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LOW ENERGY PION-NUCLEON PHASE SHIFTS
FROM A PADE APPROXIMANTS' APPROACH

E. Remiddi

CERN - Geneva

M. Pusterla

University di Padova
INFN, Sezione di Padova

J.A. Mignaco ⁺⁾

Université de Genève, Genève

A B S T R A C T

In the framework of the Padé approximants' technique, a parametrization for low energy πN elastic scattering is given. A fit of the corresponding phase shifts up to 1600 MeV of c.m.s. energy is discussed.

⁺⁾ Fellow of Consejo Nacional de Investigaciones Cientificas y Técnicas de la Republica Argentina.

We present some results on pion-nucleon elastic scattering, obtained in the framework of the Padé approximants' technique ¹⁾ for summing the (renormalized) formal perturbative development of the S matrix in a Lagrangian theory of strong interactions.

To start with, we consider the sum of the following interaction Lagrangians:

- 1) the pseudoscalar pion-nucleon interaction, with coupling constant g ($g^2/4\pi = 14.6$);
- 2) the four-pion point interaction with coupling constant $\lambda = 6.5$, value obtained in Ref. ¹⁾.

The expansion of the S matrix obtained for πN elastic scattering up to fourth order in the coupling constants reads:

$$S = 1 + g^2 S_2 + g^2 \lambda T_3 + g^2 \lambda^2 T_4 + g^4 S_4 \quad (1)$$

In this expression the relation of each term to the Feynman diagrams of the Figure is the following:

$$S_2 = \text{I} + \text{crossed}$$

$$T_3 = \text{II}$$

$$T_4 = \text{III} + \text{crossed} + \text{IV}$$

$$S_4 = \text{V} + \text{VI} + \text{VII} + \text{VIII} + \text{crossed}$$

We will indicate by W , p , E the c.m.s. total energy, momentum and proton energy, by m and μ the proton and pion masses.

Consider Eq. (1) as written in the isospin and total angular momentum representation (even without writing for simplicity the relevant indices). The corresponding diagonal Padé approximant is

$$S^P(W) = 1 + g^2 S_2(W)/J(W) \quad (2)$$

where

$$J(W) = 1 - \frac{T_3(W)}{S_2(W)} \lambda + \left[\left(\frac{T_3(W)}{S_2(W)} \right)^2 - \frac{T_4(W)}{S_2(W)} \right] \lambda^2 - \frac{S_4(W)}{S_2(W)} g^2 \quad (3)$$

Note that due to the absence of terms of the first order in the perturbative expansion, Eq. (1), this is the first diagonal Padé approximant not identically equal to 1.

We recall from Ref. ¹⁾ some of the main general properties of the diagonal Padé approximants:

- they are unitary in the elastic region $m + \mu \leq W \leq m + 2\mu$, i.e., $|S^P(W)|^2 = 1$;
- they have the same analytic properties in W as the terms of the perturbative expansion (1), plus possibly poles from the zeros of $J(W)$ (corresponding in principle to resonances or ghosts);
- they can be continued analytically from the elastic to the inelastic region, where $|S^P(W)| \leq 1$.

Let us make some comments on the shortcomings of the concerned first Padé approximant.

First, although some inelasticity will be present outside the elastic region, only diagram III in Fig. 1 can account for it, so that we cannot expect an adequate representation of inelasticity effects.

Furthermore, effects from two-pion exchange are present in Eq. (1) only through terms which are at most of second order in λ . According to Ref. ¹⁾, higher orders in λ are necessary to describe the pion-pion dynamics, so that effects due to two-pion exchange are likely to be accounted for very roughly within this first Padé approximant. Finally, contributions due to heavier mesons or baryons are absent in Eq. (1). We have calculated explicitly ^{*)} the contributions to the first partial waves (S, P, D) of all the graphs of Fig. 1. According to the above remarks, we take the attitude that the contributions of those graphs to Eq. (4) give the long range part (i.e., the contributions most rapidly varying with the energy) of the πN scattering. To include also short range contributions we ascribe them to a new term U and modify Eqs. (2) and (3) as follows:

