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PARTICLE PRODUCTION IN NUCLEI AND UNSTABLE PARTICLE CROSS-SECTIONS

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ABSTRACT

We present a theory of coherent and incoherent production of particles in medium and heavy nuclei at very high energies where threshold effects are negligible. The modification necessary for coherent production at lower energies (of the order of a few GeV or less) is made. Both coherent and incoherent production are sensitive to the produced particle nucleon total cross-section. Tables are given of the "effective nucleon numbers" involved in production. These are useful in the determination of the produced particle cross-section on a nucleon. This study offers the possibility for determining the cross-sections of virtually all unstable particles or resonances. Detailed angular distribution calculations are presented for the case of photoproduction of ρ^0 mesons. A determination of the ρ^0 nucleon total cross-section is made by comparison with existing data.

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1. INTRODUCTION

In a recent letter ¹⁾, one of us has described results of the extension and application of a Glauber type ^{2),3)} multiscattering theory to the coherent and incoherent production of particles in medium and heavy nuclei at multi-GeV energies. This production proved to be reasonably sensitive to the produced particle - nucleon total cross-section. The nucleus acts both as a generator of particles and then serves as a "nucleon matter" target. For the case of photoproduction of ρ^0 mesons, coherent production has already been discussed previously by other authors ⁴⁾. The incoherent scattering of protons has been discussed by Glauber and Matthiae ³⁾. These authors have analyzed the data of Belletini et al. ⁵⁾ for incident protons of 19.3 GeV/c.

In this paper we derive the results stated in Ref. 1). They depend, for detailed accuracy, in the case of coherent production, on the assumption that the longitudinal momentum transfer times the nuclear radius is small compared to unity. For energies of a few GeV or less, this assumption is sometimes not a good one. We will present a multi-interaction formula to include this case. We then show detailed calculations for photoproduction of ρ^0 mesons indicating the differences between the extreme high energy approximation, which we call the infinite energy theory, and the more general theory. For incoherent production there is no difference because phases of waves are of no importance. The longitudinal momentum transfer referred to here arises from the difference in mass between the incident and produced particle. It is given by $q_\ell = (m_2^2 - m_1^2)/(p_1 + p_2)$, where m_1 and m_2 are the masses of the incident and produced particles, respectively, and p_1 and p_2 are their momenta.

In Section 2 we present the infinite energy theory for coherent elastic production of particles in medium and heavy nuclei. In Section 3 we present the theory of incoherent production. In Section 4 we discuss briefly semi-coherent processes, the effect of correlations in the nuclear wave function, and other questions relevant to this study. In Section 5 we discuss coherent production in the energy region of a few GeV or less where threshold effects are still non-negligible. We apply this theory to the available data for photoproduction of ρ^0 mesons ⁶⁾. Section 6 contains general discussion and conclusions.

2. COHERENT PRODUCTION AT VERY HIGH ENERGY

We follow Glauber in assuming that a multiple sequence of interactions initiated in a nucleus by a high energy incident particle takes place through single interactions with one or more target particles. Multiple interaction with any given target particle is assumed negligible. We are interested here in calculating processes of the type

$$1 + N \rightarrow 2 + N \quad (1)$$

in the high energy limit where $q_\ell \rightarrow 0$. In Eq. (1), 1 is the incident and 2 the outgoing particle. N represents the target nucleus in its ground state. Equation (1) defines coherent production. The incident particle may be a photon, pion, kaon, nucleon, etc. In order for (1) to go strongly, the class of particles is restricted by selection rules. We must satisfy charge, parity and angular momentum conservation. We assume that our interactions do not depend on target particle quantum numbers. Then for a spin zero incident or produced particle

$$P(1) P(2) = (-1)^{|J_1 - J_2|} \quad (2)$$

where $P(i)$ and J_i are the parity and spin of the incident and produced particle. There may be further selection rules as well which follow from particular dynamical theories or symmetry schemes.

We furthermore assume that the two-body process

$$1 + n \rightarrow 2 + n \quad (3)$$

is a small part of the total cross-section for 1 or 2 incident on a nucleon, n . In the case of photoproduction, the restriction is that (3) is small compared with the total cross-section of 2 on n .

