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HADRONIC CONTRIBUTIONS TO THE MUON MAGNETIC MOMENT

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A B S T R A C T

We have examined the extent to which the estimate of the hadronic contribution to the muon magnetic moment can be improved using the most recent experimental information.

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Recent evaluations ¹⁾ of the leading contributions of the α^3 radiative corrections to $\frac{1}{2}(g-2)$ of the muon magnetic moment indicate that these corrections are not much larger than the expected order of magnitude $(\alpha/\pi)^3$ or 10^{-8} . Although the complete α^3 calculation has not been carried out, it now seems certain that the most important contribution to $\frac{1}{2}(g-2)$ after the α^2 radiative corrections arises from the modification of the photon propagator due to the strong interactions. Some time ago the contribution of the 2π intermediate state was estimated ²⁾ to be of the order of 10^{-7} assuming that the ρ meson alone dominated the pion electromagnetic form factor. In the meantime more experimental information has become available. Furthermore, the present data suggest important corrections to the simple assumption that the ρ meson alone dominates the pion form factor.

In this note, we examine the extent to which the calculation of the hadronic contribution to $\frac{1}{2}(g-2)$ of the muon can be improved in light of the more recent experimental information now available. For completeness we will first review the relevant formulae, all of which have been given previously ²⁾.

The effects of **strong** interactions on the photon propagator can be expressed using the Källén-Lehmann representation ³⁾

$$D(q^2) = \frac{1}{q^2} + \int_0^\infty \frac{d\lambda}{\lambda} \frac{\Pi(\lambda)}{q^2 - \lambda}, \quad (1)$$

where the spectral function $\Pi(\lambda)$ is defined by

$$\Pi(p^2) = \frac{(2\pi)^3}{3p^2} \sum_n \delta^4(p - p_n) \langle 0 | j_\mu(0) | n \rangle \langle n | j_\mu(0) | 0 \rangle_{(2)}$$

The resulting modification of the muon magnetic moment is given by ²⁾

$$\frac{1}{2}(g-2)_{\text{strong}} = \frac{\alpha}{\pi} \int_0^\infty \frac{d\lambda}{\lambda} \Pi(\lambda) G(\lambda), \quad (3)$$

where

$$G(\lambda) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (\lambda/m_\mu^2)(1-x)}. \quad (4)$$

In principle, the contribution of any intermediate state to the spectral function $\Pi(\lambda)$ can be determined by measuring the cross-section for electron-positron annihilation into this state. However, this information is not available at present. Therefore we shall proceed to calculate first the 2π contribution to $\Pi(\lambda)$ in terms of the pion electromagnetic form factor and then to estimate the other contributions. The 2π contribution to $\Pi(\lambda)$, which is no doubt the most important, can be expressed as ²⁾

$$\Pi(\lambda) = \frac{\alpha}{12\pi} \left(1 - \frac{4m_\pi^2}{\lambda}\right)^{3/2} |F_\pi(\lambda)|^2 \theta(\lambda - 4m_\pi^2), \quad (5)$$

where $F_\pi(\lambda)$ is the pion electromagnetic form factor. If we assume further that only the ρ meson pole contributes to $F_\pi(\lambda)$, we have ^{*)}

$$F_\pi(\lambda) = \frac{-m_\rho^2}{\lambda - m_\rho^2 + i m_\rho \Gamma_\rho(\lambda)}, \quad (6)$$

where

$$\Gamma_\rho(\lambda) = \Gamma_\rho \left(\frac{m_\rho^2}{\lambda}\right)^{1/2} \left(\frac{\lambda - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} \theta(\lambda - 4m_\pi^2). \quad (7)$$

*) This is the p wave Breit-Wigner formula used in determining the ρ meson parameters m_ρ and Γ_ρ appearing in Ref. ⁴⁾.

Using the recent values ⁴⁾ $m_\rho = 770$ MeV and $\Gamma_\rho = 140$ MeV, one finds after integration that the ρ meson contribution is ^{*)}

$$\frac{1}{2} (g-2)_\rho = 3.9 \times 10^{-8} = 3.1 \left(\frac{\alpha}{\pi} \right)^3. \quad (8)$$

If one makes the narrow width approximation

$$|F_\pi(\lambda)|^2 \simeq \pi m_\rho^3 \Gamma_\rho^{-1} \delta(\lambda - m_\rho^2), \quad (9)$$

one obtains

$$\frac{1}{2} (g-2)_\rho = 3.5 \times 10^{-8} = 2.8 \left(\frac{\alpha}{\pi} \right)^3, \quad (10)$$

indicating that the dependence on the finite width of the peak is only about 10%.

