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COHERENT AND INCOHERENT PRODUCTION IN NUCLEI AND UNSTABLE
PARTICLE CROSS-SECTIONS

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A B S T R A C T

Theoretical expressions for the coherent and incoherent production of particles in nuclei in the GeV energy range are presented. They are both found to be sensitive to the produced particle-nucleon total cross-sections. Data for photoproduction of φ^0 mesons are analyzed and the total φ^0 nucleon cross-section is deduced. The results indicate a very general method for getting unstable particle-nucleon cross-sections.

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Symmetry schemes and dynamical theories of strong interactions make it extremely valuable to know as many cross-sections as possible for the elementary particles. Most of these particles are extremely unstable with lifetimes of the order of 10^{-23} seconds. A method of measuring the cross-sections of these particles is to produce them in nuclei. They are produced on one nucleon of the nucleus and can then interact with others before decaying. Indeed if the particles are produced with GeV energies their lifetime lengthens and they can travel many fermis before decaying.

At the smallest angles the particles will be produced coherently, by which we mean the nucleus acts as a whole to effect the production, remaining mostly in the ground state, if selection rules allow. At larger angles, i.e., for larger momentum transfers, the nucleus will not be able to support coherent production and each nucleon will interact independently with the incident particle, there being no phase relation between different particle amplitudes. We have calculated both coherent elastic production and incoherent production for medium to heavy nuclei by extending the multiple scattering theory of Glauber ¹⁾. The results of the calculation are as follows. For elastic coherent production we find, using a product wave function for the target ¹⁾ and assuming spin and isospin independent interactions

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt}\right)_0 \frac{4}{(\sigma_1 - \sigma_2)^2 + (\alpha_1 \sigma_1 - \alpha_2 \sigma_2)^2} \left| \int J_0(qb) \left[e^{-\frac{1}{2}(1-i\alpha_1)\sigma_1 T(b)} - e^{-\frac{1}{2}(1-i\alpha_2)\sigma_2 T(b)} \right] db \right|^2$$

where 1 and 2 refer to the incident and produced particles respectively, σ_i are their total cross-sections in interaction with nucleons, and α_i the ratio of real to imaginary parts of the scattering amplitudes on nucleons, $(d\sigma_0/dt)$ is the corresponding two-body production cross-section and $t = -q^2$, $\vec{q}$ being the momentum transfer to the nucleus.

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$(d\sigma_0/dt)_0 \equiv (d\sigma_0/dt)_{t=0}$. $J_0(qb)$ is the cylindrical Bessel function of order zero.

$$T(b) = A \int_{-\infty}^{\infty} \rho(r) dz \quad (2)$$

where $\rho(r)$ is the one-particle density function in the target which we will take to have Woods-Saxon form.

For purely imaginary scattering amplitudes (1) becomes

$$\frac{d\sigma^{(c)}}{dt} = \left(\frac{d\sigma_0}{dt} \right)_0 \frac{4}{(\sigma_1 - \sigma_2)^2} \left| \int J_0(qb) \left[e^{-\frac{1}{2}\sigma_1 T(b)} - e^{-\frac{1}{2}\sigma_2 T(b)} \right] d^2b \right|^2 \quad (3)$$

For $t=0$ we have that (3) becomes a simple multiple of the two-body production cross-section,

$$\left(\frac{d\sigma^{(c)}}{dt} \right)_0 = \left(\frac{d\sigma_0}{dt} \right)_0 N^2 \left(\frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2 \right). \quad (4)$$

where

$$N\left(\frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2\right) = \frac{2}{\sigma_2 - \sigma_1} \int \left[e^{-\frac{1}{2}\sigma_1 T(b)} - e^{-\frac{1}{2}\sigma_2 T(b)} \right] d^2b \quad (5)$$

Incoherent production corresponds to summing production amplitudes squared over all final nuclear states in the region qR considerably greater than unity. We find to good accuracy for $|t| \lesssim 0.1(\text{GeV}/c)^2$ that the incoherent production cross-section

$$\begin{aligned} \frac{d\sigma^{(I)}}{dt} \cong & \frac{d\sigma_0}{dt} \frac{1}{(\sigma_2 - \sigma_1)} \int [e^{-\sigma_1 T(b)} - e^{-\sigma_2 T(b)}] d^2b \\ & + \frac{1}{(\sigma_2 - \sigma_1)} \frac{1}{k_2^2} \int d^2q' |f_{12}(q')|^2 \int d^2b \left\{ \frac{1}{\sigma_2 - \sigma_1} (e^{-\sigma_2 T(b)} - e^{-\sigma_1 T(b)}) (|f_{11}(q-b)|^2 - |f_{22}(q-b)|^2) \right. \\ & \left. + T(b) (e^{-\sigma_1 T(b)} |f_{11}(q-b)|^2 - e^{-\sigma_2 T(b)} |f_{22}(q-b)|^2) \right\} \end{aligned}$$

where f_{ii} are the particle-nucleon scattering amplitudes and k_2 is the produced particle wave number.

