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SIMPLE MODELS FOR FORWARD SCATTERING AMPLITUDES
OF STRONGLY INTERACTING PARTICLES

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A B S T R A C T

Some well-known relations between forward scattering amplitudes of mesons and baryons, originally derived from a quark model and the additivity assumption, are discussed in the framework of a Regge pole model with universal coupling of the trajectories. For the vector contributions the two models are completely equivalent and experimentally indistinguishable. Concerning the tensor trajectories, we investigate in particular their conjectured relation to the energy-momentum tensor. It is found that if there is any such relation, the tensor residues at $t = 0$ should be connected with definite pieces of the general (linear) $SU(6)$ decomposition of meson and baryon masses.

The Regge pole model ¹⁾ on the one hand and the independent quark model ²⁾ on the other hand have been rather successful in the description of hadron-hadron collisions at high energies. From the quark model and additivity of quark scattering amplitudes one obtains various relations for elastic and inelastic scattering processes which are in many cases in remarkable agreement with experiment ^{2),3)}. The model does of course not predict, in this crude version, the energy and momentum dependence of scattering amplitudes and assumes tacitly that the s and t dependence of related amplitudes is the same.

The Regge pole model, on the other hand, yields a good parametrization of the s and t dependence of scattering amplitudes but does not give relations between different processes unless further assumptions on the residue functions are introduced. A hint to a deeper connection of the two models was given very early by Kokkedee and Van Hove ⁴⁾ who showed that the quark model with additivity yields (for processes involving non-strange quarks) the factorization theorem usually derived from the Regge pole model. In fact, if the residue functions are related by further assumptions such as symmetries and/or universality (Refs. 5),6),7),8)), some of the quark model relations are recovered.

In this note we restrict ourselves to the study of elastic forward scattering amplitudes. As is well known, these are well described in terms of the exchange of two nonets of trajectories with opposite signature plus the Pomeranchuk trajectory which may or may not belong to the tensor nonet. It has been proposed to couple the vector trajectories universally to the conserved currents ^{6),7),8)}. This assumption is re-examined more carefully by Reggeizing properly the one-particle exchange diagram corresponding to the vector trajectory under consideration. The vector trajectories may either be coupled to

the external particles or to quarks. The relations obtained for total cross-sections, as well as their s dependence, are identically the same in both models. In particular, a definite prescription at which dynamical variables to compare related cross-sections emerges from the Reggeization procedure chosen.

Concerning the tensor trajectories, it has been conjectured that the sources of the corresponding tensor mesons might be related to the energy-momentum tensor or certain parts of it ^{9),10)} [and the appropriate $U(3)$ generalizations]. Invoking models for the energy momentum tensor in terms of quark fields, the authors find indeed expressions for the tensor residues at $t = 0$ of the type assumed in Ref. ⁶⁾. There are, however, serious difficulties in these approaches as will be discussed below. In particular it does not seem to be possible to define the near universality and approximate conservation of the scalar densities obtained for the residues. On the other hand, if one accepts the conjecture that the source tensors of the tensor mesons can be identified with certain parts of the full stress tensor, then the tensor residues at zero momentum transfer are proportional to certain parts of the masses of the external particles - independently of any model for the stress tensor. We discuss this question in some detail and show that if such a connection between the tensor sources and the stress tensor exists, the tensor residues at $t = 0$ must be identified with the $(1,1)$ and $(35,8)$ pieces of the general $[SU(6), SU(3)]$ decomposition of the (linear) particle masses.

In Section 2, we briefly review the quark model relations for total cross-sections. Following a suggestion by Daboul ⁸⁾, we shall cast them in a form which makes the relation to the Regge pole description considerably more transparent. In Section 3, we discuss the universal coupling of the vector trajectories and the equivalence of the two models. Sections 4 and 5 are devoted to a discussion of the tensor contributions and their possible relation to the energy-momentum tensor.

2. - QUARK MODEL DESCRIPTION OF TOTAL CROSS-SECTIONS

In the quark model, any hadron-hadron scattering amplitude is assumed to be given (in first approximation) by the sum of all contributing quark-quark and quark-antiquark scattering amplitudes ¹¹⁾. From this assumption and the well-known SU(6) wave functions of mesons and baryons follow a number of relations for elastic forward scattering amplitudes ¹¹⁾. Through the optical theorem, these relations hold then as well for the total cross-sections. In view of the connection to the Regge pole model to be discussed below, we shall make use of the particularly convenient notation introduced by Daboul ⁸⁾ and discussed further by Lipkin ¹²⁾. Using additivity and invariance under charge conjugation the linear combinations

$$\begin{aligned}\Sigma_{AB} &\equiv \sigma_{tot}(AB) + \sigma_{tot}(\bar{A}B) \\ \Delta_{AB} &\equiv \sigma_{tot}(AB) - \sigma_{tot}(\bar{A}B)\end{aligned}\quad (1)$$

of particle-particle and particle-antiparticle cross-sections are given in the quark model by

$$\begin{aligned}\Sigma_{AB} &= \sum_{ij} (n_i^A + n_i^{\bar{A}})(n_j^B + n_j^{\bar{B}}) \Sigma_{ij} \\ \Delta_{AB} &= \sum_{ij} (n_i^A - n_i^{\bar{A}})(n_j^B - n_j^{\bar{B}}) \Delta_{ij}\end{aligned}\quad (i,j = p, n, \lambda) \quad (2)$$

where Σ_{ij} and Δ_{ij} are the corresponding sums and differences (1) of quark cross-sections and where n_i^X is the number of quarks of type i in the particle X ^{*)}.

*) Since spin-flip amplitudes vanish in the forward direction, it is easy to show that forward scattering amplitudes are independent of spin couplings. The spin indices can therefore be omitted.

4.

Using isospin invariance the symmetric matrix Δ_{ij} takes the form ^{*)}

$$\Delta_{ij} = \begin{pmatrix} \Delta_{pp} & \Delta_{pn} & \Delta_{p\lambda} \\ \Delta_{pn} & \Delta_{nn} & \Delta_{n\lambda} \\ \Delta_{p\lambda} & \Delta_{n\lambda} & \Delta_{\lambda\lambda} \end{pmatrix}$$

It is easily diagonalized with the aid of the orthogonal matrix

$$C_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ x & x & -y\sqrt{2} \\ y & y & x\sqrt{2} \end{pmatrix} \quad (3)$$

where

$$X = \left(\frac{1}{2} + \frac{\Delta_{pp} + \Delta_{pn} - \Delta_{\lambda\lambda}}{2[(\Delta_{pp} + \Delta_{pn} - \Delta_{\lambda\lambda})^2 + 8\Delta_{p\lambda}^2]} \right)^{1/2}; \quad y = \sqrt{1 - X^2} \quad (4)$$

Its eigenvalues are

$$\lambda_1 = \Delta_{pp} - \Delta_{pn}$$

$$\left. \begin{matrix} \lambda_2 \\ \lambda_3 \end{matrix} \right\} = \frac{1}{2} \left\{ \Delta_{pp} + \Delta_{pn} + \Delta_{\lambda\lambda} \pm \sqrt{(\Delta_{pp} + \Delta_{pn} - \Delta_{\lambda\lambda})^2 + 8\Delta_{p\lambda}^2} \right\} \quad (5)$$

and the combination Δ_{AB} of particle cross-sections becomes in this diagonal representation

$$\Delta_{AB} = \sum_{\nu=1}^3 \lambda_{\nu} d_{\nu}^A d_{\nu}^B \quad (6)$$

with

$$d_{\nu}^X = \sum_i C_{\nu i} (n_i^X - \bar{n}_i^X)$$

^{*)} We denote the proton and neutron quarks by p and n while the physical proton and neutron are denoted by P and N respectively.