It is natural to question the approximation that only the ρ meson pole contributes to the pion form factor [Eq. (6)] since, as is well known, such a one-pole formula does not account for the nucleon isovector form factor. To examine this point we write the more general form

$$F_\pi(\lambda) = \frac{\gamma_{\rho\pi\pi}}{2\gamma_\rho} \frac{-m_\rho^2}{\lambda - m_\rho^2 + i m_\rho \Gamma_\rho(\lambda)} + \left(1 - \frac{\gamma_{\rho\pi\pi}}{2\gamma_\rho} \right) f_\pi(\lambda), \quad (11)$$

*) This integration emphasizes the low energy contribution. Thus this result could well be increased if the low energy p wave $\pi\pi$ scattering amplitude is significantly larger than that given by the tail of the Breit-Wigner formula [Eq. (6)].

where $f_\pi(\lambda)$ represents all non- ρ contributions normalized so that $f_\pi(0) = 1$. The residue of the ρ pole is the product of the effective $\rho\pi\pi$ coupling constant, $\gamma_{\rho\pi\pi}$, which is related to the $\rho \rightarrow \pi\pi$ width by

$$\Gamma(\rho^0 \rightarrow \pi^+\pi^-) = \frac{m_\rho}{12} \frac{\gamma_{\rho\pi\pi}^2}{4\pi} \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{3/2} \quad (12)$$

and the effective $\rho - \gamma$ coupling constant $m_\rho^2/2 \gamma_\rho$ which is related to the width for the leptonic decay $\rho^0 \rightarrow \ell^+ + \ell^-$ by ⁵⁾

$$\Gamma(\rho^0 \rightarrow \ell^+ + \ell^-) = \frac{\alpha^2}{12} \frac{4\pi}{\gamma_\rho^2} m_\rho \left(1 - \frac{4m_\ell^2}{m_\rho^2}\right)^{1/2} \left(1 + \frac{2m_\ell^2}{m_\rho^2}\right). \quad (13)$$

While the $\rho\pi\pi$ width is relatively well known ⁴⁾ the experimental values ⁶⁾ for the lepton pair branching ratio $\Gamma(\rho^0 \rightarrow \ell^+ + \ell^-)/\Gamma(\rho^0 \rightarrow \pi^+ + \pi^-)$ vary from $(3.9 \pm 1.2) \times 10^{-5}$ to $(9.7 \pm 2.0) \times 10^{-5}$ each with considerable uncertainty, part of which arises from the possible presence of $\omega \rightarrow \ell^+ + \ell^-$ events in the data. Hence these experiments only indicate that

$$1.04 \lesssim \left| \frac{\gamma_{\rho\pi\pi}}{2\gamma_\rho} \right| \lesssim 1.64, \quad (14)$$

which is not actually inconsistent with the assumption $\gamma_{\rho\pi\pi} = 2\gamma_\rho$ that underlies Eq. (6) when one takes account of the experimental uncertainties. Corresponding to this range [Eq. (14)] one finds for the ρ contribution to $\frac{1}{2}(g-2)$

$$4.2 \times 10^{-8} \lesssim \frac{1}{2}(g-2)_\rho \lesssim 10.5 \times 10^{-8}. \quad (15)$$

If one averages these data weighting each measurement by the inverse of its quoted error, one finds

$$\frac{1}{2}(g-2)_\rho = 6.1 \times 10^{-8} = 4.9 \left(\frac{\alpha}{\pi}\right)^3, \quad (16)$$

which corresponds to

$$|\gamma_{\rho\pi\pi}/2\gamma_{\rho}| = 1.25. \quad (17)$$

Regarding the additional term $f_{\pi}(\lambda)$ in Eq. (11), there is very little direct experimental information. If $f_{\pi}(\lambda)$ is dominated by a second $\pi\pi$ resonance this additional contribution could be quite important, particularly if $\gamma_{\rho\pi\pi}/2\gamma_{\rho}$ turns out to be negative [c.f., Eq. (14)]. However, since no such resonance has been found in searches⁶⁾ extending up to about 1500 MeV, we will ignore this contribution.

To estimate the contributions of intermediate states in Eq. (2) other than the 2π state, we can compute the effects of the isoscalar mesons ω and φ . According to the current ideas based on SU(3), one can obtain the ω and φ contributions by multiplying the contribution by $\frac{1}{3}\sin^2\theta$ and $\frac{1}{3}\cos^2\theta$, respectively, where θ is the φ - ω mixing angle, and then correcting for the mass differences. Taking the value⁴⁾ $\theta = 40.1^\circ$ and using the average result [Eq. (16)] for the ρ contribution, we find

$$\frac{1}{2}(g-2)_{\omega} = 0.72 \times 10^{-8} = 0.58 \left(\frac{\alpha}{\pi}\right)^3, \quad (18)$$

$$\frac{1}{2}(g-2)_{\varphi} = 0.63 \times 10^{-8} = 0.50 \left(\frac{\alpha}{\pi}\right)^3. \quad (19)$$

In summary, the best estimate of the hadronic contributions to $\frac{1}{2}(g-2)$ of the muon, allowed by the present experimental information, is the sum of the ρ , ω , and φ contributions [Eqs. (16), (18) and (19)] which amounts to

*) For comparison, the weak vector boson, if it exists, is expected to contribute⁷⁾ about -2×10^{-8} to $\frac{1}{2}(g-2)$ depending on the value of its magnetic moment.

$$\frac{1}{2}(g-2)_{strong} = 7.5 \times 10^{-8} = 6.0 \left(\frac{\alpha}{\pi} \right)^3. \quad (20)$$

We emphasize that this value is subject to all the uncertainties mentioned above. A more reliable estimate will be possible when the spectral function $\Pi(\lambda)$ is measured directly, for example, in electron-positron colliding beam experiments.

Although the hadronic effects estimated above are well below the present uncertainties in the recently published experimental value ⁸⁾

$$\frac{1}{2}(g-2)_{exp} = (1165 \pm 3) \times 10^{-6}, \quad (21)$$

this experiment is being continued and a substantial improvement in the precision can be expected in the near future.

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