at the end of Section IV.3, and $\tilde{f}_{p.t.}^{(+)}(t)$ is the leading term of $\tilde{f}_{p.t.}^{(+)}(\nu,t)$ for $\nu \rightarrow \infty$. This result is obtained by putting in the dispersion integral (D.8) corresponding to $\text{Im} \tilde{f}^{(+)}$,

$$\frac{2\nu^2}{\nu'} \frac{1}{\nu'^2 - \nu^2} = \frac{1}{\nu' - \nu} + \frac{1}{\nu' + \nu} - \frac{2}{\nu'} \quad ; \quad (D.11)$$

the third term gives rise to the integral in (D.10), while the first two terms contribute to $\phi_1(\nu^{-\epsilon};t)$ in (D.9). Equation (D.10) is a superconvergence type relation for $\tilde{f}^{(+)}$. Since $\phi(t)$ is of order ν^0 , observables at high energy should be sensitive to this quantity according to our assumption stated after (D.7), i.e., we use $\phi(t)$ to fit high energy experiments. The corresponding form of $\phi(t)$ has to be consistent with (D.10). Note that $\tilde{f}^{(+)}(0,t)$ and the low energy part of the integral (D.10) are determined from lower energy experiments.

We introduce now the assumption that $\tilde{f}_{HE}^{(+)}(\nu,t)$ is given by a sum of pure powers ¹⁹⁾, as was motivated at the end of Section IV.3, i.e.,

$$\tilde{f}_{HE}^{(+)}(\nu,t) = \sum_j \phi_j(t) \left(\frac{\nu}{m^2}\right)^{\alpha_j(t)} \quad (D.12)$$

Since $\tilde{f}_{HE}^{(+)}$ is even under crossing, we have

$$\phi_j(t) = \pm |\phi_j(t)| e^{-i \frac{\pi \alpha_j(t)}{2}} \quad (D.13)$$

For $j = P, P', P''$, with $\alpha_P = 1$, $\alpha_{P'} = \alpha$, $\alpha_{P''} = 0$, ϕ_P becomes imaginary and $\phi_{P''}$ real. Therefore

$$\text{Im } f_{HE}^{(j)}(v, t) = \text{Im } \phi_P(t) \frac{v}{M^2} + \text{Im } \phi_{P'}(t) \left(\frac{v}{M^2} \right)^{\alpha(t)}, \quad (\text{D.14})$$

where $\text{Im } \phi_P$ and $\text{Im } \phi_{P'}$ are determined by identifying the terms of order s/M^2 and $(s/M^2)\alpha(t)$ with (IV.7). If we put (D.14) into the dispersion integral (D.9), we find for the term of order s/M^2 the result (IV.16), for the term of order $(s/M^2)\alpha(t)$ the expression (IV.17). For $f_{P''}(s, t)$ we have

$$\begin{aligned} f_{P''}(s, t) = & \phi(t) + \frac{2}{\pi \alpha(t)} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \gamma(t) \left(\frac{v_0}{M^2} \right)^{\alpha(t)} + \\ & + \left(\frac{2}{\pi} \frac{v_0}{M^2} + i \frac{\frac{1}{2}t - M^2 - \mu^2}{M^2} \right) M^2 B \gamma(Bt) + O_1(v^{-\varepsilon}, t). \end{aligned} \quad (\text{D.15})$$

The function $\phi_{P''}(t)$ is then given by

$$\phi_{P''}(t) = \phi(t) + \frac{2}{\pi \alpha(t)} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi}} \varphi(t) \gamma(t) \left(\frac{v_0}{M^2} \right)^{\alpha(t)} + \frac{2}{\pi} M^2 B \gamma(Bt) \frac{v_0}{M^2} \quad (\text{D.16})$$

and corresponds to the contribution of a Pomeranchuk daughter P'' which carries exchanged angular momentum $\alpha = 0$ and which acts as ghost compensator, as was already mentioned before. We recall that similar results hold for spin-flip.