The matrix $\sum_{ij}$ is diagonalized similarly. One finds

$$\sum_{AB} = \sum_{\nu=1}^3 \rho_{\nu} s_{\nu}^A s_{\nu}^B \quad (7)$$

where the eigenvalues ρ_{ν} are given by the expressions obtained by replacing in Eq. (5) all Δ_{ij} by $\sum_{ij}$ and where the quantities s_{ν}^X are defined in analogy to the d_{ν}^X . The analogy of the expressions (6) and (7) to the Coulomb scattering amplitude of two charged particles

$$a(AB) = e_A e_B F(s, t)$$

(e_A, e_B being the charges of the particles A and B, $F(s, t)$ being the kinematical amplitude) has led Lipkin to interpret the quantities d_{ν}^X and s_{ν}^X as generalized "charges" of the hadrons and the eigenvalues λ_{ν} and ρ_{ν} as generalized kinematical amplitudes¹²⁾. Each hadron is then characterized by six charges describing its (universal) coupling to the six independent amplitudes ρ_{ν} and λ_{ν} which in turn should contain the energy dependence of the scattering amplitudes. This interpretation will indeed turn out to be very plausible as will become clear from the model to be discussed in the next Section.

By calculating the "charges" d_{ν}^X and s_{ν}^X from (3) and (4) (and from the corresponding quantities for the sums) and by eliminating the unknown amplitudes λ_{ν} and ρ_{ν} one obtains the well-known quark model relations

$$\Delta_{K^+P}(S_{KP}) - \Delta_{K^+N}(S_{KP}) = \Delta_{\pi^+P}(S_{\pi P}) \quad (8)$$

$$\Delta_{pP}(S_{pP}) - \Delta_{nP}(S_{pP}) = \Delta_{\pi^+P}(S_{\pi P}) \quad (9)$$

$$\sum_{pP}(S_{pP}) - \sum_{nP}(S_{pP}) = \sum_{K^+P}(S_{KP}) - \sum_{K^+N}(S_{KP}) \quad (10)$$

$$\sum_{pP}(S_{pP}) + \sum_{nP}(S_{pP}) = 3 \sum_{\pi^+P}(S_{\pi P}) \quad (11)$$

In particular, for asymptotic energies, relation (10) goes over into

$$\left(\frac{\sigma_{\text{tot}}(\pi P)}{\sigma_{\text{tot}}(PP)} \right)_{S \rightarrow \infty} = 2/3 \quad (11')$$

Unfortunately there is no unique prescription, in the quark model, at which dynamical variable related cross-sections should be compared. In view of the flatness of the total cross-sections at high energies when plotted versus energy or beam momentum, the relations (8)-(11') were originally compared (arbitrarily) at equal centre-of-mass energies in all related cross-sections ¹¹⁾. Since then, James and Watson have given the following argument ¹³⁾. The quark-quark scattering amplitudes contributing to a two-body collision are assumed to be independent of the nature of the colliding particles. When eliminating them from two different particle-particle total cross-sections, say $\sigma_T(AB)$ and $\sigma_T(A'B)$, the quark amplitudes should be taken at equal values of the invariant energy squared of the two quarks $s_{ij} = (p^i + p^j)^2$, p^i being the four-momentum of quark i belonging to particle A (or A'), p^j being the four-momentum of quark j belonging to particle B . Since

$$\frac{S_{AB} - m_A^2 - m_B^2}{m_A m_B} = \frac{S_{ij} - m_i^2(A) - m_j^2(B)}{m_i(A) m_j(B)} \quad (12)$$

this implies that two related cross-sections $\sigma_T(AB)$ and $\sigma_T(A'B)$ should be compared at invariant energies squared $S_{A'B}$ and S_{AB} respectively with the ratio ^{*)}

$$\frac{S_{A'B} - m_{A'}^2 - m_B^2}{m_{A'} m_B} = \frac{m_{A'} m_i(A)}{m_A m_i(A')} \frac{S_{ij} - m_i^2(A) - m_j^2(B)}{S_{ij} - m_i^2(A') - m_j^2(B)}$$

$m_i(A)$ being the mass of the quark i bound in particle A .

*) One shows easily that this prescription amounts to compare at equal relative velocity of A and B and of A' and B respectively.

There are then two possibilities :

- a) if the quark masses are effective masses, i.e., $m_1(A) \approx m_A/Z_A$, Z_A being the number of quarks in particle A, and for $S \gg m^2$, then

$$S_{A'B}/S_{AB} = Z_{A'}/Z_A \quad (13)$$

This is the proposal of Kokkedee and Van Hove ⁴⁾.

- b) if the quark masses are fixed and independent of their bound state, then

$$\frac{S_{A'B} - m_{A'}^2 - m_B^2}{S_{AB} - m_A^2 - m_B^2} = \frac{m_{A'}}{m_A} \quad (14)$$

This is the prescription of James and Watson ¹³⁾.

The relations (8), (9) and (10) involve differences of cross-sections with similar s dependence and are therefore rather insensitive to the prescription chosen when comparing with experiment. In relation (11), however, the left-hand side is about 20 % bigger than its right-hand side when compared at equal centre-of-mass energy ¹¹⁾. The agreement with experiment is slightly improved if one compares according to the prescription (13). When using the rule (14), however, the discrepancy seems to be completely removed. This is illustrated by Fig. 1, where relation (11) is compared with experiment both at equal centre-of-mass energies and according to the prescription (14). In the model for Regge pole residues to be discussed below, it will be shown that a unique choice of the prescription is imposed by the model itself and that in fact the rule (14) is the appropriate one for comparing the relations (8)-(11') with experiment.

3. - A MODEL FOR REGGE RESIDUES IN TOTAL CROSS-SECTIONS

As is well known, total cross-sections, at high energies, are well described by the exchange of two nonets of Regge trajectories with opposite signature, and the Pomeron which may or may not be itself member of the even-signature nonet ¹⁴⁾⁻¹⁶⁾. The Regge poles contributing to elastic forward scattering amplitudes carry the quantum numbers of the ρ^0 , ω and φ mesons (odd signature) and of the A_2^0 , f and f' mesons (even signature), plus the Pomeron P which some authors like to identify with f or f' (or a mixture of them).

