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LOW ENERGY THEOREM CONNECTING WEAK PION PRODUCTIONAND TWO PION PHOTOPRODUCTIONP. Narayanaswamy ^{*)} and B. Renner ^{**)}

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A B S T R A C T

($N^*\pi$) photoproduction from nucleons is connected to the matrix elements of the weak axial current between nucleons and $N^*(1236)$ states as a low energy theorem by current algebras. Using predictions for these matrix elements, partly again from current algebras, the threshold cross-section of $\gamma N \rightarrow \pi N^*$ is worked out, so providing a possible consistency test.

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1. DERIVATION OF THE LOW ENERGY THEOREM

Many remarkable results from current algebras ¹⁾ take the form of low energy theorems. One class of these, with the Callan-Treiman relation ²⁾ as its first example, connects matrix elements of an axial vector current $\langle a | J_{\mu 5} | b \rangle$ to matrix elements of a vector current with an additional pion emitted (or absorbed) : $\langle a \pi | J_{\mu} | b \rangle$. In this paper we will compare the matrix element of the weak axial current between nucleon and $N^*(1236)$ states $\langle N^{*\alpha} | J_{\mu 5}^{\beta} | N^{\gamma} \rangle$, which is under investigation in neutrino experiments, to the matrix element of the electromagnetic current $\langle N^{*\alpha} \pi^{\beta} | J_{\mu}^e | N^{\gamma} \rangle$, describing $(N^* \pi)$ photoproduction from nucleons. We will treat the N^* resonance as quasi-stable, using the conventional formalism of a spin $\frac{3}{2}$ particle. α, β, γ describe isospin states.

As usual, we start from the identity

$$\begin{aligned} 0 &= \int d^4x \delta(x_0) e^{iq \cdot x} \langle N^{*\alpha}(p^*) | [J_{05}^{\beta}(x), J_{\mu}^e(0)] | N^{\gamma}(p) \rangle \\ &+ \int d^4x \theta(x_0) i q_{\lambda}^{\gamma} e^{iq \cdot x} \langle N^{*\alpha}(p^*) | [J_{\lambda 5}^{\beta}(x), J_{\mu}^e(0)] | N^{\gamma}(p) \rangle \\ &+ \int d^4x \theta(x_0) e^{iq \cdot x} \langle N^{*\alpha}(p^*) | [\partial^{\lambda} J_{\lambda 5}^{\beta}(x), J_{\mu}^e(0)] | N^{\gamma}(p) \rangle \end{aligned}$$

(definition)

$$= I_{\mu}^{(1)}(q^2, s, t) + I_{\mu}^{(2)}(q^2, s, t) + I_{\mu}^{(3)}(q^2, s, t). \quad (1)$$

The integrals $I_{\mu}^{(i)}$ are vectors under Lorentz transformations and may contain 16 independent kinematic forms, of which 12 have the vector q as a factor. A low energy theorem can be derived only for the first four invariants $A_n^{(i)}$: $(n=1-4)$

$$\begin{aligned}
I_{\mu}^{(i)}(q^2, s, t) = & (\bar{u}_\nu k^\nu \gamma_\mu u) A_1^{(i)}(q^2, s, t) \\
& + (\bar{u}_\mu u) A_2^{(i)}(q^2, s, t) \\
& + (\bar{u}_\nu k^\nu p_\mu^* u) A_3^{(i)}(q^2, s, t) \\
& + (\bar{u}_\nu k^\nu p_\mu u) A_4^{(i)}(q^2, s, t) \\
& + O_\mu(q) .
\end{aligned} \tag{2}$$

Here we have denoted the N^* spinor by $\bar{u}_\nu$, the N spinor by u ; we have introduced $k = p^* + q - p$ as the momentum attached to $J_\mu^e(0)$; $s = (p+k)^2 = (p^*+q)^2$, $t = (p^*-p)^2 = (q-k)^2$. We will fix $k^2 = 0$ to make contact with photoproduction.

Choosing the vector q timelike we can always find a frame of reference where $I_{\mu}^{(1)}$ can be written as an equal-time commutator of an axial charge with $J_\mu^e(0)$:

$$I_{\mu}^{(1)} = \langle N^{*\alpha}(p^*) | \left[\int J_{05}^\beta(x) d^3x, J_\mu^e(0) \right] | N^\gamma(p) \rangle \tag{3}$$

by the $SU(2) \times SU(2)$ current algebra ¹⁾. By covariance we generalize Eq. (3) to arbitrary frames, taking the point of view ³⁾ that possible non-covariant contributions to Eq. (1) are paired off with possible Schwinger terms in Eq. (3) and are irrelevant for physical applications ⁴⁾.

