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REMARKS CONCERNING THE POSSIBILITY OF FINITE CHARGE
RENORMALIZATION IN SPINOR ELECTRODYNAMICS

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A B S T R A C T

The analysis of spinor electrodynamics, performed by Johnson, Willey and Baker, is examined by use of the renormalization group. It is concluded that the vanishing of the coefficient of the $\log(m/M)$ in Z_3^{-1} is necessary, but not sufficient, for Z_3^{-1} to be finite. The mass assumption of the renormalization group is discussed, and explicitly verified through sixth order.

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1. INTRODUCTION

The question whether Z_3 , the charge renormalization factor of spinor electrodynamics, can be finite has been recently re-examined by Johnson, Willey and Baker ¹⁾. (Although their investigation is performed in the context of their broader programme of rendering all the renormalization parameters of spinor electrodynamics finite, it can be viewed independently of this programme.) These authors study the unrenormalized vacuum polarization tensor (which determines Z_3) under the approximation that the photon propagator can be replaced by its uninteracting (or equivalently high energy asymptotic) expression. They find that in this model the divergent part of $Z_{3,model}^{-1}$ has the form

$$\text{divergent part } Z_{3,model}^{-1} = f(\alpha_0) \log \frac{M^2}{m^2}. \quad (1.1)$$

Here m is the fermion mass, M is an ultra-violet cut-off, and α_0 is related to the unrenormalized coupling constant: $\alpha_0 = e_0^2/8\pi^2 > 0$. Thus if α_0 can be chosen to be a positive zero of f , $Z_{3,model}$ becomes finite and cut-off independent. Johnson et al. then argue that Z_3 in the exact theory will also be finite and cut-off independent if α_0 is chosen to be a positive zero of f , viz, of the function determined by the model and defined by (1.1). (It is also necessary that the slope of f at this zero be negative.)

No information about f or about its zeros is available. In perturbation theory, $f(\alpha_0)$ has been calculated through sixth order by Rosner ²⁾, who obtains

$$f(\alpha_0) = \frac{2}{3}\alpha_0 + \alpha_0^2 - \frac{1}{4}\alpha_0^3 + O(\alpha_0^4). \quad (1.2)$$

It is then argued that the fact that the coefficient of the cubic term is negative gives hope that the complete f possesses an appropriate zero.

The idea that charge renormalization in spinor electrodynamics is finite due to the vanishing of a particular function of the unrenormalized coupling constant, is an old one, going back to Pauli. More recently, the methods of the renormalization group were used by Gell-Mann and Low ³⁾ to demonstrate the feasibility of such a state of affairs. The purpose of this paper is to use the renormalization group to study the behaviour of Z_3^{-1} in perturbation theory, thus approaching the results of Johnson et al. from another point of view. Our conclusions, however, are different from those of the above investigation. We find that when the asymptotic form of the unrenormalized photon propagator is just its free form, then the vanishing of the coefficient of the first power of the cut-off dependent logarithm in Z_3^{-1} is necessary but not sufficient for Z_3^{-1} to be finite. The appropriate necessary and sufficient condition which follows from the renormalization group is such that a study of $f(\alpha_0)$ cannot shed any light on the finiteness of charge renormalization. By reference to perturbation theory, we demonstrate that the occurrence of the negative number in Rosner's sixth order calculation has little bearing on the finiteness of Z_3^{-1} . In Section 2 we present the method of the renormalization group in a non-standard fashion, particularly suited for our purposes. In Section 3, we derive our result for Z_3^{-1} . Section 4 is devoted to a discussion of the validity of our application of the renormalization group.

2. RENORMALIZATION GROUP FOR THE PHOTON PROPAGATOR

A. The complete, renormalized photon propagator of spinor electrodynamics has the form

$$D_{\mu\nu}(k) = \left(g_{\mu\nu} - k_\mu k_\nu / k^2 \right) \frac{d\left(\frac{k^2}{m^2}, \alpha\right)}{k^2} + k_\mu k_\nu S(k) / k^4. \quad (2.1)$$