As is well known, only one helicity amplitude of the t channel contributes to the elastic forward scattering amplitude, both for meson-nucleon as well as for nucleon-nucleon collisions, provided one averages and sums over initial and final helicities ¹⁷⁾. The angular dependence of this amplitude is given by simple Legendre polynomials $P_J(\cos \vartheta_t)$ where ϑ_t denotes the centre-of-mass scattering angle in the t channel. After Reggeization, the spin-averaged forward scattering amplitude therefore has the standard form

$$\sum_{\text{helicities}} f_{AB}(s, t=0) = \sum_i C_i \beta_{AB}^{(i)}(0) \frac{1 + \varepsilon_i e^{-i\pi\alpha_i(0)}}{\sin \pi\alpha_i(0)} \times \frac{\Gamma(\alpha_i(0) + \frac{3}{2})}{\sqrt{\pi} \Gamma(\alpha_i(0) + 1)} \left(\frac{s - m_A^2 - m_B^2}{s_0} \right)^{\alpha_i(0)} \quad (15)$$

where ε_i and $\alpha_i(0)$ denote signature and intercept respectively of the Regge trajectory (i). According to the factorization theorem ¹⁸⁾ the residue $\beta_{AB}^{(i)}$ factorizes at the two vertices

$$\beta_{AB}^{(i)}(0) = \gamma_{AA}^{(i)}(0) \gamma_{BB}^{(i)}(0) \quad (16)$$

In Table 1, we have listed, for a number of meson-baryon and baryon-baryon amplitudes, the residues (16) making use of the charge conjugation and G parity properties of the corresponding trajectories. As can be seen from the table, the quantities Δ_{AB} , Eq.(1), depend only on the vector trajectories, the quantities Σ_{AB} only on the tensor trajectories.

In order to reduce the number of parameters entering the parametrization in terms of Regge poles, several additional assumptions were introduced such as for instance : (a) SU(3) invariance at the particle-trajectory vertices ^{14),15),19)} ; (b) SU(3) invariance for the coupling of the trajectories to quarks, with definite symmetry breaking by the Pomeranchon ²⁰⁾ ; (c) universal, U(6,6) invariant coupling ⁵⁾ ; (d) degeneracy of ρ and A_2 trajectory (exchange-degeneracy) ²¹⁾, etc. In particular, Cabibbo et al. ⁶⁾ assumed that the nine vector residues and the nine tensor residues are given, at zero momentum transfer, by the matrix elements of two nonets of vector and scalar densities respectively which generate the non-chiral group $U(3) \times U(3)$. ₄

Some workers in the field choose to consider the reference energies s_0^i in the expression (15) as additional free parameters which may be different for each trajectory. Others ^{1),22)} require that the expression (15), for unphysical values of t in the neighbourhood of $t = m_1^2$, goes over into the corresponding Born term for the exchange of the physical particle (i), thus fixing the quantity s_0^i . In our approach we choose the latter alternative. Since mass factors are important in our discussion we discuss this point in some detail and consider first the Feynman diagram for the exchange of a vector or tensor particle corresponding to the Regge pole under consideration.

A - Vector meson exchange and vector trajectories

The exchange of a vector meson V gives the following contribution to the scattering amplitude

$$\langle q' | j_\mu^{(V)}(0) | q \rangle \frac{-q^{\mu\nu} + e^\mu e^\nu / m_V^2}{t - m_V^2} \langle p' | j_\nu^{(V)}(0) | p \rangle \quad (17)$$

where $e^\mu = q^\mu - q'^\mu = p'^\mu - p^\mu$ is the momentum, m_V the mass and $j_\mu^{(V)}(x)$ the source of the exchanged vector particle; $t = e_\mu e^\mu$.

In what follows we shall use Sakurai's hypothesis ²³⁾ according to which the ρ , the ω and the φ mesons are coupled to the isospin current and to two orthogonal linear combinations of the baryon and the hypercharge current. We shall in particular follow the suggestion of Levinson, Lipkin and Wall ⁷⁾ who propose to couple the ω and the φ meson to the particular mixture of baryon and hypercharge current corresponding to the well-known mixture of the physical ω and φ . If $\phi_\mu^{(V)}(x)$ denotes the interpolating field for the vector meson V , we then have ^{*)}

$$\phi_\mu^{(\rho)}(x) = - \frac{C_\rho}{m_\rho^2} j_\mu^{(3)}(x) \quad (18a)$$

$$\phi_\mu^{(\omega)}(x) = - \frac{C_\omega}{m_\omega^2} \left\{ \sqrt{\frac{1}{3}} j_\mu^{(8)}(x) + \sqrt{\frac{2}{3}} j_\mu^{(0)}(x) \right\} \quad (18b)$$

$$\phi_\mu^{(\varphi)}(x) = - \frac{C_\varphi}{m_\varphi^2} \left\{ -\sqrt{\frac{2}{3}} j_\mu^{(8)}(x) + \sqrt{\frac{1}{3}} j_\mu^{(0)}(x) \right\} \quad (18c)$$

*) We consider the conserved vector currents as interpolating fields of the vector mesons. In Sakurai's model the vector currents appear as the sources of the vector mesons rather than as their interpolating fields. For a discussion of these questions see, for example, Ref. ²⁴⁾. In our discussion, identical conclusions are reached if one identifies the vector currents with the sources.

where $j_{\mu}^{(0)}$, $j_{\mu}^{(3)}$ and $j_{\mu}^{(8)}$ denote the normalized conserved vector currents $j_{\mu}^{(*)}$. The constants C_V characterize the coupling to the currents. In the limit of exact $SU(3)$ symmetry one has $C_{\rho} = C_{\omega} = C_{\varphi}$.

As an example let us consider the contribution of ρ exchange to the amplitude for elastic pion-nucleon scattering. From the assumption (18a) we obtain

$$\langle q' | j_{\mu}^{(\rho)}(0) | q \rangle = C_{\rho} \left(\frac{t}{m_{\rho}^2} - 1 \right) \langle q' | j_{\mu}^{(3)}(0) | q \rangle$$

Using a standard decomposition for the matrix elements of the conserved current $j_{\mu}^{(3)}$ into covariants,

a) for mesons A with spin 0

$$\langle q' | j_{\mu}^{(3)}(0) | q \rangle = (q + q')_{\mu} F^{(3)A}(t) \quad (19a)$$

b) for baryons B with spin $\frac{1}{2}$

$$\langle p' | j_{\mu}^{(3)}(0) | p \rangle = \overline{u}(p') \left\{ \gamma_{\mu} F_1^{(3)B}(t) + \frac{F_2^{(3)B}(t)}{2m_B} \sigma_{\mu\lambda} (p - p')^{\lambda} \right\} u(p) \quad (19b)$$

one easily calculates the expression (17). The part of the amplitude which does not vanish for $t = 0$ is found to be

$$A_{\pi p}^{(b)}(s, t) = -C_{\rho}^2 \left(\frac{t}{m_{\rho}^2} - 1 \right)^2 F^{(3)\pi}(t) F_1^{(3)p}(t) \frac{\overline{u}(p') u(p)}{2m_p} \times \frac{\sqrt{(t - 4m_{\pi}^2)(t - 4m_p^2)} P_1(\cos \vartheta_t)}{t - m_{\rho}^2} \quad (20)$$