Extending the Glauber formalism ^{2),3)} in a straightforward way we have, with the above assumptions, that the production amplitude at small angles is given by, (ignoring Coulomb distortion)

$$F_{II}(\vec{p}_2, \vec{p}_1) = \frac{ik}{2\pi} \sum_{j=1}^A \int e^{i(\vec{p}_1 - \vec{p}_2) \cdot \vec{b}} d^2b |u_I(\vec{r}_1, \dots, \vec{r}_A)|^2 \Gamma_{12}(\vec{b} - \vec{s}_j) \\ \times \prod_{\substack{i < j \\ z_i < z_j}} [1 - \Gamma_{11}(\vec{b} - \vec{s}_i)] \prod_{\substack{i > j \\ z_k > z_j}} [1 - \Gamma_{22}(\vec{b} - \vec{s}_k)] d\vec{r}_1 \dots d\vec{r}_A \quad (4)$$

where $u_I(\vec{r}_1, \dots, \vec{r}_A)$ is the target wave function, $\vec{p}_1$ and $\vec{p}_2$ are the incident and outgoing particle momenta, with common magnitude k in the high energy limit. The vector $\vec{b}$ is the impact vector of 1 or 2 and $\vec{s}_i$ is the impact vector of the i th target nucleon. The quantities Γ_{xy} , with $x, y = 1, 2$ are given by

$$\Gamma_{xy}(\vec{b}) = \frac{1}{2\pi i k} \int f_{xy}(\vec{q}) e^{-i\vec{q} \cdot \vec{b}} d^2q \quad (5)$$

where f_{xy} is a two-body scattering or production amplitude. For example, $f_{11}(\vec{q})$ is the amplitude for scattering of particle 1 from a nucleon, at momentum transfer $\vec{q}$, $f_{12}(\vec{q})$ is the amplitude for producing 2 with 1 incident. Inverting, we get

$$f_{xy}(\vec{q}) = \frac{ik}{2\pi} \int e^{i\vec{q} \cdot \vec{b}} \Gamma_{xy}(\vec{b}) d^2b \quad (6)$$

We now assume, since we are dealing with medium to heavy nuclei, that a simple approximation for the nuclear wave function, which has no correlations, will be adequate for small angle production. Following Glauber²⁾ we take

$$|u_I(\vec{r}_1, \dots, \vec{r}_A)|^2 = \prod_{i=1}^A \rho(\vec{r}_i) \quad (7)$$

where $\rho(\vec{r})$ describes the single particle density function of the nucleus with

$$\int \rho(\vec{r}) d\vec{r} = 1 \quad (8)$$

We now define

$$T(\vec{b}) \equiv A \int_{-\infty}^{\infty} \rho(\vec{b}, z) dz \quad (9)$$

Using the product wave function (7) in (4), we are led to consider quantities

$$\begin{aligned} \int \Gamma_{xy}(\vec{b}-\vec{s}) \rho(\vec{s}, z) dz &\simeq \int \Gamma_{xy}(\vec{b}-\vec{s}) d^2s \int_{-\infty}^{\infty} \rho(\vec{b}, z) dz \\ &= -\frac{2\pi i}{k} f_{xy}(0) T(\vec{b})/A \end{aligned} \quad (10)$$

where we have used Eqs. (5) and (9). The approximate equality in (10) follows if $\rho(\vec{s}, z)$ is slowly varying compared to $\Gamma(\vec{b}-\vec{s})$ as a function of $\vec{s}$. This follows if $(d/R)^2 \ll 1$ where d is the range of two-body interactions and R is the nuclear radius.

We find then, using (10),

$$\begin{aligned} F_{II}(\vec{p}_2, \vec{p}_1) &= \frac{ik}{2\pi} \frac{f_{12}(0)}{f_{11}(0) - f_{22}(0)} \times \\ &\times \int e^{i\vec{r}\cdot\vec{b}} d^2b \left\{ \left(1 + \frac{2\pi i}{k} \frac{T(\vec{b})}{A} f_{11}(0) \right)^A - \left(1 + \frac{2\pi i}{k} \frac{T(\vec{b})}{A} f_{22}(0) \right)^A \right\} \end{aligned} \quad (11)$$

where $\vec{q} = \vec{p}_1 - \vec{p}_2$. Now

$$\left(1 + \frac{2\pi i}{k} \frac{T(\vec{b})}{A} f(0) \right)^A \simeq e^{\frac{2\pi i}{k} T(\vec{b}) f(0)} \quad (12)$$

so that

$$F_{II}(\vec{p}_2, \vec{p}_1) = \frac{ik}{2\pi} \frac{f_{12}(0)}{f_{11}(0) - f_{22}(0)} \times \int e^{i\vec{q} \cdot \vec{b}} \left[e^{\frac{2\pi i}{k} T(\vec{b})} f_{11}(0) - e^{\frac{2\pi i}{k} T(\vec{b})} f_{22}(0) \right] d^2 b \quad (13)$$

