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PRODUCTION OF PIONS BY PROTONS ON NUCLEI AND NUCLEAR SHAPE

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A B S T R A C T

An isobar model for the production of pions by 600 MeV protons on nuclei is studied. The higher energy pions are predicted to come from incoherent nucleon-nucleon production of the (3,3) isobar in the nuclear surface. One finds considerable sensitivity of the production to the nuclear surface properties. Agreement is found with nuclear surface properties determined by multi GeV incoherent quasi-elastic scattering of nucleons by nuclei. The results here can be easily generalized for neutron projectiles. It is possible that the incoherent production model studied here will be useful in determining high energy unstable particle-nucleon total cross-sections.

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INTRODUCTION

Recently there have been detailed measurements of the production of charged pions in nuclei by incident protons of energy 600 MeV ¹⁾. The measurements are of interest for the planning of meson factories and perhaps for more fundamental reasons as well. A striking feature of the measurements is that the cross-section for producing positive pions (Fig.1) is several times bigger than that for producing negative pions (Fig.2). The π^+ cross-sections show strong peaking for outgoing pions of kinetic energy slightly over 200 MeV. The π^- production cross-sections show a suggestion of such a peak on a spectrum with more strength at lower energies.

For protons of several hundred MeV, the wave-length is small enough that it is a good approximation in calculating reactions to consider the target nucleons as acting more or less independently with the incident proton. A knowledge of nucleon-nucleon cross-sections as well as pion-nucleon cross-sections together with Monte Carlo techniques should then be sufficient to reproduce the measured spectra. On the other hand, it is of interest to discuss pion production by protons on nuclei from another point of view to gain physical understanding. To this end we examine the possibility that an isobar model is applicable ^{2),3)}.

THE ISOBAR MODEL

We assume that the primary reaction resulting in pion production in nuclei is

where n_i and n_f are nucleons and where Δ is a charge state of the (3,3) nucleon isobar. We can then produce positive pions through the reactions :

2.

$$p + p \rightarrow \Delta^+ + n \rightarrow \pi^+ + p + n \quad (1.a)$$

$$p + p \rightarrow \Delta^+ + p \rightarrow \pi^+ + n + p \quad (1.b)$$

$$p + n \rightarrow \Delta^+ + n \rightarrow \pi^+ + n + n \quad (1.c)$$

and negative pions through the reaction

$$p + n \rightarrow \Delta^0 + p \rightarrow \pi^- + p + p \quad (1.d)$$

We neglect double isobar production which is barely possible because of energetics.

Using charge independence and Clebsch-Gordan algebra, the relative strengths ³⁾ of the matrix elements squared for reactions (1.a) to (1.d) are respectively 9, 1, 1 and 1. Neglecting interference terms, one then finds that the ratio for the production of positive, relative to negative mesons, in nuclei, is given by $(10Z+N)/N$. Positive meson production is then strongly favoured.

The processes $(p; \Delta, n)$ and $(p; \Delta, p)$ in nuclei are similar to reactions that have been studied extensively by nuclear physicists ⁴⁾, namely $(p; 2p)$ and $(p; p, n)$ reactions. In these reactions the final baryons can be emitted with a range of energies and momenta.

For isobar production, with incident protons of 600 MeV, the available energy for the isobar will be 300 MeV less the amount shared by the struck nucleon and the residual nucleus. There will be a small amount of nuclear rearrangement energy to take into account as well.

Simple kinematic considerations show that it is reasonable on this basis to expect a peak in the pion energy spectrum, resulting from isobar decay, at around 200 MeV kinetic energy. The width of the peak reflects the width of the $(3,3)$ resonance and the angular and energy distribution of the emitted isobars. The positive pion spectrum of Fig. 1 indeed shows strong peaking at around 200 to 225 MeV kinetic energy. The lighter target nuclei yield negative pion spectra with a similar peak several times weaker. The heavier nuclei have the semblance of a negative pion peak superimposed on a spectrum with more strength at lower energies. The ratio of positive to negative pion production, $(10Z+N)/N \sim 7$ to 11, predicted on the basis of the isobar model seems to be borne out for the higher energy spectra. The lower energy pions are on this model expected to come mainly from secondary reactions. There should be a similar low energy positive pion spectrum as well. Recent preliminary measurements ⁵⁾ indicate that this is so.