*) This type of universal coupling of the vector trajectories is, of course, implied by the assumptions of Cabibbo et al.⁶⁾ However, in their algebra of residues, the two $I = 0$ vector trajectories are necessarily coupled to pure octet and singlet densities. The authors loose therefore relation (25) below which depends on the particular mixture (18b), (18c).

where ϑ_t is the scattering angle in the t channel ^{*)}, with

$$\cos \vartheta_t = \frac{2(s + t/2 - m_\pi^2 - m_P^2)}{\sqrt{(t - 4m_\pi^2)(t - 4m_P^2)}}$$

We now Reggeize this amplitude in the standard way by demanding that the obtained Regge amplitude, when expanded in t around the unphysical point $t = m_P^2$, gives back again the amplitude (20). In particular, we assume that the coupling factors $C_P(t/m_P^2 - 1) \times F^{(3)A}(t)$ remain unchanged and describe, for physical values of t in the s channel, the coupling of the ρ trajectory to the external particles. The residues $\gamma_{AA}^{(\rho)}(t)$ can then be identified with the form factors $F^{(3)A}(t)$.

Using the well-known asymptotic expansion of the Legendre polynomial $P_\alpha(z)$ for large values of the energy

$$P_\alpha(z) \underset{z \rightarrow \infty}{\sim} \frac{\Gamma(\alpha + 1/2)}{\sqrt{\pi} \Gamma(\alpha + 1)} (2z)^\alpha$$

this leads finally to the following contribution of the ρ trajectory to the forward scattering amplitude, for large s ,

$$A_{\pi P}^{(\rho)}(s, t=0) = C_P^2 \gamma_{\pi\pi}^{(\rho)}(0) \gamma_{PP}^{(\rho)}(0) \frac{4\pi\Delta_P}{3} \frac{1 - e^{-i\pi\alpha_P(0)}}{\sin \pi\alpha_P(0)} \times \\ \times \frac{\Gamma(\alpha_P(0) + 3/2)}{\sqrt{\pi} \Gamma(\alpha_P(0) + 1)} m_\pi m_P \left(\frac{s - m_\pi^2 - m_P^2}{m_\pi m_P} \right)^{\alpha_P(0)} \quad (21)$$

with $\gamma_{\pi\pi}^{(\rho)}(0) = F^{(3)\pi}(0)$ and $\gamma_{PP}^{(\rho)}(0) = F_1^{(3)P}(0)$ and $\Delta_P = [d\alpha_P(t)/dt]_{t=m_P^2}$.

^{*)} We use the relativistic normalization $\langle p' | p \rangle = 2p_0 (2\pi)^3 \delta(\vec{p}' - \vec{p})$, for mesons and baryons, and the metric $g_{\mu\nu} = (1, -\vec{1})$.

One easily shows that a similar expression holds for the case of nucleon-nucleon scattering.

With the analogous assumptions for the ω and the φ trajectory and using the optical theorem

$$\sigma_{tot}^{AB}(s) = \frac{1}{2k\sqrt{s}} \text{Im} A(s, 0) \approx \frac{\text{Im} A(s, 0)}{s - m_A^2 - m_B^2} \quad (\text{for } s \text{ large})$$

any of the three vector trajectories (V) gives a contribution to the total cross-section of particles A and B , of the form

$$C_V^2 \gamma_{AA}^{(V)}(0) \gamma_{BB}^{(V)}(0) \frac{4\pi\Delta_V}{3} \frac{\Gamma(\alpha_V + 3/2)}{\Gamma(\alpha_V + 1)} \left(\frac{s - m_A^2 - m_B^2}{m_A m_B} \right)^{\alpha_V - 1} \quad (22)$$

With the assumption (18) the residues $\gamma_{AA}^{(V)}(0)$ are easily expressed in terms of the quantum numbers I_3 , B and Y of hadron A :

$$\gamma_{AA}^{(\rho)}(0) = I_3^A \quad (23a)$$

$$\gamma_{AA}^{(\omega)}(0) = 2B^A + Y^A \quad (23b)$$

$$\gamma_{AA}^{(\varphi)}(0) = \sqrt{2} (B^A - Y^A) \quad (23c)$$

As a first consequence one notes that the φ trajectory does not contribute to any total cross-section with non-strange targets. Furthermore, the relations between residues $\beta^{(i)}(0)$, Eq. (16) and Table I, which follow from (23), will hold as well for the appropriate combinations of total cross-sections, provided the dynamical factor

$$\left(\frac{s - m_A^2 - m_B^2}{m_A m_B} \right)^{\alpha - 1}$$

is the same in all related cross-sections. This, however, is precisely the prescription (14) advocated by James and Watson ¹³⁾ for the quark model from an entirely different argument.

From (23) and Table I one finds the well-known relations ⁷⁾

$$\begin{aligned}\Delta_{PP}(S_{PP}) - \Delta_{NP}(S_{PP}) &= \Delta_{K^+P}(S_{KP}) - \Delta_{K^+N}(S_{KP}) = \Delta_{\pi^+P}(S_{\pi P}) = \\ &= 2 C_P^2 \frac{4 \Delta_P \sqrt{\pi}}{3} \frac{\Gamma(\alpha_P + 3/2)}{\Gamma(\alpha_P + 1)} \left(\frac{S_{\pi P} - m_\pi^2 - m_P^2}{m_\pi m_P} \right)^{\alpha_P - 1}\end{aligned}\quad (24)$$

and

$$\begin{aligned}\Delta_{PP}(S_{PP}) + \Delta_{NP}(S_{PP}) &= 3 \left[\Delta_{K^+P}(S_{KP}) + \Delta_{K^+N}(S_{KP}) \right] = \\ &= 3 C_\omega^2 \frac{4 \Delta_\omega \sqrt{\pi}}{3} \frac{\Gamma(\alpha_\omega + 3/2)}{\Gamma(\alpha_\omega + 1)} \left(\frac{S_{KP} - m_K^2 - m_P^2}{m_K m_P} \right)^{\alpha_\omega - 1}\end{aligned}\quad (25)$$

with the energies s_{PP} , s_{KP} , $s_{\pi P}$ chosen such as to fulfil the prescription (14).