The matrix element of the axial current in Eq. (3) is independent of q_μ , so $I_{\mu}^{(1)}$ gives contributions in Eq. (2) only in the forms $A_n^{(1)} = A_n^{(1)}(t)$ for $n = 1, 2, 3, 4$ and $A_n^{(1)} \equiv 0$ for $n > 4$.

To dispose of the integral $I_{\mu}^{(2)}$, one considers the limit $q^{\lambda} \rightarrow 0$, where only the Born terms of Fig. 1 and Fig. 2 may survive. These Born terms can always be combined with the corresponding Born terms of $I_{\mu}^{(3)}$: $I_{\mu}^{(2)} + I_{\mu}^{(3)}$ (definition) $I_{\mu}^{(B)} + I_{\mu}^{(R)}$, with $(iq^{\lambda} J_{\lambda 5})$ replaced by $(\partial^{\lambda} J_{\lambda 5})$, to produce a well defined expression⁵⁾, irrespective of how the limit $q^{\lambda} \rightarrow 0$ is taken.

This can be seen quite generally by recalling⁶⁾ that this class of Born terms is generated by $(iq^{\lambda} J_{\lambda 5})$ or $(\partial^{\lambda} J_{\lambda 5})$ attached to external baryon lines (E in Fig. 3); the baryon propagator of the internal line (I) becomes singular in the limit $q \rightarrow 0$. If both baryon lines are on the mass shell, then the Dirac equation (and with higher spins, also transversality conditions) can be used to show that the insertion of $(\partial^{\lambda} J_{\lambda 5})$ or $(-iq^{\lambda} J_{\lambda 5})$ into the vertex produces equivalent expressions. Consequently at the pole the sum of the two residues cancels in Eq. (1) and only a continuous subtraction polynomial remains in $I_{\mu}^{(B)}$. We note in passing that this subtraction polynomial is not sensitive to small mass shifts and hence the device of using electromagnetic or even fictitious mass differences as a regularization^{7),8)} will never lead to any incorrect results.

In calculating the Born terms at $q^{\lambda} \rightarrow 0$, $s = M^2$, $t = 0$, we use the vertices :

$$\langle N^{*\alpha}(p^*) | J_{\mu}^e(0) | N^{\beta}(p) \rangle = (\bar{u}_{\nu} \gamma_{\mu} \gamma_5 K^{\nu} u - \bar{u}_{\mu} \gamma_{\nu} K^{\nu} \gamma_5 u) C_1(K^2) \quad (4)$$

$$\langle N^{*\alpha} | \partial^{\lambda} J_{\lambda 5}^{\beta}(0) | N^{\gamma} \rangle = \bar{u}_{\lambda} \gamma_5 [g^{\lambda\nu} B_1(\Delta^2) + \Delta^{\nu} \Delta^{\lambda} B_2(\Delta^2)] 2M u_{\nu}^{\gamma} \quad (5)$$

$$\langle N^{\alpha} | \partial^{\lambda} J_{\lambda 5}^{\beta}(0) | N^{\gamma} \rangle = \bar{u}^{\alpha} \gamma_5 u^{\beta} D(\Delta^2) 2m \quad (6)$$

where m and M are the masses of N and N^* . In Eq. (4) we have retained only the dominant one of the three couplings⁹⁾. For later detailed computations, the isospin dependence of the vertices has to be put in by Clebsch-Gordan coefficients which, for convenience, has been absorbed into the definition of the coupling constants here. By Goldberger-Treiman relations we have :

$$2m D(0) = -f G \quad (7a)$$

$$2M B_1(0) = -f G^* \quad (7b)$$

where G and G^* are the πNN and $\pi N^* N^*$ coupling constants and f is the pion decay constant in the (unconventional) normalization :

$$\frac{1}{\sqrt{2}} \langle 0 | (\partial^\lambda J_{\lambda 5}^1 - i \partial^\lambda J_{\lambda 5}^2) | \pi^+ \rangle = f \mu^2$$

μ being the pion mass. The N^* propagator has been taken in the form of Salin¹⁰⁾

$$\frac{\gamma p + M}{p^2 - M^2} \left[-g_{\lambda\nu} + \frac{1}{3} \gamma_\lambda \gamma_\nu + \frac{(\gamma p)_\lambda p_\nu + \gamma_\nu p_\lambda (\gamma p)}{3 p^2} \right] \quad (8)$$

In the limit $q \rightarrow 0$, we obtain :

$$I_\mu^{(B)}(0, M^2, 0) = i(\bar{u}_\nu \gamma_\mu \gamma_\nu u - \bar{u}_\mu \gamma_\nu \gamma_\mu u) C_1(0) [B_1(0) + D(0)] \quad (9)$$