$$J_{ph}(W) = J(W) - U(W)/S_2(W) \quad (4)$$

$$S_{ph}^P(W) = 1 + g^2 S_2(W)/J_{ph}(W) \quad (5)$$

^{*)} We used dispersive methods for expressing the various partial waves as integrals from a threshold to infinity of the discontinuity of scalar amplitudes times Legendre functions. In such a way we are left with two numerical integrations for graphs III and IV of the Figure, and a single numerical integration for the other graphs.

Among the other checks, the resulting figures agree with the corresponding ones evaluated by Dr. J.L. Gammel and Dr. J.J. Kubis (preprint).

The U term will be treated phenomenologically, with the only assumptions that it is real and satisfies the threshold conditions ^{*}).

Quite in general, we can consider the contribution to a specific partial wave written as ²⁾

$$\frac{1}{i} U_{j=l \pm \frac{1}{2}}^{(I)}(W) = p \frac{E+m}{4\pi W} \left[A_l^{(I)}(W) + (W-m) B_l^{(I)}(W) \right] \\ + p \frac{E-m}{4\pi W} \left[-A_{l \pm 1}^{(I)}(W) + (W+m) B_{l \pm 1}^{(I)}(W) \right]$$

the superscript I ($=1, 3$) referring to isospin ($\frac{1}{2}, \frac{3}{2}$). In an exact theory the $A^{(I)}(W)$ depend on I and l only, and not on j [the same $A_1^{(3)}(W)$, for instance, appears in the waves S_{31} , P_{31} , P_{33} and D_{33}]. In the present phenomenological approach to the U term we approximate the corresponding $A_l^{(I)}$, $B_l^{(I)}$ by a factor $\left(\frac{2q^2}{\mu^2}\right)^l$ (which gives the threshold behaviour) time real constants. These constants will be determined a posteriori through a fit to the experimental phase shifts ³⁾. As they are just approximations to the $A_l^{(I)}(W)$, we let them depend on the total angular momentum j , so that our phenomenological U reads:

$$U_{ph, j=l \pm \frac{1}{2}}^{(I)}(W) = p \frac{E+m}{4\pi W} \left(\frac{2p^2}{\mu^2}\right)^l \left[\frac{a_l^{(I,j)}}{\mu} + (W-m) \frac{b_l^{(I,j)}}{\mu^2} \right] \\ + p \frac{E-m}{4\pi W} \left(\frac{2p^2}{\mu^2}\right)^{l \pm 1} \left[-\frac{a_{l \pm 1}^{(I,j)}}{\mu} + (W+m) \frac{b_{l \pm 1}^{(I,j)}}{\mu^2} \right] \quad (6)$$

the $a_l^{(I,j)}$, $b_l^{(I,j)}$ being the parameters we want to fit. In such a way, for instance, we have four parameters $a_0^{(31)}$, $a_1^{(31)}$, $b_0^{(31)}$,

^{*}) One can consider U for instance as representing the contributions to the perturbative S matrix coming from more general interaction Lagrangians involving heavier mesons and baryons.

$b_1^{(31)}$ for the S_{31} and P_{31} waves, the corresponding parameters for P_{33} and D_{33} being $a_1^{(33)}$, $a_2^{(33)}$, $b_1^{(33)}$, $b_2^{(33)}$.

It is important to note that the properties of $S_{PH}^P(W)$, [Eq. (5)], are the same as those of $S^P(W)$, [Eq. (2)].