Here α is related to the renormalized charge : $\alpha = e^2/8\pi^2$. The function d , which will be our main object of study, is a dimensionless, Lorentz and gauge invariant, and therefore depends on k only through the combination k^2/m^2 . It has the property that $d(0, \alpha) = 1$. The function δ is the gauge dependent part of the photon propagator with which we shall not be concerned. The unrenormalized quantities, which are calculated in perturbation theory, can be obtained from the renormalized quantities in the following fashion. Since the unrenormalized quantities diverge in perturbation theory, a cut-off parameter M must be introduced to render them finite. Setting the cut-off dependent charge renormalization constant Z_3^{-1} to be

$$Z_3^{-1} = d\left(\frac{M^2}{m^2}, \alpha\right), \quad (2.2)$$

we define the unrenormalized cut-off dependent propagator function d_0 by

$$d_0\left(\frac{k^2}{M^2}, \frac{m^2}{M^2}, \alpha_0\right) = Z_3 d\left(\frac{k^2}{m^2}, \alpha\right) \quad (2.3a)$$

$$\alpha_0 = Z_3^{-1} \alpha. \quad (2.3b)$$

Explicitly, Eqs. (2.3) give the following rule for calculating d_0 . According to (2.3b), form the quantity $\alpha d(M^2/m^2, \alpha) \equiv \alpha_0$ and invert this to give α in terms of α_0 and M^2/m^2 . (A unique inverse is determined by perturbation theory.) Substitute this value for α in the ratio $d(k^2/m^2, \alpha)/d(M^2/m^2, \alpha)$, obtaining, according to (2.3a), $d_0(k^2/M^2, m^2/M^2, \alpha_0)$. Thus we have defined a three-variable object d_0 from a two-variable object d . However, we may now consider the three variables in d_0 to be independent. Evidently d_0 is normalized to unity at $k^2 = M^2$.

4.

Z_3 , d and d_0 will have terms that go to zero as k^2 and M^2 go to infinity. Let us ignore such terms, concentrating only on the asymptotic portions which remain non-zero in the large k^2, M^2 limit. We denote these asymptotic terms by the superscript ∞ . However, we do not modify the notation for Z_3^{-1} , since conventionally this quantity is defined as the asymptotic part of d . Next we make the crucial assumption wherein lies all the physics: we assume that d_0^∞ is independent of m^2/M^2 . This assumption is based on experience with perturbation theory. Although Gell-Mann and Low give an argument for the validity of this assumption to all orders of perturbation theory, explicit calculation provides a verification only through fourth order. We shall present in Section 4 an explicit sixth order calculation which verifies the assumption to this order, and we shall comment on the status of this assumption to all orders.

B. The above mass assumption converts the definitions (2.2) and (2.3) into functional equations

$$d_0^\infty\left(\frac{k^2}{M^2}, 0, \alpha_0\right) = \frac{d^\infty\left(\frac{k^2}{m^2}, \alpha\right)}{d^\infty\left(\frac{M^2}{m^2}, \alpha\right)} \quad (2.4a)$$

$$\alpha_0 = \alpha d^\infty\left(\frac{M^2}{m^2}, \alpha\right) \quad (2.4b)$$

$$Z_3^{-1} = d^\infty\left(\frac{M^2}{m^2}, \alpha\right). \quad (2.4c)$$

Equations (2.4a) and (2.4b) are solved by Gell-Mann and Low. Particularly useful for us are the integral representations given by them

$$\log \frac{k^2}{M^2} = \int_{\alpha_0}^{\alpha_0} \alpha_0 d_0^{\infty} \left(\frac{k^2}{M^2}, 0, \alpha_0 \right) \frac{dx}{x g(x)} \quad (2.5a)$$

$$g(x) \equiv \frac{\partial}{\partial z} d_0^{\infty}(z, 0, x) \Big|_{z=1} \quad (2.5b)$$

$$\log \frac{k^2}{m^2} = \int_{q(\alpha)}^{\alpha} \alpha d^{\infty} \left(\frac{k^2}{m^2}, \alpha \right) \frac{dx}{x g(x)} \quad (2.6a)$$

$$q(\alpha) \equiv \alpha d^{\infty}(1, \alpha). \quad (2.6b)$$

It is seen that the unrenormalized, asymptotic propagator function d_0^{∞} is completely determined by one function $g(x)$, while the renormalized, asymptotic propagator function d^{∞} requires two functions for a complete determination: $g(x)$ and $q(\alpha)$.

The expression (2.6a) exhibits the important result of Gell-Mann and Low about the possible finiteness of the unrenormalized charge. As k^2 approaches infinity, the left-hand side of (2.6a) diverges, while the right-hand side becomes:

$$\int_{q(\alpha)}^{\alpha_0} \frac{dx}{x g(x)}$$

The integral must diverge; the only way this can happen for finite α_0 is if $g(\alpha_0) = 0$. Thus charge renormalization can be finite when g develops a zero.