Using the optical theorem

$$\text{Im} f_{ii}(0) = \frac{k}{4\pi} \sigma_i \quad (14)$$

where σ_i is the total cross-section for particle $i = 1, 2$ on a nucleon, we get for the coherent elastic production cross-section in the high energy limit when $\rho(\vec{r}) = \rho(r)$

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} \frac{4}{(\sigma_1 - \sigma_2)^2 + (\alpha_1 \sigma_1 - \alpha_2 \sigma_2)^2} \times \left| \int J_0(q_V b) \left[e^{-\frac{1}{2}(1-i\alpha_1)\sigma_1 T(b)} - e^{-\frac{1}{2}(1-i\alpha_2)\sigma_2 T(b)} \right] d^2 b \right|^2 \quad (15)$$

where the α_i are the ratio of real to imaginary parts of the scattering amplitudes on nucleons, $(d\sigma_0/dt)$ is the two-body cross-section corresponding to Eq. (3) and J_0 is the cylindrical Bessel function of the first kind of order zero. The quantity $t = -q^2$.

At small angles the momentum transfer in the high energy limit is essentially transverse. One therefore has the total momentum transfer in the argument of the Bessel function in Eq. (15). As we will indicate below, one can use (15) to good accuracy for energies above 10 GeV, unless one is producing a particle which is very heavy. The criterion for the validity of (15) is $q_\ell R$ considerably less than unity.

In photoproduction of particles, Eq. (15) becomes

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} \frac{4}{\sigma_2^2 (1 + \alpha_2^2)} \left| \int_0^\infty J_0(qb) \left[1 - e^{-\frac{1}{2}(1 - i\alpha_2)\sigma_2 T(b)} \right] d^2b \right|^2 \quad (16)$$

which is a multiple of the cross-section for elastic scattering of the produced particle from the target nucleus. At finite energies this scattering will be slightly off the mass shell.

It should be pointed out that formulae (15) and (16) could be derived somewhat more simply using optical model considerations ^{4),7)}. However, we have presented an approach here, which allows for the calculation, in a unified way, of inelastic production where the nucleus is left excited, either in a particle stable or unstable state. Further, in the case of light nuclei, the above approach is essential in order to take into account the details of the nuclear wave function and perhaps the finite range of the two-body interactions. These refinements may be desirable eventually for heavier nuclei as well.

At multi-GeV energies, scattering amplitudes are predominantly imaginary, in all cases where they are known, and so to good accuracy we can write

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} \frac{4}{(\sigma_1 - \sigma_2)^2} \left| \int_0^\infty J_0(qb) \left[e^{-\frac{\sigma_1}{2} T(b)} - e^{-\frac{\sigma_2}{2} T(b)} \right] d^2b \right|^2 \quad (17)$$

For $t=0$ we have that (17) becomes a simple multiple of the two-body cross-section

$$\left(\frac{d\sigma^{(c)}}{dt} \right)_{t=0} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} N^2 \left(A; \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2 \right) \quad (18)$$

where

$$N(A; \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2) = \frac{2}{\sigma_2 - \sigma_1} \int [e^{-\frac{1}{2}\sigma_1 T(b)} - e^{-\frac{1}{2}\sigma_2 T(b)}] d^2b \quad (19)$$

In the limit where σ_1 and σ_2 approach zero, we get

$$N(A; 0, 0) = \int T(b) d^2b = A \quad (19')$$

which corresponds to first Born approximation production. In the limit where $\sigma_1 \rightarrow \sigma_2 = \sigma$, equation (17) becomes

$$\frac{d\sigma^{(C)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} \left| \int_0^\infty T(q, b) T(b) e^{-\frac{1}{2}\sigma T(b)} d^2b \right|^2 \quad (20)$$

Note that this is not an expression for elastic scattering but rather for coherent production when $\sigma_1 = \sigma_2$.

3. INCOHERENT PRODUCTION OF PARTICLES

From the practical point of view, for incident particles in the multi-GeV range, one can easily be limited by an energy resolution of the order of several tens of MeV. It may then be convenient to consider cross-sections summed over final states of the target. It has been shown ²⁾ for scattering of particles by a nucleus that if one subtracts the elastic scattering from this sum over final states for $qR \gg 1$, the resulting cross-section corresponds to scattering incoherently from individual target nucleons. For short enough mean free paths of the incident particles, ($\sigma_{\text{tot}} \gtrsim 30$ mb) there is strong scattering from only a few surface nucleons lying in a ring at the surface of the nucleus, the plane of the ring being normal to the incident direction. In fact, it turns out ^{1), 2)} that incoherent scattering and reactions are a sensitive probe of the nuclear surface density. (In photoproduction processes, it is the back part of the nucleus that contributes.)