The dependence of the cross-sections on the atomic number is discussed now. The initial and final baryons will experience strong damping due to nuclear interactions with the target and residual nuclei. It is therefore mainly surface nucleons ⁶⁾ in a ring with polar angle $\theta = \pi/2$, (taking the incident beam in the z direction), that will be effective in producing reactions (1.a) to (1.d) in nuclei. The number of such nucleons is roughly $A^{1/3}$ and so the isobar production cross-section should vary with target mass number in this way. In detail we shall see that the cross-section depends sensitively on the nuclear shape. The low energy pions, in this model, will be produced mainly by secondary reactions throughout the nucleus. The production of these should show a different and stronger dependence on A , more like $A^{2/3}$. This stronger A dependence is evident in Fig. 2.

We now turn to more quantitative consideration of reactions (1.a) to (1.d) in nuclei. Glauber ⁶⁾ has derived, on the basis of multiple scattering theory, the cross-section for incoherent, quasi-elastic scattering of nucleons from nuclei. Provided one considers only single quasi-elastic scattering, Glauber finds

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{inel.}} = |f(q)|^2 \int e^{-\sigma T(\vec{b})} T(\vec{b}) d^2B \quad (2)$$

where σ is the total nucleon-nucleon cross-section, assumed charge and spin independent,

$$T(\vec{b}) = A \int_{-\infty}^{\infty} \rho(r^2) dz \quad (2.a)$$

The quantity ρ is the one particle nuclear density, assumed equal for neutrons and protons; $(\vec{b}, z)$ are cylindrical co-ordinates, b playing the role of impact parameter. $T(\vec{b})$ is called the surface thickness of the target. Formula (2) is obtained by summing high energy inelastic scattering cross-sections over final target states at fixed momentum transfer. One has in mind then a high energy scattering experiment of imperfect energy resolution. The quantity $f(q)$ is the nucleon-nucleon scattering amplitude. The integral on the right side of (2) represents the effective number of nucleons contributing to this slightly inelastic or quasi-elastic incoherent scattering.

Figure 3 shows the quantity N_1 , calculated by Glauber⁶⁾, for 19.3 GeV/c scattering of protons by nuclei assuming a square well shape and a Wood-Saxon shape for $\rho(r)$. Also shown are results for N_1 deduced by Glauber from measurements of Belletini et al.⁷⁾. The sensitivity of quasi-elastic scattering to nuclear density is evident. The energy resolution in the experiment was about 50 MeV and the data used correspond to momentum transfers too large to support coherent scattering ($qR \gg 1$).

The considerations of Glauber are readily generalized to the case of resonance production. This has already been done for coherent production⁸⁾.

The generalization of (2) to (3,3) isobar production, with incident protons, taking into account the relative weighting of the cross-sections (1.a) to (1.d) is, in the impulse approximation,

$$\frac{d\sigma}{d\Omega} \cong \frac{1}{A} \left(Z + \frac{1}{10} N \right) N_1^I |f^*(q)|^2 \quad (3)$$

where $|f^*(q)|^2$ is the differential cross-section for isobar production in p-p interaction corresponding to a momentum transfer q .

$$N_1^I = \int d^2\ell \left[e^{-\sigma_1 T(\ell)} - e^{-\sigma_2 T(\ell)} \right] / (\sigma_2 - \sigma_1) \quad (4)$$

Here σ_1 and σ_2 are, respectively, the nucleon-nucleon and isobar nucleon total cross-sections assumed spin and isospin independent. For $\sigma_1 = \sigma_2$, $N_1^I = N_1$. The factor $1/A(Z + \frac{1}{10}N)$ takes into account all isobar production processes which have a positive pion as a decay product.