The relations (24) are identical with the relations (8) and (9) derived from the quark model ; as noted by Levinson, Lipkin and Wall ⁷⁾, relation (25) is new and is obtained in the quark model only with the additional assumption $\Delta_{\lambda_P} = 0$, [cf. Eq. (5)]. In Fig. 2, relation (25) is compared with experiment, both at equal centre-of-mass energies and according to the prescription (14). Its agreement with experiment is good ; the comparison is, however, insensitive to the precise prescription chosen. In addition, the Regge pole model allows for a reasonable description of the s dependence of relations (24) and (25), with $\alpha_P \approx \alpha_\omega \approx 0.5$ ^{15) *)}, in contrast to the quark model

*) SU(3) symmetry would imply $C_P^2 = C_\omega^2$. A numerical evaluation of (24) and (25) shows that this is violated to about 15 %. One finds, assuming $\Delta_P = \Delta_\omega$ and with $\alpha_P = \alpha_\omega = 0.5$, $C_P^2/C_\omega^2 \approx 1.15$. Furthermore, with $\Delta_P \approx 1(\text{GeV}/c)^{-2}$ one obtains $C_P^2/4\pi \approx 0.4$ which agrees in order of magnitude with the numbers found from the vector-dominance model ²⁴⁾.

which in the simple version presented in Section 2 says nothing about the s dependence of scattering amplitudes. One may, however, try to apply the Regge pole analysis to the quark-quark scattering amplitudes rather than to the particle-particle scattering amplitudes, using the same universal coupling of vector trajectories. We shall briefly discuss this point in the next Section.

B - Quark scattering amplitudes and vector trajectories

In the quark model the total cross-section for the collision of two hadrons A and B is given by the sum of all contributing quark-quark and quark-antiquark total cross-sections. If we now apply all considerations of the last Section to the total cross-sections of quarks and antiquarks, we find for the contribution of a vector trajectory (V) to the total cross-section of quarks (or antiquarks) i and j

$$C_V^2 \gamma_{ii}^{(V)}(0) \gamma_{jj}^{(V)}(0) \frac{4\Delta_V \sqrt{\pi}}{3} \frac{\Gamma(\alpha_V + 3/2)}{\Gamma(\alpha_V + 1)} \left(\frac{S_{ij} - m_i^2 - m_j^2}{m_i m_j} \right)^{\alpha_V - 1}$$

With the assumption (18) [or (23)] none of the three vector trajectories contributes to $\sigma_T(p\lambda)$ or $\sigma_T(p\bar{\lambda})$. On the other hand, as follows from charge-conjugation and G parity, the tensor trajectories contribute equally to both cross-sections; hence $\sigma_T(p\lambda) = \sigma_T(p\bar{\lambda})$, or $\Delta_{p\lambda} = 0$. We thus recover relation (25). Furthermore, using relation (12), it is easy to show that one obtains exactly the same Regge expressions on the right-hand sides of (24) and (25), including all dynamical factors. We thus arrive at the result that, as far as the contributions of the vector trajectories are concerned, there is no way to distinguish experimentally between the two models. The relations (24) and (25) do not need quarks but, on the other hand, do by no means exclude their existence.

With $\Delta_{\lambda p} = 0$, the eigenvalues λ_i and the quantities x and y defined in Eq. (5) and (4) respectively, take the particularly simple form

where P and Q are the sum and difference of the momenta respectively.

The part of the amplitude (27) that does not vanish at $t = 0$ is then easily computed to be, for the scattering of a meson A on a baryon B , for instance,

$$\begin{aligned} & \frac{\overline{u(p)} u(p)}{2m_B} \left\{ G_1^{(T)A} G_1^{(T)B} \frac{(t - 4m_A^2)(t - 4m_B^2)}{4m_A m_B} \frac{P_2(\cos \vartheta_t)}{t - m_T^2} + \right. \\ & + \left[G_2^{(T)A} G_2^{(T)B} \frac{m_A m_B}{3m_T^4} (2t + 4m_T^2) + G_1^{(T)A} G_2^{(T)B} \frac{m_B}{6m_A} \frac{(4m_A^2 - t)}{m_T^2} + \right. \\ & \left. \left. + G_2^{(T)A} G_1^{(T)B} \frac{m_A}{6m_B} \frac{(4m_B^2 - t)}{m_T^2} \right] P_0(\cos \vartheta_t) \right\} \quad (29) \end{aligned}$$

The pole term in the curly brackets, $P_2(\cos \vartheta_t)/(t - m_T^2)$, is Reggeized in the standard manner. If we again assume that the form factors $G_1^{(T)}(t)$ remain unchanged and describe, at $t \leq 0$, the coupling of the trajectory (T) to the external particles, we obtain the Regge amplitude

$$\begin{aligned} A_{AB}^{(T)}(s, t=0) &= C_T^2 \gamma_{AA}^{(T)}(0) \gamma_{BB}^{(T)}(0) \frac{4\pi \Delta_T}{5} \frac{1 + e^{-i\pi\alpha_T}}{\sin \pi\alpha_T} \times \\ &\times \frac{\Gamma(\alpha_T + 3/2)}{\sqrt{\pi} \Gamma(\alpha_T + 1)} m_A m_B \left(\frac{s - m_A^2 - m_B^2}{m_A m_B} \right)^{\alpha_T} \quad (30) \end{aligned}$$

We have set

$$\gamma_{AA}^{(T)}(0) = C_T G_1^{(T)A}(0) \quad (31)$$

where C_T characterizes the coupling to the trajectory.

The second term in the curly brackets of (29) which at $t = m_T^2$ is regular, gives under the same assumptions a constant at $t = 0$ which might then be neglected as compared to the term (30) increasing with s like $s^{\alpha_T^*}$. An expression analogous to (29) is found for the case of nucleon-nucleon scattering.

From the expression (30) and the optical theorem follows again that any relation between the residues (16) holds for the appropriate combinations of total cross-sections, provided the values of s are chosen according to the prescription (14).

In the model of Cabibbo, Horwitz and Ne'eman⁶⁾, the form factors $G_1^{(T)}(0)$ at $t = 0$ are identified with the matrix elements of scalar densities $S^{(i)}$ with the transformation behaviour, in terms of quark fields,

$$S^{(i)} = \int d^3x \bar{\Psi}(x) \lambda^i \Psi(x) \quad (32)$$

The operators (32) are obviously not conserved quantities. Only if one makes the further assumption that mesons and baryons fall into simple irreducible representations of the group $U(3) \times U(3)$,⁴⁾ can their matrix elements be computed⁶⁾. Indeed, if the mesons belong to the representation $(6, 6^*)$ and the baryons to $(56, 1)$ (with possibly some admixture resulting in a non-zero d coupling), one recovers all remaining quark-model relations, in particular relations (10), (11) and (11').