We consider now incoherent production of particles. The amplitude for production in the j^{th} nucleon, the nucleus being left in a state F , is, in the high energy limit

$$F_{FIj}(\vec{p}_2, \vec{p}_1) = \frac{ik}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} d^2b \int u_F^*(\vec{r}_1, \dots, \vec{r}_A) \Gamma_{12}(\vec{b} - \vec{s}_j) \times \frac{i}{\prod_{z_i < z_j} [1 - \Gamma_{11}(\vec{b} - \vec{s}_i)]} \frac{k}{\prod_{z_k > z_j} [1 - \Gamma_{22}(\vec{b} - \vec{s}_k)]} u_I(\vec{r}_1, \dots, \vec{r}_A) d\vec{r}_1 \dots d\vec{r}_A \quad (21)$$

where u_F is the final state wave function. The cross-section for production of particle 2 summed over all nucleon states is then given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sum_F \left| \sum_j F_{FIj}(\vec{p}_2, \vec{p}_1) \right|^2 \\ &= \sum_F \sum_{j, j'} F_{FIj'}^*(\vec{p}_2, \vec{p}_1) F_{FIj}(\vec{p}_2, \vec{p}_1) \end{aligned} \quad (22)$$

We consider separately in (22) the terms with $j' = j$. This defines the incoherent production cross-section

$$\frac{d\sigma^{(I)}}{d\Omega} = \sum_{F,j} |F_{FI_j}(\vec{p}_2, \vec{p}_1)|^2 \quad (23)$$

We now proceed to evaluate this sum. Using closure, we get, writing $u_I(\vec{r}_1, \dots, \vec{r}_A) \equiv |I\rangle$,

$$\begin{aligned} \frac{d\sigma^{(I)}}{d\Omega} &= \frac{k^2}{4\pi} \sum_{j=1}^A \int d^2b d^2b' e^{i\vec{q} \cdot (\vec{b} - \vec{b}')} \langle I | \Gamma_{12}(\vec{b} - \vec{s}_j) \Gamma_{12}^*(\vec{b}' - \vec{s}_j) \\ &\times \prod_{\substack{i \\ z_i < z_j}} [1 - \Gamma_{11}(\vec{b} - \vec{s}_i)] [1 - \Gamma_{11}^*(\vec{b}' - \vec{s}_i)] \prod_{\substack{k \\ z_k > z_j}} [1 - \Gamma_{22}(\vec{b} - \vec{s}_k)] [1 - \Gamma_{22}^*(\vec{b}' - \vec{s}_k)] | I \rangle \end{aligned} \quad (24)$$

Using the product wave function (7) for $|I\rangle$ and the fact that

$$\int \Gamma_{xy}(\vec{b} - \vec{s}) \Gamma_{xy}^*(\vec{b}' - \vec{s}) d^2s = \int |f_{xy}(\vec{\lambda})|^2 e^{-i\vec{\lambda} \cdot \vec{\beta}} d^2\lambda / k^2 \quad (25)$$

we find under the condition $(d/R)^2 \ll 1$

$$\begin{aligned} \frac{d\sigma^{(I)}}{d\Omega} &= \frac{d\sigma_0}{d\Omega} N(A; \sigma_1, \sigma_2) + \\ &\int \frac{d\sigma_0(q')}{d\Omega} G(\vec{q}, \vec{q}'; A; \sigma_1, \sigma_2) d^2q' + \dots \end{aligned} \quad (26)$$

where

$$N(A; \sigma_1, \sigma_2) = \frac{1}{\sigma_2 - \sigma_1} \int [e^{-\sigma_1 T(\vec{B})} - e^{-\sigma_2 T(\vec{B})}] d^2B \quad (27)$$

and

$$\begin{aligned}
 G(\vec{q}, \vec{q}'; A; \sigma_1, \sigma_2) = & \frac{1}{\sigma_2 - \sigma_1} \frac{1}{k^2} \int d^2 B \left\{ \frac{1}{\sigma_2 - \sigma_1} \left(e^{-\sigma_2 T(\vec{B})} - e^{-\sigma_1 T(\vec{B})} \right) \right. \\
 & \times \left(|f_{11}(\vec{q} - \vec{q}')|^2 - |f_{22}(\vec{q} - \vec{q}')|^2 \right) + \\
 & \left. + T(\vec{B}) \left(e^{-\sigma_1 T(\vec{B})} |f_{11}(\vec{q} - \vec{q}')|^2 - e^{-\sigma_2 T(\vec{B})} |f_{22}(\vec{q} - \vec{q}')|^2 \right) \right\}
 \end{aligned} \quad (28)$$