For the crossing odd amplitude $\tilde{f}^{(-)}$ [again the discussion of $\tilde{f}^{(-)}$ is analogous], we define the high energy part similarly to (D.6) and (D.7), together with the same assumption following the latter equation (ε has not necessarily to be the same). At high energy, the (exact) dispersion relation for $\tilde{f}^{(-)}$,

$$\operatorname{Re} \tilde{f}^{(-)}(\nu, t) = \tilde{f}_{\text{p.t.}}^{(-)}(\nu, t) + \frac{2}{\pi} \nu \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} \tilde{f}^{(-)} \frac{1}{\nu'^2 - \nu^2} \quad (D.17)$$

where the pole term $\tilde{f}_{\text{p.t.}}^{(-)}$ is $\mathcal{O}(\nu^{-1}; t)$, becomes

$$\operatorname{Re} \tilde{f}^{(-)}(\nu, t) = \frac{2}{\pi} \nu \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} \tilde{f}_{\text{HE}}^{(-)}(\nu', t) \frac{1}{\nu'^2 - \nu^2} + \mathcal{O}_2(\nu^{-\varepsilon}; t) \quad (D.18)$$

This result is found by introducing into (D.17) the decomposition analogous to (D.6) and putting in the dispersion integral corresponding to $\operatorname{Im} \tilde{f}^{(-)}$,

$$2\nu \frac{1}{\nu'^2 - \nu^2} = \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \quad (D.19)$$

If $\varepsilon > 1$, we now get superconvergence type relations for $\tilde{f}^{(-)}$.

Finally, (D.18) shows that there are no terms of order s^0 for $\pi^- p \rightarrow \pi^0 n$, as was already remarked in Section III.2; furthermore, if the leading order in $\tilde{f}_{\text{HE}}^{(-)}$ is s^α , there are well-determined corrections of order $s^{\alpha-1}$.

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FIGURE CAPTIONS

- Figure 1 : $\mathcal{P}(t)$ and $(\Delta B/B) \mathcal{P}^{(1)}(t)$ at 6, 11 and 18 GeV/c [according to Section III.3) the value $\mathcal{P}_1 = 1.77 \text{ (GeV/c)}^2$ is taken for the slope of $\mathcal{P}(t)$; the other parameters have the values given in I].
- Figure 2 : $\overline{\mathcal{P}}(t)$ and $(\Delta B/B) \overline{\mathcal{P}}^{(1)}(t)$ at 6, 11 and 18 GeV/c (parameter values from I).
- Figure 3 : $\pi^- p \rightarrow \pi^0 n$ polarization at 6(a) and 11(b) GeV/c (Ref. 2) compared to the values calculated from (III.35).
- Figure 4 : $\pi^\pm p \rightarrow \pi^\pm p$ polarization at 6(a), 8(b), 10(c) and 12(d) GeV/c (Ref. 3) compared to the contribution (IV.20) to polarization (parameter values from I and Appendix C).
- Figure 5 : Predictions for $\pi^\pm p \rightarrow \pi^\pm p$ polarization from the contribution (IV.20) to $P^{(0)}_1$ at 10, 18 and 25 GeV/c.
- Figure 6 : Vectors used to define the Wolfenstein parameters in (IV.22), (IV.23).
- Figure 7 : $R_{\text{recoil,as.}}$ (a) and $A_{\text{recoil,as.}}$ (b) for $\pi^\pm p \rightarrow \pi^\pm p$ predicted by (IV.30) and (IV.31) for $\overline{\mathcal{R}}'/M = 0, \pm 0.02, \pm 0.04$.
- Figure 8 : Predictions for R_{recoil} (a),(c) and A_{recoil} (b),(d) in $\pi^\pm p \rightarrow \pi^\pm p$ from (IV.24), (IV.25) and the contributions (IV.28), (IV.29) to R_{CM} and A_{CM} for the example of $\overline{\mathcal{R}}'/M = +0.04$.

FIG.1

FIG.2

FIG. 3a

FIG. 3b

FIG. 4a

FIG. 4b

FIG. 4c

FIG. 4d

FIG 5

FIG. 6

FIG.7a

FIG.7b

FIG. 8a

FIG. 8b

FIG. 8c

FIG. 8d