Several attempts have been made to justify this model, by assuming a certain connection between the sources $t_{\mu\nu}$ of the tensor mesons and the energy-momentum tensor and certain $U(3)$ generalizations of it^{9), 10)}. In particular, Delbourgo, Salam and Strathdee¹⁰⁾ identify the source tensors $t_{\mu\nu}^{(i)}$ with the following tensors, written in terms of quark fields Ψ ,

*) If $t_{\mu\nu}^{(T)}$ is conserved, then $G_2^{(T)}(t) \equiv 0$.

$$\mathcal{V}_{\mu\nu}^{(i)} = \frac{i}{4} \bar{\Psi} (\gamma_{\mu} \overleftrightarrow{\partial}_{\nu} + \gamma_{\nu} \overleftrightarrow{\partial}_{\mu}) \lambda^i \Psi - g_{\mu\nu} \bar{\Psi} (\frac{i}{2} \not{\partial} - m^*) \lambda^i \Psi \quad (33)$$

where m^* is some renormalized quark mass, λ^i are the well-known Gell-Mann matrices. The authors assume that the tensors (33) are nearly conserved, provided the renormalization of the quark mass takes care of an essential part of the interaction of the quarks. The tensors (33) would then describe the stress tensor [and its U(3) generalizations] of quasi-free quarks. If introduced into our formulas (29), (30) and (31), this would indeed lead to a coupling via the operators (32) as postulated by Cabibbo et al. ^{6),9)}. This interpretation, however, leads to difficulties : consider the limit where the tensors (33) are exactly conserved. The conservation of the $\mathcal{V}_{\mu\nu}^{(i)}$ implies the invariance of the quark Lagrangian under the combined SU(3) and space-time transformations $e^{i\alpha P \otimes \lambda^i}$. It is then easy to show that such an invariance leads to a trivial S matrix, i.e., no scattering ²⁵⁾. Since in this limit particles (quarks) do not scatter, it seems difficult to define an approximate conservation of the quantities (33) ^{*)}.

On the other hand, if the sources $t_{\mu\nu}$ of the tensor mesons are indeed related to the true stress tensor or suitable parts of it, we would expect that the coupling factors $G_1^{(T)A}(0)$ [and hence the tensor residues, Eq. (31)] are related to the masses or certain components of the masses of mesons and baryons. In the following Section we shall examine this question in more detail, without recurring to a particular model of the stress tensor itself, e.g., in terms of quark fields.

*) I am indebted to P. Hertel for clarification of this point.

5. -

TENSOR RESIDUES AND PARTICLE MASSES

The full stress tensor $\Theta_{\mu\nu}(x)$ is conserved, symmetric but not traceless. For one-particle matrix elements of $\Theta_{\mu\nu}$ we choose the following decomposition into covariants,

a) for mesons A with spin 0,

$$\langle q | \Theta_{\mu\nu}(0) | q \rangle = \frac{1}{2m_A} F_1^A(t) P_\mu P_\nu + \frac{1}{2m_A} F_3^A(t) (Q_\mu Q_\nu - t g_{\mu\nu}) \quad (34a)$$

b) for baryons B with spin $\frac{1}{2}$

$$\begin{aligned} \langle p | \Theta_{\mu\nu}(0) | p \rangle = & \frac{u(p)}{2m_B} \left\{ \frac{F_1^B(t)}{2m_B} P_\mu P_\nu + \frac{F_3^B(t)}{2m_B} (Q_\mu Q_\nu - t g_{\mu\nu}) + \right. \\ & \left. + \frac{F_5^B(t)}{2m_B} (\sigma_{\mu\lambda} Q^\lambda P_\nu + P_\mu \sigma_{\nu\lambda} Q^\lambda) \right\} u(p) \end{aligned} \quad (34b)$$

With our normalization (see footnote on page 12) one then has for any particle A, meson or baryon,

$$F_1^A(0) = m_A \quad (35)$$

Suppose the full stress tensor $\Theta_{\mu\nu}(x)$ is decomposed into a sum of SU(3) components $\Theta_{\mu\nu}^{(i; IY)}$,

$$\Theta_{\mu\nu}(x) = \sum_{i_n} \Theta_{\mu\nu}^{(i_n, I=Y=0)}(x) \quad (36)$$

where (i) denotes the dimension of the contributing SU(3) multiplets. Depending on the argument leading to a decomposition of the form (37), it is possible that for a given SU(3) dimension (i) there is more than one term in the sum of (36). If this is the case such terms shall be distinguished by an additional index n.

The tensors $\Theta_{\mu\nu}^{(i; IY)}$ will in general not be conserved. The decomposition of their one-particle matrix elements into covariants will therefore be of the form (28). We have, for mesons with spin zero for instance,

$$\begin{aligned} \langle q' | \Theta_{\mu\nu}^{(\alpha)}(0) | q \rangle = & \frac{1}{2m_A} F_1^{(\alpha)A}(t) P_\mu P_\nu + m_A F_2^{(\alpha)A}(t) g_{\mu\nu} + \\ & + \frac{F_3^{(\alpha)A}(t)}{2m_A} (Q_\mu Q_\nu - t g_{\mu\nu}) \quad ; \quad \alpha \equiv (i_m, IY) \end{aligned} \quad (37)$$

and an analogous decomposition for baryons.

From (36) and (34) follows, for any meson or baryon A,

$$\sum_{\alpha} F_1^{(\alpha)A}(t) = F_1^A(t) \quad (38a)$$

$$\sum_{\alpha} F_2^{(\alpha)A}(t) \equiv 0 \quad (38b)$$

in particular at $t = 0$,

$$\sum_{\alpha} F_1^{(\alpha)A}(0) = m_A \quad (39)$$

If we now identify, tentatively, the source $t_{\mu\nu}^{(\alpha)}(x)$ of the tensor meson (α) with a particular term $\Theta_{\mu\nu}^{(\alpha)}$ in Eq. (36), having the right SU(3) labels (α) , according to

$$t_{\mu\nu}^{(\alpha)}(x) = \frac{1}{M_0} \Theta_{\mu\nu}^{(\alpha)}(x) \quad (40)$$

where M_0 is some universal constant with the dimension of a mass, the coupling factors $G_1^{(\alpha)A}(0)$ and the residues $\gamma_{AA}^{(\alpha)}(0)$ in Eqs. (30) and (31) are given by

$$\gamma_{AA}^{(\alpha)}(0) = C_{\alpha} G_1^{(\alpha)A}(0) = \frac{C_{\alpha}}{M_0} F_1^{(\alpha)A}(0) \quad (41)$$

Since we only need the form factors $F_1^{(\alpha)}$ at zero momentum transfer and looking at Eq. (39), the decomposition (36) amounts to a decomposition of the mass of the particles :

$$m_A = \sum_{\alpha_n} m_A^{(\alpha_n)} \quad (39')$$

with $m_A^{(\alpha_n)} \equiv F_1^{(\alpha_n)A}(0)$, where again n distinguishes different SU(3) components with the same dimension α . A hint to the possible choice of the decomposition is again given by the quark model : if the success of the quark model, like in the case of the vector trajectories, is due to a universal coupling of the tensor trajectories, then pseudoscalar and vector mesons as well as octet and decimet baryons should be treated on the same footing. For instance, the quark model predicts equal total cross-sections, for the scattering on the same target, of any pseudoscalar and the corresponding vector meson with same isospin and hypercharge, (similarly for octet and decimet baryons). The most natural choice is therefore an SU(6) decomposition of the particle masses.