The first term on the right-hand side of (26) is simply a multiple of the two-body production cross-section. It thus exhibits the momentum transfer dependence of the two-body process. This term corresponds to production preceded and followed by absorption in the nucleus. The second term on the right-hand side of (26) gives corrections due to single scattering before or after production. It is a relatively small correction for multi-GeV energies for $t \gtrsim 0.1 \text{ (GeV/c)}^2$. Roughly, its size relative to the first term is of order $\sigma_{\text{el}}/\sigma_{\text{tot}}$ for the particle nucleon systems under study. We can see this explicitly by taking Gaussian forms for the amplitudes, $f_{ij} = f_{ij}(0)e^{-\frac{1}{2}a_{ij}q^2}$, and using the optical theorem. We get then that (26) becomes

$$\begin{aligned}
 \frac{d\sigma^{(I)}}{dt} = & \frac{d\sigma_0}{dt} \left\{ N(A; \sigma_1, \sigma_2) + \right. \\
 & \left. \sum_{i=1}^2 (-1)^i \left[N(A; \sigma_1, \sigma_2) - N_1(A; \sigma_i) \right] \frac{a_{ii}}{a_{ii} + a_{12}} \frac{\sigma_i^{(d)}}{\sigma_2 - \sigma_1} e^{\frac{a_{12}^2}{a_{12} + a_{ii}} t} + \dots \right\}
 \end{aligned} \quad (29)$$

where

$$N_1(A; \sigma_i) = \int e^{-\sigma_i T(\vec{B})} T(\vec{B}) d^2 B = N(A; \sigma_i, \sigma_i) \quad (30)$$

To see in some detail the size of the sum on the right-hand side of (29) relative to unity, we consider the limit as $\sigma_2 \rightarrow \sigma_1 = \sigma$; $a_{ii} = a_{12} = a$; $\alpha_1 = \alpha_2 = \alpha$. We find then that (29) (including double scattering corrections) reduces to ³⁾

$$\begin{aligned}
 \frac{d\sigma^{(I)}}{dt} = & \left(\frac{d\sigma_0}{dt} \right)_{t=0} \left\{ N_1(A; \sigma) e^{at} + \frac{\epsilon}{2} N_2(A; \sigma) e^{\frac{a}{2}t} + \right. \\
 & \left. \frac{\epsilon^2}{3} N_3(A; \sigma) e^{\frac{a}{3}t} + \dots \right\}
 \end{aligned} \quad (31)$$

where

$$N_m(A; \sigma) = \frac{1}{m!} \int_0^\infty (\sigma T(\vec{k}))^{m-1} e^{-\sigma T(\vec{k})} T(\vec{k}) d^2k \quad (32)$$

$m = 1, 2, \dots$

and

$$\varepsilon = \frac{1+\alpha^2}{4b\pi} \frac{\sigma}{a} \simeq \frac{\sigma_{el}}{\sigma} \quad (33)$$

It is to be noted that N_m decreases as m increases, so that the second term on the right-hand side is small if $\sigma_{el}/\sigma \ll 1$ and if $q^2 a \lesssim 1$.

We note that in the high energy limit, if $\sigma_{el}/\sigma_{tot} \rightarrow 0$

$$\frac{d\sigma^{(I)}}{dt} = \frac{d\sigma_0}{dt} N(A; \sigma_1, \sigma_2) \quad (34)$$

which we compare with the coherent production cross-section for $t=0$,

$$\left(\frac{d\sigma^{(C)}}{dt} \right)_{t=0} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} N^2 \left(A; \frac{1}{2} \sigma_1, \frac{1}{2} \sigma_2 \right) \quad (35)$$

Further, as σ_1 and $\sigma_2 \rightarrow 0$, Eq. (34) becomes

$$\frac{d\sigma^{(I)}}{dt} = A \frac{d\sigma_0}{dt} \quad (36)$$

which is the first Born approximation result for $qR \gg 1$.

Both formulae (34) and (35) may be produced using classical probability arguments which we leave to the reader⁸⁾.