In the remaining integral $I_{\mu}^{(R)}$ we assume that the most significant contribution is the pion pole at $q^2 = \mu^2$ and we will represent suitable invariant matrix elements $A_r^{(R)}$ of $I_{\mu}^{(R)}$ by the corresponding invariants of the photoproduction amplitude $\langle N^{*\alpha} \pi^{\beta} | J_{\mu}^e(0) | N^{\gamma} \rangle^{(R)}$, where the superscript (R) indicates that the Born terms of Figs. 1 and 2 have been separated out

$$I_{\mu}^{(R)} \approx (-i) \mu^2 \langle N^{*\alpha} \pi^{\beta} | J_{\mu}^e(0) | N^{\gamma} \rangle^{(R)} \frac{1}{\mu^2 - q^2} \quad (10)$$

Equation (10) expressing pion pole dominance ¹¹⁾ or PCAC ¹²⁾ should be accepted only under certain precautions :

- a. For different choices of invariant amplitudes, pole models will in general give inequivalent results. With the choice ¹³⁾ as in Eq. (2) we have not found any pathologies, but this does not imply, of course, that it is the most appropriate one.
- b. In representing a function of several variables (q^2, s, t) by a pole in one of them, there is an ambiguity as to which combinations of the others should be kept fixed in the analytic continuation ¹⁴⁾. By current algebras we estimate $I_{\mu}^{(R)}$ at $q=0$ which can at best be used to estimate the amplitude $\gamma N \rightarrow \pi N^*$ at the reaction threshold. So we have not only a change from $q^2=0$ to $q^2=\mu^2$, but also an extrapolation in s, t and u . For this purpose, the Born terms of Figs. 1 and 2 have been separated out and the amplitudes $\langle N^{*\alpha} \pi^{\beta} | J_{\mu}^e | N^{\gamma} \rangle^{(R)}$ have been formed. Neglecting in these amplitudes the variations caused by extrapolating s, t, u from $q^2=0$ to threshold would imply, for instance, neglecting the variations produced by the graphs of Figs. 4, 5 and 6.

Graph 4 can contribute only to A_3 and A_4 , graph 5 does not contribute at all to $A_1 - A_4$ and graph 6 contributes only $O(q)$. This means that for A_1 and A_2 the extrapolation may be approximately valid. Later we will see that we just need these two amplitudes.

Inserting Eqs. (3), (9) and (10) into Eq. (2) and specializing to $q^\lambda = 0$, we derive for $n=1$ to 4, the low energy theorem :

$$0 \approx A_n^{(1)} + A_n^{(B)} + (-i) \int \langle N^{*\alpha} \pi^\beta | J_\mu^e(0) | N^\gamma \rangle_n^{(R)} \quad (11)$$

representing the contributions coming from Eqs. (3), (9) and (10).

2. DEDUCTIONS FROM THE LOW ENERGY THEOREM

In Eq. (11), the matrix elements $A_n^{(1)}$ of the weak axial current between nucleons and N^* states are under experimental investigation. At present we have to use the theoretical predictions from static model calculations and dispersion theory ^{15),16)}, higher symmetries ^{17),18)} and truncated current algebra sum rules ^{18),19)}. These are given in the Table.

The Born terms $A_n^{(B)}$ are given in Eq. (9) for $q=0$ and in Eq. (12) below for the physical amplitude :

$$2 G C_1 (m^2 - u)^{-1} [\bar{u}_\nu(\gamma q) \gamma_\mu k^\nu u - \bar{u}_\mu(\gamma q) \gamma_\nu k^\nu u] \\ + [G^* C_1 (s - M^2)^{-1} - G C_1 (m^2 - u)^{-1}] [\bar{u}_\nu(\gamma q) \gamma_\mu k^\nu u - \bar{u}_\mu(\gamma q) \gamma_\nu k^\nu u] \quad (12)$$

At threshold, we have the simplification : $\bar{u}_\nu (\gamma q) = \mu \bar{u}_\nu$. Further, we drop the first term as being $O(\mu)$. The terms of Eq. (12) have to be subtracted from the amplitudes T_μ as measured in $\gamma N \rightarrow \pi N^*$, to obtain $\langle N^* \pi | J_\mu^e | N \gamma \rangle_n^{(R)}$ in Eq. (11).