The consequence of this state of affairs for perturbation theory can be exhibited by using an expansion for d_0^∞ which follows from (2.5a) and which is given by Gell-Mann and Low

$$d_0^\infty\left(\frac{k^2}{M^2}, 0, \alpha_0\right) = \frac{1}{\alpha_0} \sum_{n=0} \frac{\log^n \frac{k^2}{M^2}}{n!} \left[\alpha_0 g(\alpha_0) \frac{d}{d\alpha_0} \right]^n \alpha_0. \quad (2.7)$$

It is seen that all terms beyond the first in the right-hand side of (2.7) have as a common factor g . Hence, when this vanishes, $d_0^\infty = 1$, which is what one expects for the unrenormalized asymptotic propagator, if it is finite when the cut-off goes to infinity. We give another useful representation for d_0^∞ . We define the proper part ρ by

$$d_0^\infty\left(\frac{k^2}{M^2}, 0, \alpha_0\right) = \left[1 - \rho\left(\frac{k^2}{M^2}, \alpha_0\right) \right]^{-1}. \quad (2.8a)$$

Then the form for ρ , which follows from (2.5a), is determined by inverting the series

$$\log \frac{k^2}{M^2} = \sum_{m=1} \frac{\rho^m}{m!} \left(\frac{d}{d\alpha_0} \right)^{m-1} \frac{\alpha_0^{m-1}}{g(\alpha_0)}. \quad (2.8b)$$

Accordingly, we get :

$$\rho\left(\frac{k^2}{M^2}, \alpha_0\right) = g(\alpha_0) \log \frac{k^2}{M^2} \times \quad (2.8c)$$

$$\left[1 + \frac{1}{2!} \alpha_0^2 \frac{d}{d\alpha_0} \frac{g(\alpha_0)}{\alpha_0} \log \frac{k^2}{M^2} + \frac{1}{3!} \left(\alpha_0^2 \frac{d}{d\alpha_0} \frac{g(\alpha_0)}{\alpha_0} \right)^2 + \alpha_0^2 g''(\alpha_0) g(\alpha_0) \log^2 \frac{k^2}{M^2} + \dots \right].$$

Again it is seen that when $g = 0$, $\rho = 0$.

3. Z_3^{-1} ACCORDING TO THE RENORMALIZATION GROUP

A. We now use the above formalism to obtain an expression for Z_3^{-1} . A formula for Z_3^{-1} in terms of renormalized quantities is given by (2.4c). For present purposes, we want Z_3^{-1} in terms of unrenormalized parameters. To arrive at such an expression, we use (2.3) (in the asymptotic region and making the usual assumption about mass dependence) to get

$$\alpha_0 d_0^\infty\left(\frac{k^2}{M^2}, 0, \alpha_0\right) = \alpha d^\infty\left(\frac{k^2}{M^2}, \alpha\right) \quad (3.1a)$$

$$Z_3^{-1} = \frac{\alpha_0}{\alpha} \quad (3.1b)$$

From (2.6b) and (3.1a), we have

$$\alpha_0 d_0^\infty\left(\frac{M^2}{M^2}, 0, \alpha_0\right) = g(\alpha). \quad (3.2)$$

8.

Defining Q to be the function inverse to q , $Q(q(\alpha)) = \alpha$, we have from (3.1b) :

$$Z_3^{-1} = \frac{\alpha_0}{Q\left[\alpha_0 d_0^{\infty}\left(\frac{m^2}{M^2}, 0, \alpha_0\right)\right]} \quad (3.3)$$

It is seen therefore that Z_3^{-1} is determined by the two functions $q(\alpha)$ and $\phi(x)$.