Tables I, II, III and IV give values of $N(A; \sigma_1, \sigma_2)$ for $\sigma_1 = 0$ (photoproduction processes), $\sigma_1 = 17$ mb (kaons), $\sigma_1 = 26$ mb (pions) and $\sigma_1 = 39$ mb (incident nucleons) respectively and various values of σ_2 . One can interpolate readily from these tables for intermediate cases. The nuclear density has been taken, following findings of Glauber and Matthiae ³⁾ to be

$$\rho = \rho_0 [1 + e^{(r-c)/a}]^{-1} \quad (37)$$

with $c = 1.14 A^{1/3}$ fm and $a = 0.545$ fm. It is to be noted that $N(A; \sigma_1, \sigma_2) = N(A; \sigma_2, \sigma_1)$ which is the result of detailed balance. This makes the tables more generally useful.

In practice it may be desirable to consider the nuclear radius as a parameter making slight variations in the values of a and c quoted below formula (37).

4. SEMI-COHERENT PROCESSES, CORRELATIONS

In Section 2 we have calculated coherent elastic production of particles which is very strong for $qR \ll 1$. In Section 3 we have calculated incoherent production which is dominant for $qR \gg 1$. We now focus attention briefly on the intermediate region of qR where neither of these components of production is dominant. In principle, the answer to what happens in this region can be obtained by summing formula (22) over all states and all particles including values of $j' \neq j$. The predominant terms for small qR correspond to coherent elastic production as given by formulae (13) and (15). There are corrections which depend in detail on correlations in the nuclear wave functions. (They will of course have their own characteristic momentum transfer dependence.) We leave consideration of this question for later study⁹⁾. One can gain direct experimental information on semi-coherent terms by studying processes for which there is no elastic coherent production.

We note finally that coherent spin flip terms may play some non-negligible role in the intermediate range of values of qR .

5. FINITE ENERGY CORRECTIONS TO PARTICLE PRODUCTION

For finite energies, in the GeV energy range, the momentum transfer to the nucleus at small angles has longitudinal and transverse components given by

$$q_e \cong \frac{m_z^2 - m_1^2}{2 p_1} \quad (38)$$

$$q_t \cong p_1 \Theta \quad (39)$$

The finite longitudinal momentum transfer makes it necessary to modify the previous calculations for coherent production. For incoherent production the results are not affected since the phases of the incident and outgoing waves are of no importance. Coherent photoproduction when q_e is appreciable has been discussed by Ross and Stodolsky ⁴⁾ and by Drell and Trefil ⁴⁾.

Following general optical model methods in the WKB approximation one easily finds for coherent production

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_{t=0} \left| A \int J_0(q_t b) e^{iq_e z} \varphi(b, z) e^{-\frac{1}{2}\sigma'_1 T(b)} \times e^{-\frac{1}{2}(\sigma'_1 - \sigma'_2) \int_z^\infty \varphi(b, z') dz'} d^2 b dz \right|^2 \quad (40)$$

where

$$\sigma'_i = \sigma_i (1 - i\alpha_i) \quad (41)$$

We ignore any t dependence of $d\sigma_0/dt$ in (40).

This expression for coherent production reduces to (15) when $q_{\perp} = 0$. In practice the corrections will be small unless $q_{\perp} R$ is of the order of unity or greater. Figures 1 and 2 show calculations for photo-production of ρ^0 mesons on Cu and Pb in the high energy limit using formula (15) and at finite energies using (40). One can see that the high energy limit is reached effectively at around 10 GeV/c except at the diffraction zero. We have taken $\alpha_2 = 0$. For $\alpha_2 \neq 0$ the diffraction minima would be less pronounced.

Figures 3 and 4 show experimental data ⁶⁾ for photoproduction of ρ^0 mesons on Cu and Pb for $p_2 = 2.7$ GeV/c, as well as some data for Pb at $p_2 = 4.5$ GeV/c. Figures 3 and 4 also show calculated values of $(d\sigma^{(C)}/dt)$ and $(d\sigma^{(I)}/dt)$ divided by $(d\sigma_0/dt)_{t=0}$ using formulae (40) and (29), with $\alpha_2 = 0$, $\sigma_1 = 0$ and $\sigma_2 = 26$ mb, as well as the sum of coherent and incoherent production. The experimental data have also been normalized to the two-body data, using ¹⁰⁾

$$\frac{d\sigma_0}{dt} = .147 e^{6.9 t} \text{ mb/(GeV/c)}^2/\text{ster} \quad (42)$$