We expect a parametrization of that type to be able to agree qualitatively with the experimental phases up to energies where the detailed structure of multiparticle and two-body inelastic processes becomes important. As far as λ is concerned, we have taken throughout $\lambda = 6.5$ consistently with Ref. ¹⁾. However, all the good features discussed below are fairly independent of the value of λ . In fact, it turns out that the terms in λ and λ^2 in Eq. (3) can be almost completely absorbed in the U term, for $1 \leq \lambda \leq 10$ and for the partial waves and energy range considered in the following. Indeed, a careful analysis of the various terms appearing in Eq. (3) shows that, in general, only the term in g^2 [coming from the term in g^4 in Eq. (1)] has a noticeable structure, the others being in most cases relatively flat.

We discuss separately the two isospin channels.

$$I = \frac{3}{2}$$

The first two-body inelastic channel is ΣK , which opens at 1690 MeV of c.m.s. energy. Therefore, if multipion channels are not too important for our purposes, we expect to have a good fit to the experimental phases up to an energy close to that value. This seems to be the case since our phases and the experimental ones agree within 1 up to 1600 MeV. (See Figs. 2 and 3 for the values of the phases and of the relevant parameters.)

$$I = \frac{1}{2}$$

The first two-body inelastic channels are $N\eta$ (1500 MeV) and ΛK (1610 MeV), so our parametrization is presumably no longer good for energies larger than 1400 MeV. On the other hand, at energies less than 1400 MeV the experimental phases are in general small (with the only exception of the S_{11}), the structure beginning just in the region where our model is expected to become more and more approximate. We may try to have a quantitative fit for the low energies, or a qualitative fit for the higher energies. [The main feature of the theory really tested in this last case is just the occurrence of the unitarized Born term in Eq. (2).] A fit to the S_{11} and P_{11} up to 1700 MeV is shown in Fig. 4. As the fit to the P_{11} is rather unsatisfactory we also tried to decouple S_{11} and P_{11} . A good fit to the S_{11} in terms of a_0 , b_0 only, and an improved fit to the P_{11} in terms of four independent parameters a_0 , b_0 , a_1 , b_2 , are shown in Fig. 5.

The experimental P_{11} phases are small and negative at low energies, becoming positive and then resonating at higher energies. Such a feature could not be reproduced by the first Padé approximant we are discussing, independently of U , so we must ascribe it, if definitely established, to "higher order effects".

P_{13} and D_{13} are almost zero up to 1400 MeV, and are well fitted by us up to that energy. However, we are not able to reproduce, even qualitatively, the D_{13} resonance (which lies outside the range of validity of our model).

As a last remark, we have some inelasticity in some waves (especially P_{11} and P_{31}), not shown in the Figures, but the inelasticities, as expected, do not agree in general with the observed ones. We recall that, within the present first Padé approximant, only graph III of Fig. 1 accounts for two-pion effects.

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R E F E R E N C E S

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- 3) A. Donnachie, R.G. Kirsopp and C. Lovelace, CERN preprint TH. 838.

FIGURE CAPTIONS

Figure 1 :

Feynman graphs evaluated up to fourth order.

CAUTION : In the following figures, square blocks are centered on points taken from Ref. 2), their height being always 2^0 to exhibit changes of scale. This is not the error quoted by Ref. 2).

Figure 2 :

Waves S_{31} and P_{31} with four common parameters.

Figure 3 :

Waves P_{33} and D_{33} with four common parameters.

Figure 4 :

Waves S_{11} and P_{11} with four common parameters.

Figure 5 :

Waves S_{11} with two parameters and P_{11} with four parameters.

FIG. 1

$$\alpha_0^{(31)} = 1.16$$

$$\alpha_1^{(31)} = 9.64 \times 10^{-3}$$

$$b_0^{(31)} = -0.135$$

$$b_1^{(31)} = -1.32 \times 10^{-4}$$

$$\lambda = 6.5$$

I: Points from Ref. (2)

FIG.2

FIG. 3

$\alpha_0^{(11)} = -2.14$; $\alpha_1^{(11)} = -3.25 \times 10^{-2}$; $b_0^{(11)} = 0.362$; $b_1^{(11)} = 3.25 \times 10^{-3}$; $\lambda = 6.5$; ■: Points from Ref. (2)

FIG.4

FIG. 5