To obtain some understanding of this formula, recall that Z_3 is also given by the residue at $k^2 = 0$ of the unrenormalized photon propagator

$$Z_3 = d_0\left(0, \frac{m^2}{M^2}, \alpha_0\right) \quad (3.4)$$

Note that d_0 here is not in its asymptotic form. In using the asymptotic formulae of the renormalization group, we set the second variable in d_0 to zero, viz., $m^2 = 0$. Therefore one might expect that

$$d_0\left(0, \frac{m^2}{M^2}, \alpha_0\right) \approx d_0^{\infty}\left(\frac{m^2}{M^2}, 0, \alpha_0\right) \quad (3.5a)$$

$$Z_3 \approx d_0^{\infty}\left(\frac{m^2}{M^2}, 0, \alpha_0\right) \quad (3.5b)$$

But (3.5b) is precisely the expression that follows from (3.3) when $d^{\infty}(1, \alpha) = 1$, i.e., $Q(\alpha) = \alpha$:

$$Z_3 \Big|_{d^{\infty}(1, \alpha) = 1} = d_0^{\infty}\left(\frac{m^2}{M^2}, 0, \alpha_0\right) \quad (3.5c)$$

Therefore the formula (3.3) includes the corrections to (3.5b) which arise from the fact that (3.5a) does not hold exactly. It is interesting that these corrections manifest themselves as departures from unity of $d^0(1, \alpha)$.

Substituting the series (2.7) into (3.3), we get

$$Z_3^{-1} = \frac{\alpha_0}{Q(\alpha_0)} + g(\alpha_0) \log \frac{m^2}{M^2} \\ \times \left[\alpha_0^2 \frac{d}{d\alpha_0} \frac{1}{Q(\alpha_0)} + \frac{1}{2!} \alpha_0^2 \frac{d}{d\alpha_0} \left\{ \alpha_0 g(\alpha_0) \frac{d}{d\alpha_0} \frac{1}{Q(\alpha_0)} \right\} \right. \\ \left. \times \log \frac{m^2}{M^2} + \dots \right]. \quad (3.6)$$

This is our final result for the charge renormalization constant as determined by the renormalization group.

B. We wish to make the following observations concerning (3.6). As expected, when $g(\alpha_0)$ vanishes for a special value of α_0 , then all the cut-off dependent terms in (3.6) disappear and $Z_3^{-1} = \alpha_0/Q(\alpha_0)$. It then follows that $\alpha = Q(\alpha_0)$. In general, when $g(\alpha_0) \neq 0$, the divergent part of Z_3^{-1} contains terms proportional to all powers of $\log(m^2/M^2)$, not just to the first power, as in the model of Johnson et al. We conclude therefore the subset of graphs considered by Johnson et al. is not renormalizable. Further, note that although the coefficient of the single logarithm in (3.6) must vanish for Z_3^{-1} to be cut-off independent, this is not a sufficient condition for the vanishing of the higher powers of the logarithms in (3.6). The reason for this is that the coefficient of the first logarithm could vanish by virtue of a zero in $(d/d\alpha_0)Q^{-1}(\alpha_0)$ and not in $g(\alpha_0)$. In that case Z_3^{-1} would have the form

$$Z_3^{-1} = \frac{\alpha_0}{Q(\alpha_0)} + \frac{\alpha_0^3 g^2(\alpha_0)}{2!} \frac{d^2}{d\alpha_0^2} \frac{1}{Q(\alpha_0)} \log^2 \frac{m^2}{M^2} + \dots \quad (3.7)$$

Only when $\alpha_0^2(d/d\alpha_0)Q^{-1}(\alpha_0)$ has no zeros, can one be sure that the vanishing of the coefficient of the first logarithm signals that all the higher powers of the logarithms disappear. [A particularly simple situation would occur if $\alpha_0^2(d/d\alpha_0)Q^{-1}(\alpha_0)$ is constant, viz., if $d^\infty(1, \alpha)$ is independent of α . However, according to perturbation theory, this does not happen, see below.]

C. We examine explicitly the situation in perturbation theory through sixth order. The renormalized, asymptotic propagator function $d^\infty(k^2/m^2, \alpha)$ has the form

$$d^\infty\left(\frac{k^2}{m^2}, \alpha\right) = \left[1 - \alpha A - \alpha^2 B - \alpha^3 C\right]^{-1} + O(\alpha^4) \quad (3.8a)$$

$$\begin{aligned} A &= a_1 \log \frac{k^2}{m^2} + a_0 \\ B &= b_1 \log \frac{k^2}{m^2} + b_0 \\ C &= c_2 \log^2 \frac{k^2}{m^2} + c_1 \log \frac{k^2}{m^2} + c_0. \end{aligned} \quad (3.8b)$$