at $p_2 = 2.7$ GeV/c. The determination of the ρ^0 nucleon total cross-section σ_2 as 26 mb rests somewhat on the normalization given by Eq. (42). Asbury et al. ⁶⁾ using their forward photoproduction data and a formula of the form given by (40) with square shape for $\rho(r)$, determine $\sigma_2 = \sigma_{\text{tot}}(\rho^0 n)$ to be 31.2 ± 2.3 mb. The procedure of these authors is to normalize data for different target nuclei to measurements for ^9Be . In this way they eliminate the need for knowing the two-body cross-section (42). However, one then uses Eq. (40) for light nuclei such as ^9Be and ^{12}C and this is open to question. Some evidence that this is so comes from the work of Corley et al. ⁵⁾ who find that incoherent scattering of protons from ^{40}Ca at 1 GeV is well explained by (31), whereas for ^{12}C there are apparent deviations.

It might be expected that correlation effects for example will be relatively more important in light nuclei. We note in passing that in fact for Cu and Pb the measured photoproduction distributions in Figs. 3 and 4 are somewhat broader than the theory. It will be interesting to see the results of further studies. This could be the result of semi-coherent processes. A more definitive determination of $\sigma_{\text{tot}}(\gamma^0 n)$ depends on a more certain determination of the two-body photoproduction cross-section and measurements for more medium and heavy nuclei.

The results of the extreme high energy theory for coherent production [Eq. (17)] may be used simply to get the average total cross-section in the GeV region. One simply takes measurements of $d\sigma/dt$ for $\theta=0$ at different energies and plots these as a function of $t=t_{\text{min}}$. One then extrapolates to $t=0$ which is the minimum value of t (t_{min}) for infinite energy. One can then make use of Tables I to IV employing formula (18). This was the procedure used in Ref. 1) for the coherent data.

6. GENERAL DISCUSSION AND CONCLUSIONS

Symmetry schemes and dynamical theories of strong interactions make it extremely valuable to know cross-sections of particles whether they be stable or resonances. The theory presented here shows great promise as a method of analyzing data for particle production in nuclei. A parameter in this theory is the total produced particle nucleon cross-section. It appears that the nucleon structure is well enough known in heavy or medium nuclei that this unknown cross-section is to first approximation the only unknown. This is already evident in the work of Glauber and Matthiae³⁾ on incoherent scattering of protons. Put otherwise, the nucleus acts to good approximation as a piece of "nucleon matter". Further refinements due to nuclear structure should be studied, however. A further detail to be studied is the effect of the width of the produced unstable particles.

Although light nuclei are rather special, they may have an important role to play in the study of unstable particle nucleon cross-sections. The studies of Formanek and Trefil⁴⁾ show that by selecting pure coherent elastic production with recoil techniques, it may be possible to determine the elastic scattering amplitudes of unstable particles. This will depend on having a good enough wave function for ^4He , the most promising target for such experiments. This in turn is information that one can get best from experiments¹¹⁾ on elastic scattering of particles from ^4He .

We note finally that studies of incoherent phenomena should not be ignored in analyses of particle production. Where selection rules forbid coherent production, one can still study incoherent production. In fact in this case semi-coherent processes should be detectable at smaller momentum transfers.

Further, at lower energies¹⁾ threshold considerations may make coherent production negligible even when selection rules are not violated. This region of incident energies, less than or of the order of 1 GeV, may provide interesting nucleon structure information through incoherent production studies.

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TABLE I - $\sigma_1 = 0$ mb

Effective nucleon number $N(A; 0, \sigma_2)$ from formula (27) for various targets as a function of produced particle total cross-section, σ_2 . This corresponds to photo-production processes. The nucleon shape is given by Eq. (37).

σ_2 (mb)	0	10	20	30	40	50	60
$N(^{20}\text{Ne})$	20.0	16.2	13.4	11.5	9.98	8.83	7.93
$N(^{27}\text{Al})$	27.0	21.0	17.0	14.3	12.2	10.7	9.55
$N(^{40}\text{Ca})$	40.0	29.6	23.1	18.8	15.9	13.7	12.1
$N(^{64}\text{Cu})$	64.0	44.0	32.8	26.0	21.5	18.4	16.1
$N(^{108}\text{Ag})$	108	67.9	48.1	36.9	30.0	25.3	22.0
$N(^{140}\text{Ce})$	140	83.9	57.8	43.8	35.3	29.7	25.6
$N(^{208}\text{Pb})$	208	115	76.3	56.7	45.2	37.7	32.5

TABLE II - $\sigma_1 = 17$ mb

Effective nucleon number $N(A; \sigma_1, \sigma_2)$ from formula (27).