In doing so, it is important to notice that the decomposition (39') has to be linear in the mass, both for baryons and mesons. This is physically so because the eigenvalue of $\int d^3x \Theta_{0\nu}(x)$ is p_ν and not any other power of the momentum. Formally, it can easily be seen by noting that, for instance,

$$F_1^A(0) = m_A = \sqrt{\sum_{\alpha} (m_A^2)^{(\alpha)}}$$

is not easily decomposed into a sum of terms with simple SU(3) behaviour. Thus the usual ambiguity of whether linear or quadratic mass formulas should be chosen is avoided ²⁶⁾.

The decomposition in terms of the most general mass operator in SU(6) leads to a system of linear equations connecting the empirical masses of the particles to the strength and reduced matrix elements of the appropriate SU(6) tensors. This linear system is easily

inverted. Defining the reduced matrix elements as in Ref. ²⁷⁾, one finds from the empirical masses ^{*)},

a) for baryons

$[SU(6), SU(3)]$	(1,1)	(35,8)	(405,1)	(405,8)	(405,27)	(2695,8)	(2695,27)	(2695,64)
r.m.e. (MeV)	1316	-142	104	23	0	-4	1	0

b) for mesons

$[SU(6), SU(3)]$	(1,1)	(35,8)	(189,1)	(189,8)	(189,27)	(405,1)	(405,8)	(405,27)
r.m.e. (MeV)	743	-101	126	28	0	155	-48	0.1

In identifying the coupling factors $G_1^{(\alpha)}(0)$ with suitable parts of the masses we are guided by two requirements : in order for the coupling to be universal for mesons and baryons, these parts should appear in both decompositions a) and b). Furthermore, one would expect the relations

$$\sigma_{tot}^{\pi p} = \sigma_{tot}^{pp}, \quad \sigma_{tot}^{Kp} = \sigma_{tot}^{K^*p} \text{ etc.} \quad (42)$$

to hold. They follow from the quark model and seem to be in agreement with experiment ^{28), 29)}.

The only parts in the decompositions a) and b) which fulfil these requirements are the components (1,1) and (35,8).

Let (T_8) and (T_1) denote the pure octet member with $I = Y = 0$ and the pure singlet of the tensor nonet respectively. The f trajectory and the Pomeron F will in general be mixtures of the $SU(3)$ states (T_1) and (T_8) . We then identify, in the sense of the discussion given above, (T_1) with the singlet (1,1), (T_8) and (A_2) with the $I = 0$ and $I = 1$ members respectively of the octet (35,8) of the decomposition (39'). This leads to the results listed in Table II.

^{*)} The numbers for baryons are taken from Ref. ²⁷⁾.

The residues (31) are then given by

$$\begin{aligned}
 \gamma_{AA}^{(A_2)}(0) &= C_{A_2} G_1^{(A_2)A}(0) \\
 \gamma_{AA}^{(F)}(0) &= C_F \left\{ G_1^{(T_8)A}(0) \cos \phi + G_1^{(T_1)A}(0) \sin \phi \right\} \\
 \gamma_{AA}^{(T)}(0) &= C_T \left\{ -G_1^{(T_8)A}(0) \sin \phi + G_1^{(T_1)A}(0) \cos \phi \right\}
 \end{aligned} \tag{43}$$

and may be inserted into our expression (30).

As a first result, Table II shows that the proton and the K^+ meson are equally coupled to the A_2 trajectory. Using Table I, this leads to the relation (10), first derived from the quark model, and is in agreement with experiment. Furthermore, both couplings $G_1^{(T_1)}(0)$ and $G_1^{(T_8)}(0)$ give an approximate 3:2 ratio for baryons compared to mesons *) thus leading at least approximately to relations (11) and (11'), independently of the angle ϕ . This mixing angle is expected to be small since the Pomeron should not be too different from a pure singlet. Since $G_1^{(T_1)}(0)$ is in all cases sensibly greater than $G_1^{(T_8)}(0)$, the asymptotic ratios for pion-nucleon and kaon-nucleon to nucleon-nucleon total cross-sections are rather insensitive to the exact value of ϕ , as long as ϕ is small. As an example, a mixing angle of about 31° , taken from the masses of the tensor mesons, would lead to

$$\begin{aligned}
 \left(\sigma_{\text{tot}}^{\text{PP}} / \sigma_{\text{tot}}^{\text{TP}} \right)_{s \rightarrow \infty} &\approx 1.74 \\
 \left(\sigma_{\text{tot}}^{\text{TP}} / \sigma_{\text{tot}}^{\text{KP}} \right)_{s \rightarrow \infty} &\approx 1.13
 \end{aligned} \tag{44}$$

*) In fact, the ratio of the numbers in each column of Table II corresponds almost exactly to the simple ratios obtained from the model of Ref. 6) and from the quark model.

These numbers are in reasonable agreement with the experimental situation ³⁰⁾. The relations (11) and (11') are thus recovered only in an approximate way, depending on the precise value of the mixing angle.

We thus find the remarkable result that the tensor trajectories, at $t = 0$, seem to be coupled universally to mesons and baryons via the $(1,1)$ and the $(35,8)$ parts of the meson and baryon masses. Note that the numbers displayed in Table II are determined from the empirical masses of mesons and baryons and that there is no a priori reason for the simple ratios found when comparing baryons to mesons.

We conclude with a few remarks on this result.

1. - It is clear that the source tensors $\Theta_{\mu\nu}^{(\alpha)}$, Eq. (40), and hence the Lorentz scalar form factors $F_1^{(\alpha)}(t)$, can only carry SU(3) quantum numbers but not SU(6) labels. It should be clear from our discussion, however, that we are not attaching SU(6) labels to these form factors: the SU(6) decomposition (39') is only done at zero momentum transfer, where the form factors $F_1^{(\alpha)}(0)$ are identified with certain components of the mass with the right SU(3) quantum numbers α . For baryons, for instance, the full SU(3) singlet part of the masses is given by

$$m^1 = m_1^1 + m_{405}^1$$

the full SU(3) octet part by

$$m^8 = m_{35}^8 + m_{405}^8 + m_{2695}^8$$

The SU(6) decomposition of the masses thus serves merely to define different SU(3) singlet and different SU(3) octet contributions to the masses, [in Eq. (39') these were denoted by the additional index $\underline{n}$], some of which are then identified

with the form factors at zero momentum transfer. Away from $t = 0$, the form factors $F_1^{(\alpha)}(t)$ of course only keep these SU(3) labels α *).