The forms chosen for A and B are those that have been found by explicit calculation⁴⁾. The form for C is chosen without powers of the logarithm higher than the second, in anticipation of the prediction of the renormalization group, (see below). From (3.8) it follows that the two basic functions $q(\alpha)$ and $\mathcal{G}(x)$ have the form

$$q(\alpha) = \alpha + \alpha^2 a_0 + \alpha^3 (a_1^2 + b_0) + \alpha^4 (a_1^3 + 2a_1 b_1 + c_0) + O(\alpha^5) \quad (3.9a)$$

$$\mathcal{G}(x) = a_1 x + b_1 x^2 + (c_1 - a_0 b_1) x^3 + O(x^4). \quad (3.9b)$$

The values of a_0 , a_1 and b_1 are known

$$a_0 = -\frac{10}{9} \quad a_1 = \frac{2}{3} \quad b_1 = 1 \quad (3.10)$$

To this order we have for ρ , from (2.8c)

$$\begin{aligned} \rho\left(\frac{k^2}{M^2}, \alpha_0\right) = & \left[a_1 \alpha_0 + b_1 \alpha_0^2 + (c_1 - a_0 b_1) \alpha_0^3 \right] \log \frac{k^2}{M^2} \\ & + \frac{a_1 b_1}{2} \alpha_0^3 \log^2 \frac{k^2}{M^2} + O(\alpha_0^4). \end{aligned} \quad (3.11)$$

When $d^\omega(k^2/M^2, \alpha)$ is evaluated by (3.1) and then cast into the form (3.8a), the quantities A, B, C are as in (3.8b), except that c_2 is determined

$$c_2 = \frac{a_1 b_1}{2} \quad (3.12)$$

Therefore through fourth order the renormalization group merely predicts that ρ contains no powers of the logarithm higher than the first. This has been verified by explicit calculation. In sixth order, the renormalization group predicts that ρ contains no powers of the logarithm higher than the second, and evaluates the coefficient of the $\log^2(k^2/M^2)$ term. These results are well known, however we present them here explicitly so that we may make reference to them later.

Let us now calculate Z_3^{-1} . We have from (3.9a)

$$\begin{aligned} Q(\alpha_0) = & \alpha_0 - a_0 \alpha_0^2 + (a_0^2 - b_0) \alpha_0^3 \\ & - (a_0^3 - 3a_0 b_0 + c_0) \alpha_0^4 + O(\alpha_0^5) \end{aligned} \quad (3.13)$$

$$\begin{aligned} Q^{-1}(\alpha_0) = & \frac{1}{\alpha_0} + a_0 + b_0 \alpha_0 + (c_0 - a_0 b_0) \alpha_0^2 \\ & + O(\alpha_0^3). \end{aligned}$$

Therefore Z_3^{-1} becomes, by virtue of (3.6),

$$\begin{aligned} Z_3^{-1} = & 1 + \alpha_0 a_0 + \alpha_0^2 b_0 + \alpha_0^3 (c_0 - a_0 b_0) \\ & + [\alpha_0 a_1 + \alpha_0^2 b_1 + \alpha_0^3 (c_1 - a_0 b_1 - a_1 b_0)] \log \frac{M^2}{m^2} \\ & - \alpha_0^3 \frac{a_1 b_1}{2} \log^2 \frac{M^2}{m^2} + O(\alpha_0^4). \end{aligned} \quad (3.14)$$

Note that, as we have emphasized before, the coefficient of the single logarithm is not $\mathcal{G}(\alpha_0) = \alpha_0 a_1 + \alpha_0^2 b_1 + \alpha_0^3 (c_1 - a_0 b_1)$, which must vanish for a finite charge renormalization, but rather $\mathcal{G}(\alpha_0)(1 - \alpha_0^2 a_1 b_0)$. Rosner calculated the single logarithm coefficient of Z_3^{-1} , obtaining the known values for a_1 and b_1 , (3.10), and $-1/4$ for the α_0^3 term (see also below). Therefore the quantity which multiplies α_0^3 in $\mathcal{G}(\alpha_0)$ is $-\frac{1}{4} + a_1 b_0 = -\frac{1}{4} + \frac{2}{3} b_0$. If this were negative, one might hope that the complete $\mathcal{G}(\alpha_0)$ possesses an appropriate zero, thus rendering charge renormalization finite. Unfortunately b_0 is not known, and no information about the sign of the sixth order term in $\mathcal{G}(\alpha_0)$ is available.