σ_2 (mb)	0	10	20	30	40	50	60
$N(^{20}\text{Ne})$	14.2	11.3	9.34	7.93	6.88	6.09	5.47
$N(^{27}\text{Al})$	18.1	13.9	11.1	9.24	7.91	6.92	6.18
$N(^{40}\text{Ca})$	24.7	17.8	13.7	11.1	9.32	8.07	7.14
$N(^{64}\text{Cu})$	35.6	23.6	17.2	13.4	11.1	9.50	8.35
$N(^{108}\text{Ag})$	52.8	31.1	21.3	16.2	13.1	11.2	9.76
$N(^{140}\text{Ce})$	63.9	35.3	23.4	17.5	14.2	12.0	10.5
$N(^{208}\text{Pb})$	85.1	42.2	26.7	19.7	15.8	13.3	11.7

TABLE III - $\sigma_1 = 26$ mbEffective nucleon number $N(A; \sigma_1, \sigma_2)$ from formula (27).

σ_2 (mb)	0	10	20	30	40	50	60
$N(^{20}\text{Ne})$	12.2	9.70	7.99	6.77	5.88	5.21	4.69
$N(^{27}\text{Al})$	15.3	11.6	9.31	7.73	6.62	5.81	5.19
$N(^{40}\text{Ca})$	20.3	14.6	11.2	9.02	7.59	6.59	5.85
$N(^{64}\text{Cu})$	28.4	18.6	13.5	10.6	8.77	7.54	6.67
$N(^{108}\text{Ag})$	40.7	23.7	16.2	12.3	10.1	8.64	7.62
$N(^{140}\text{Ce})$	48.5	26.4	17.5	13.2	10.8	9.20	8.12
$N(^{208}\text{Pb})$	63.2	30.8	19.5	14.5	11.8	10.1	8.93

TABLE IV - $\sigma_1 = 39$ mbEffective nucleon number $N(A; \sigma_1, \sigma_2)$ from formula (27).

σ_2 (mb)	0	10	20	30	40	50	60
$N(^{20}\text{Ne})$	10.1	8.02	6.60	5.60	4.87	4.32	3.90
$N(^{27}\text{Al})$	12.4	9.43	7.53	6.26	5.38	4.73	4.25
$N(^{40}\text{Ca})$	16.1	11.5	8.80	7.14	6.04	5.27	4.70
$N(^{64}\text{Cu})$	21.9	14.2	10.4	8.18	6.83	5.92	5.27
$N(^{108}\text{Ag})$	30.6	17.7	12.1	9.35	7.75	6.71	5.97
$N(^{140}\text{Ce})$	36.0	19.5	13.0	9.96	8.24	7.14	6.36
$N(^{208}\text{Pb})$	46.2	22.4	14.4	10.9	9.05	7.85	7.01

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FIGURE CAPTIONS

- Figure 1 : Theoretical cross-section for coherent photoproduction of ρ^0 mesons on Cu, taking $\sigma_{\text{tot}}(\rho^0 n) = 26$ mb, divided by the forward two-body cross-section, using formulae (17) for $p_2 = \infty$ and (40) for $p_2 = 2.7$ and 4.5 GeV/c.
- Figure 2 : Theoretical cross-section for coherent photoproduction of ρ^0 mesons on Pb taking $\sigma_{\text{tot}}(\rho^0 n) = 26$ mb, divided by the forward two-body cross-section, using formulae (17) for $p_2 = \infty$ and (40) for $p_2 = 2.7$ and 4.5 GeV/c.
- Figure 3 : Experimental cross-section ⁶⁾ for photoproduction of ρ^0 mesons with $p_2 = 2.7$ GeV/c on Cu. The data have been divided by the corresponding two-body cross-section ¹⁰⁾ given by Eq. (42). Also shown are theoretical fits given by the sum of coherent and incoherent cross-sections using formulae (40) and (29). The first approximation to incoherent production given by formula (34) is shown as a dotted line. We have taken the ρ^0 nucleon scattering amplitude as purely imaginary with t dependence e^{7t} . It is to be noted that we are not justified in using (29) for small qR , but this is not serious since coherent production dominates here.
- Figure 4 : The same as for Fig. 3, but with Pb as target. Also some data at $p_2 = 4.5$ GeV/c are shown.

FIG. 1

FIG. 2

FIG. 3

FIG. 4