The first integral on the right side of (6) is simply a multiple of the two-body production cross-section. It corresponds to production preceded by and followed by absorption in the nucleus. The second term gives corrections due to single scattering before or after production. It is a relatively small correction for $t \lesssim 0.1(\text{GeV}/c)^2$. Roughly, its size relative to the first term is in the ratio σ_{el}/σ_{tot} for the particles involved. We have then to a good approximation in the multi-GeV region using the definition

$$N(\sigma_1, \sigma_2) = \frac{1}{\sigma_2 - \sigma_1} \int [e^{-\sigma_1 T(b)} - e^{-\sigma_2 T(b)}] d^2b \quad (6')$$

$$\frac{d\sigma^{(I)}}{dt} = \frac{d\sigma_0}{dt} N(\sigma_1, \sigma_2) \quad (7)$$

Formulae (6) and (7) can also be obtained from simple probability considerations. We leave this to the reader. We have calculated $N(\sigma_1, \sigma_2)$ for incident pions ($\sigma_1 = 26$ mb) and incident γ rays ($\sigma_1 = 0$) for several nuclei as a function of σ_2 . The results are in Tables 1 and 2.

These results indicate reasonable sensitivity to the unknown produced particle-nucleon cross-section, σ_2 . A study of Tables 1 and 2 and formulae (4) and (7) show clearly that for reasonable sized total cross-sections σ_i , it is only surface nucleons, mainly near $z=0$, (incident beam in z direction), that contribute appreciably to the production processes. Incident or produced particles with small impact parameters have a small chance of being effective in peripheral production such as is studied here. This is less true for incident photons of course. Since the nuclear surface is such an important region here it is important that we represent it accurately in our calculations. A square or Gaussian shape is often inadequate ^{2),3)}. Used together, formulae (3), (4) and (6) or (7) provide good possibility for the determination of many unstable particle cross-sections. When coherent elastic production is not possible because of selection rules, formula (6) is still available. Its use requires knowledge of the production angular distribution in a reasonable t range near $|t|=0.1(\text{GeV}/c)^2$ where one would see an angular distribution similar to $d\sigma_0/dt$, the two-body distribution.

We have analyzed the available data ⁴⁾ for photo-production of ρ^0 mesons in Cu and Pb using both coherent data extrapolated to $t=0$ and incoherent production data in the range $|t|=0.05$ to $0.07(\text{GeV}/c)^2$. We take the two-body production cross-section in the energy range $E_\gamma = 3-6 \text{ GeV}$ to be ⁵⁾

$$\frac{d\sigma}{dt} = (142 \pm 12) e^{-7|t|} \mu\text{b}/(\text{GeV}/c)^2/\text{Ster} \quad (8)$$

Our results are as follows: from coherent production on Cu and Pb, $\sigma_{\text{tot}}(\rho^0\text{N}) = 25$ and 26 mb respectively; from the ratio of the two coherent cross-sections, which eliminates the need for the results of Eq. (8), $\sigma_{\text{tot}}(\rho^0\text{N}) = 27 \text{ mb}$; and from incoherent production on Pb, $\sigma_{\text{tot}}(\rho^0\text{N}) = 26 \text{ mb}$. We deduce the value $N(\text{Pb}) = 63 \pm 5$ from the incoherent region data. It is to be noted that it is necessary to include the scattering corrections in formula (6) as they are of the order of 20%.

There is a small (3%) double scattering correction in this case which we do not discuss in detail. We further remark that in accordance with the theory above, the full production cross-section at t values corresponding to qR considerably greater than unity ($|t| \approx 0.05 \text{ (GeV/c)}^2$) is taken to be incoherent. Taking the four values of $\sigma_{\text{tot}}(\rho^0 N)$ together, we find $\sigma_{\text{tot}}(\rho^0 N) = 26 \pm 3 \text{ mb}$ for ρ^0 mesons of about 4 GeV/c. The agreement of the coherent and incoherent production results is some evidence that the neglect of the real part of the ρ nucleon scattering amplitude in formula (1) is justified in this case.

We have taken the nuclear density to be

$$\rho = \rho_0 \left[1 + e^{(r-c)/a} \right]^{-1} \quad (9)$$

with $C = 1.14 \text{ A}^{1/3}$ fermis and $d = 0.545$ fermis. This form has been found accurate in high energy quasi-elastic scattering of protons ²⁾.

Our answer for $\sigma_{\text{tot}}(\rho^0 N)$ is to be compared with the result $\sigma_{\text{tot}}(\rho^0 N) = 31.3 \pm 2.3 \text{ mb}$ obtained ⁴⁾ using a forward coherent production theory ⁶⁾. A more detailed exposition of this work including coherent angular distributions is in preparation.

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TABLE CAPTIONSTABLE 1 :

Effective nucleon number N from formula (6') for various targets for incident particles with total cross-section on a nucleon, $\sigma_1 = 26$ mb, as a function of produced particle total cross-section σ_2 . The nuclear shape is given by formula (9).

TABLE 2 :

Effective nucleon number N from formula (6') for photoproduction processes as a function of σ_2 , the total cross-section of a produced particle incident on a nucleon. The nuclear shape is given by formula (9).

TABLE 1

$\sigma_2(\text{mb})$	0	15	30	45
$N(\text{Al}^{27})$	15.3	10.4	7.73	6.18
$N(\text{Cu}^{64})$	28.4	15.7	10.6	8.10
$N(\text{Ag}^{108})$	40.7	19.2	12.3	9.29
$N(\text{Pb}^{208})$	63.2	23.9	14.5	10.9

TABLE 2

$\sigma_2(\text{mb})$	10	20	30	40	50
$N(\text{Al}^{27})$	21.0	17.0	14.3	12.2	10.7
$N(\text{Cu}^{64})$	44.0	32.8	26.0	21.5	18.4
$N(\text{Ag}^{108})$	67.9	48.1	36.9	30.0	25.3
$N(\text{Pb}^{208})$	115	76.3	56.7	45.2	37.7