4. VALIDITY OF THE MASS ASSUMPTION

A. We have analyzed Z_3^{-1} in perturbation theory, using the renormalization group with the usual mass assumption. We now examine the validity of this mass assumption, taking into account the work of Johnson et al. and Rosner. From one point of view, the mass assumption is a statement about the coefficients of the powers of the logarithms in $\mathcal{P}$. Through fourth order, the consequences of this assumption have been explicitly verified, as we have stated above. We now show that the sixth order calculation of Rosner can be used to exhibit a verification to this order.

According to Rosner's calculation the sixth order diagrams contributing to the polarization tensor and not containing any photon self-energy insertions lead only to a single power of the logarithm. Therefore the second power and other powers (if any) of the logarithm must come from diagrams with photon self-energy insertions. These diagrams are represented in Fig. 1. We show now that they contribute to $\rho(k^2/M^2, 0; \alpha_0)$ only the term $\alpha_0^3(a_1 b_1/2)\log^2(k^2/M^2)$, thus verifying the renormalization group through sixth order [see (3.11) and (3.12)].

To arrive at this result, we shall need to make reference to the second and fourth order contributions to the polarization tensor. These are represented in Figs. 2 and 3, respectively. According to (3.11), the second order, unrenormalized, cut-off dependent contribution to ρ , arising from the diagram in Fig. 2, is

$$\rho_2\left(\frac{k^2}{M^2}, \alpha_0\right) = \alpha_0 a_1 \log \frac{k^2}{M^2}. \quad (4.1)$$

We have ignored terms that become zero as k approaches infinity. Next, (3.11) states that the contribution of the three fourth order diagrams of Fig. 3

$$\rho_4\left(\frac{k^2}{M^2}, \alpha_0\right) = \alpha_0^2 b_1 \log \frac{k^2}{M^2}. \quad (4.2)$$

Again we give only the non-vanishing asymptotic expression. This contribution evidently arises from an integral of the form

$$\rho_4\left(\frac{k^2}{M^2}, \alpha_0\right) = -\alpha_0^2 b_1 \int_{k^2}^{M^2} \frac{dk^2}{k^2} \times 1. \quad (4.3)$$

Note that k^2 provides an infra-red cut-off, and M^2 , an ultra-violet cut-off. Now the contribution ρ'_6 of the diagrams of Fig. 1, is just that of Fig. 3, except that photon insertions are made; viz., the factor 1 in (4.3) is replaced, according to (4.1), by $\alpha_0 a_1 \log(k^2/M^2)$ plus terms that go to zero with k . Therefore

$$\rho'_6\left(\frac{k^2}{M^2}, \alpha_0\right) = -\alpha_0^2 b_1 \int_{\frac{k^2}{M^2}}^{M^2} \frac{dk^2}{k^2} \left[\alpha_0 a_1 \log \frac{k^2}{M^2} + O\left(\frac{m^2}{k^2}\right)^\varepsilon \right] \quad (4.4a)$$

$$\varepsilon > 0.$$

Thus the diagrams of Fig. 3 give the asymptotic contribution

$$\rho'_6\left(\frac{k^2}{M^2}, \alpha_0\right) = \alpha_0^3 \frac{a_1 b_1}{2} \log^2 \frac{k^2}{M^2}, \quad (4.4b)$$

which is the desired result. Note that no single logarithms are present in this term.

B. From another point of view, the mass assumption is the statement that $d_0(k^2/M^2, m^2/M^2, \alpha_0)$, or equivalently the polarization tensor, has a well defined limit as $m \rightarrow 0$. It is sufficient that this zero mass limit exists for those vacuum polarization diagrams which contain no photon insertions. The reason for this is that a photon insertions consists of replacing 1 by a logarithmic term [see Eqs. (4.3) and (4.4)]. But logarithmic terms are always integrable. The work of Johnson et al. proves that this zero mass limit exists for the polarization tensor without photon insertions, when the argument is zero. Therefore, if this same state of affairs persists when the argument is non-zero, the mass assumption of the renormalization group is true to all orders.

5. CONCLUSION

We have shown that the renormalization group leads to a criterion for the finiteness of charge renormalization which is different from that found by Johnson et al. We also have shown that the mass assumption of the renormalization is valid through sixth order.

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FIGURE CAPTIONS

Figure 1 : Sixth order contribution to the polarization tensor
with photon self-energy insertions.

Figure 2 : Second order contribution to the polarization tensor.

Figure 3 : Fourth order contributions to the polarization tensor.

FIG.1

FIG.2

FIG.3