We substitute the Born terms from Eqs. (9) and (12) into (11) and use the Goldberger-Treiman relations (7). In the approximation $s - M^2 = 2M\mu + \mu^2 \approx 2M\mu$, we find that the Born terms of Fig. 1 cancel each other. This can be understood directly by recalling that the final (πN^*) configuration is in a $J^P = (3/2)^-$ state at threshold and therefore not appreciably influenced by the $J^P = (3/2)^+$ s channel contributions. We are left with the Born terms of Fig. 2. These are absent in the special case of $\gamma p \rightarrow N^{*++} \pi^-$ to be considered first. Equation (11) now takes the simple form

$$O \approx A_n^{(1)} - i f T_n \quad (n = 1 - 4) \quad (13)$$

At threshold, the centre of mass system is the common rest system of N^* and π ; choosing the polarization ϵ_μ of the incoming photon such that $\epsilon_0 = 0$, $\epsilon \cdot k = 0$ (consequently, $\epsilon \cdot p^* = \epsilon \cdot q = \epsilon \cdot p = 0$) we find

$$\epsilon_\mu T_\mu = (\bar{u}_\nu k^\nu \gamma_\mu \epsilon^\mu u) T_1 + (\bar{u}_\mu \epsilon^\mu u) T_2 + O(\mu) \quad (14)$$

The (unnormalized) polarization sum gives the result :

$$\frac{16}{3} M \left\{ E_\gamma^2 (E_N - m) |T_1|^2 - \frac{1}{2} E_\gamma^2 (T_1 T_2^* + T_1^* T_2) + (E_N + m) |T_2|^2 \right\} \quad (15)$$

The cross-section follows from Eq. (15) near threshold :

$$\sigma_t = \frac{e^2}{6\pi} \frac{M}{s-m^2} \frac{P^b}{\sqrt{s}} \left\{ E_\gamma^2 (E_N - m) |T_1|^2 - \frac{1}{2} E_\gamma^2 (T_1 T_2^* + T_1^* T_2) + (E_N + m) |T_2|^2 \right\} \quad (16)$$

where P^f denotes the spatial momentum of π and N^* in the centre of mass system. For the different predictions of $A_i^{(1)}$ in the Table, the threshold cross-section σ_t for $\gamma p \rightarrow N^{*++} \pi^-$ is computed. It has the form $\sigma_t = S(\frac{P^f}{\mu})$; S is given in the last column.

Unfortunately the threshold value of the cross-section is not known experimentally; data have been analyzed only from $\sqrt{s} \approx 1700$ MeV upwards, where the s wave cross-section has been measured ²⁰⁾ with about 70 μ b. An attempt to extrapolate from threshold (1375 ± 60 MeV) to 1700 MeV meets serious difficulties, because we are passing, for instance, the resonance $N^*(1525), J^P = (3/2)^-$ which is connected to the $(N_{33}^* \pi)$ system in s wave, and also the resonances $N^*(1400)$ and $N^*(1670)$ which can be reached by $(N_{33}^* \pi)$ in p wave.

It might be very difficult to extract any precise value for σ_t from the data on $\gamma p \rightarrow p \pi^+ \pi^-$ due to the finite N^* width.

This work is related to a certain extent to a recent paper by Carruthers and Huang ²¹⁾ who compared the low energy value of $\langle N \pi(\pi) | J_\mu | N \rangle$ to $\langle N \pi | J_{\mu 5} | N \rangle$, not working in the isobar approximation. Though we agree with many of their conclusions, we preferred to present some details differently.

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T A B L E

Source	Ref.	A_1	A_2	$\sigma_t \cdot \left(\frac{\mu}{P_f}\right)$ (in microbarn)
Static model and Goldberger-Treiman relations	15)	0	-0.9	3.3
SL(6,C): symmetry limit	18)	0	-1.41	8.1
SL(6,C): broken symmetry with physical masses	18)	0	-1.04	4.3
Truncated current algebra sum rule	19)	$\frac{18.7}{m+M}$	-0.64	4.05
Truncated current algebra sum rule: equal masses	18)	0	-1.41	8.1

This table is normalized to $\langle N^{*++} | \frac{1}{\sqrt{2}} (J_{\mu 5}^1 + i J_{\mu 5}^2) | p \rangle$; the references above have different normalizations and sometimes also a different metric.

The predictions of σ_t do not strongly disagree and the errors of the extrapolation may easily be of the same order as the variations among the results. We suggest to accept as a rough prediction: $\sigma_t \approx 5 \left(\frac{P_f}{\mu}\right)$ microbarn. The controversial predictions of axial form factors are made about A_3 and A_4 ; in this derivation they do not affect σ_t , but with a choice different from the kinematic forms in Eq. (2), they can enter. This point is under further study.

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FIG. 1

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG. 6