We now discuss the application of formula (3) to the data of Figs. 1 and 2. For 600 MeV incident protons, coherent production of the (3,3) isobar should be small. This is because of the large momentum transfer required even in the forward direction. It is to be noted that the outgoing isobar has less than 300 MeV in kinetic energy and hence one can question the detailed validity of (3). We shall nevertheless proceed to use it. The total p-p and p-n cross-sections at 600 MeV are roughly equal to each other, and as it happens, to the nucleon-nucleon total cross-section at 19.3 GeV/c. The values for N_1^I for isobar production at 600 MeV should then be similar to N_1 for quasi-elastic scattering at 19.3 GeV/c, if $\sigma_2 \approx \sigma_1$. Lacking information on σ_2 , we will assume it equal to σ_1 . Integrating (3) over all angles, we get

$$\sigma_{\text{prod}} \cong \frac{1}{A} \left(Z + \frac{1}{10} N \right) N_1^I \sigma^I \quad (5)$$

where σ_{prod} is the total isobar production cross-section in the nucleus (Z, N) leading to π^+ emission and σ^I is the similar cross-section in p-p interaction. It is to be noted that Eq. (3) says that the isobar production cross-section is a function of angle, given by two-body data alone, times an angle independent function of nuclear properties. This means that the decay meson distribution should have this same property. Hence it is sufficient to have meson production data at one angle. However, in order to minimize experimental uncertainty it is perhaps preferable to use total cross-section data when available. Not having complete angular distributions of the emitted pions, we sum the peak cross-sections at the two angles where they are known. In Fig. 3 we show the resulting values for N_1 using formula (3) and the positive meson production data of Heer et al. ¹⁾, along with the high energy p-p quasi-elastic results ⁶⁾.

It is to be noticed that once we know the nucleon isobar total cross-section, σ_2 , there are absolutely no parameters in the determination of N_1^I using Eq. (3) or (5). The normalization is determined by the free p-p meson production ¹⁾. This being the case, the accurate determination of the nuclear size and shape across the periodic table is nothing less than remarkable. We remark, without showing results of our calculation that the assumption of a Gaussian nuclear shape produces strong disagreement with experiment as does the assumption of a square shape.

In the investigation here we have found consistency with $\sigma_2 \approx \sigma_1$. We have then a rough determination of σ_2 , in view of the agreement of theory and experiment. In a subsequent paper we plan discuss the sensitivity of incoherent isobar and other unstable particle production to unstable particle-nucleon total cross-section variation. It is possible that incoherent production data will yield accurate total unstable particle-nucleon cross-sections at high energies assuming we know the nuclear surface shape.

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R E F E R E N C E S

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FIGURE CAPTIONS

- Figure 1 : The cross-sections for positive pion production from Ref. 1). The incident proton laboratory kinetic energy is 600 MeV.
- Figure 2 : The cross-sections for negative pion production from Ref. 2). The incident proton laboratory kinetic energy is 600 MeV.
- Figure 3 : The effective nucleon number N_1 for incoherent quasi-elastic processes calculated ⁶⁾ using formula (4). The Woods-Saxon density has a radius $c = 1.07 A^{1/3}$ fermi and surface thickness $t = 2.4$ fermi, this distance corresponding to a 90% to 10% fall off in density. The experimental points are from Refs. 1) and 7). N_1 is deduced, for the meson production data, using formula (3). The sum of the cross-sections (at the two angles 0.8° and 21.5°) for π^+ production in p-p interaction ¹⁾ is taken to be 36 microbarn/sterad/MeV.

FIG. 1

π^-
0.8°

π^-
21.5°

FIG. 2

FIG. 3