2. - In our identification of the tensor residues with the (1,1) and (35,8) parts of the masses we have tacitly assumed the Pomeranchon F to be member of the nonet of tensor trajectories. According to the conclusions of the second paper of Ref. ⁶⁾ one is then forced to accept an intercept $\alpha_F(0)$ slightly smaller than one **). This leads to vanishing total cross-sections at $s = \infty$ but does not change the asymptotic ratios (44) appreciably as long as $\alpha_F(0)$ is still close to one. On the other hand, we can of course not exclude that an additional $I = Y = 0$ tensor trajectory is present in high-energy scattering. In the spirit of our model, its residue at zero momentum transfer could for instance be related to the (405,1) part of the masses. As long as it is not too strongly coupled, it will not spoil too much the equality of pseudoscalar and vector meson total cross-sections (42) and relation (11).
3. - SU(3) symmetry between different trajectories applied to the residues (43) implies the equality $C_f = C_{A_2} = C_F$. The identification of (T_8) and (A_2) with the (35,8) part of the masses then amounts to pure f type coupling at the baryon-baryon tensor trajectory vertex. In Ref. ⁶⁾, however, a non-zero admixture of d coupling was found necessary to fit the experimental data. On the other hand, a violation of the equality $C_f = C_{A_2}$ will manifest

*) We remark that in relating the coupling constants of T_8 and A_2 in Table II as well as in relating the pion to the kaon couplings, only SU(3) is needed but not SU(6).

**) We refer to our discussion of this point in the Appendix.

itself, in the formulation of Ref. ⁶⁾, in an effective δ admixture. The result of Ref. ⁶⁾ hence indicates a violation of $SU(3)$ for the tensor residues - analogous to the $SU(3)$ violation already found for the ω and ρ couplings C_ω and C_ρ . Note, however, that such a comparison is only meaningful if the f trajectory and the A_2 trajectory have similar intercepts.

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APPENDIXREMARK ON THE INTERCEPT OF THE POMERANCHON

We define the following quantities

$$S_{\pi P} = \frac{1}{2} \sum_{\pi+P}$$

$$S_K = \frac{1}{4} \left(\sum_{K+P} + \sum_{K+N} \right) \quad (A1)$$

Suppose the Pomeron F belongs to the tensor nonet and is identical with say f' . Then only the f trajectory and the Pomeron F contribute to s_{π} and s_K , cf. Table I. Without going into the details of the fit of Ref. 6), it can then be shown, with the aid of the empirical data for s_{π} and s_K , that $\alpha_F(0)$ cannot be equal to one.

As is well known 30), the experimental data for $s_{\pi}(s)$ and $s_K(s)$ when plotted versus $1/\sqrt{s}$, seem to lie on straight lines with slightly different intercepts at $1/\sqrt{s} = 0$ or $s = \infty$. If this is accepted then one must have $\alpha_F(0) = 1$ and $\alpha_f(0) \approx 0.5$. The ratio R of $s_{\pi}(\infty)$ to $s_K(\infty)$ and the ratio D of their derivatives at $s = \infty$ are then given by

$$R \equiv \frac{S_{\pi}(\infty)}{S_K(\infty)} = \frac{-2d/\sqrt{3} \sin \phi + e \cos \phi}{d/\sqrt{3} \sin \phi + e \cos \phi} \quad (A2)$$

$$D \equiv \left(\frac{dS_{\pi}/du}{dS_K/du} \right)_{s=\infty} \sqrt{\frac{m_K}{m_{\pi}}} = \frac{2d/\sqrt{3} \cos \phi + e \sin \phi}{-d/\sqrt{3} \cos \phi + e \sin \phi} ; \quad (u = \frac{1}{\sqrt{s}}) \quad (A3)$$

where we have introduced an octet reduced matrix element d and a singlet reduced matrix element e . These reduced matrix elements can be eliminated from (A2) and (A3), giving

$$\tan^2 \phi = \frac{(D+2)(1-R)}{(D-1)(R+2)}$$

Since $\text{tg}^2\phi \geq 0$ and since R and D are both positive numbers, we conclude that we cannot have at the same time $D > 1$ and $R > 1$. From the empirical fit ³⁰⁾ in terms of $1/\sqrt{D}$ one finds $R \approx 1.17$ and $D \approx 4$ - this is in contradiction with our assumptions, whatever the values of the parameters e , d and of the angle ϕ are.

They are two ways out of this inconsistency, as discussed in Section 4 : (a) either one gives up the requirement $\alpha_F(0) = 1$; one is then led to a fit of the kind advocated in Ref. ⁶⁾ ; or else, (b) the Pomeranchon, with $\alpha_F(0) = 1$, lies outside the tensor nonet and there is a third trajectory with $I = Y = 0$. One then looses at least partly the successful relations (10), (11) and (11').

TABLE AND FIGURE CAPTIONS

- Table I : The residue functions (16) for a number of meson-baryon and baryon-baryon forward scattering amplitudes. The symbols ρ , ω , ... denote the trajectories.
- Table II : Coupling factors $G_1^{(\alpha)A}(0)$, Eq. (41), as found from the empirical masses of mesons and baryons.
- Figure 1 : Comparison of rel. (11) with experiment, where

$$\Delta = \frac{1}{4}(\sum_{PP} + \sum_{NP})$$
; $\bullet$, $\circ$ and $\boxtimes = \frac{3}{4} \sum \pi P$. The solid lines show the data when compared according to the prescription (14); the upper scale then holds for the nucleon-nucleon data, the lower one for the pion-nucleon data. For the sake of comparison the nucleon-nucleon data are also plotted at same centre-of-mass energies as the pion-nucleon data, $E_{c.m.}^{PP} = E_{c.m.}^{\pi P}$ (dashed line). In both cases the two lines should coincide. Typical errors are displayed. The data Δ and $\boxtimes$ are taken from Ref. 31), $\bullet$ from Ref. 32), $\circ$ from Ref. 33).
- Figure 2 : Comparison of relation (25) with experiment, where

$$\Delta = -(\Delta_{PP} + \Delta_{NP})$$
; $\circ = -3(\Delta_{K+P} + \Delta_{K+N})$. The comparison is done as in Fig. 1. All data are taken from Ref. 31).

- Table I -

	ρ	ω	φ	A_2	f	f'	F
(π^+p)	$\beta_{\pi p}^{\rho}$	0	0	0	$\beta_{\pi p}^f$	$\beta_{\pi p}^{f'}$	$\beta_{\pi p}^F$
(π^-p)	$-\beta_{\pi p}^{\rho}$	0	0	0	$\beta_{\pi p}^f$	$\beta_{\pi p}^{f'}$	$\beta_{\pi p}^F$
(K^+p)	β_{KP}^{ρ}	β_{KP}^{ω}	β_{KP}^{φ}	$\beta_{KP}^{A_2}$	β_{KP}^f	$\beta_{KP}^{f'}$	β_{KP}^F
(K^-p)	$-\beta_{KP}^{\rho}$	$-\beta_{KP}^{\omega}$	$-\beta_{KP}^{\varphi}$	$\beta_{KP}^{A_2}$	β_{KP}^f	$\beta_{KP}^{f'}$	$\beta_{KP}^{F'}$
(K^0p)	$-\beta_{KP}^{\rho}$	β_{KP}^{ω}	β_{KP}^{φ}	$-\beta_{KP}^{A_2}$	β_{KP}^f	$\beta_{KP}^{f'}$	β_{KP}^F
$(\bar{K}^0p)$	β_{KP}^{ρ}	$-\beta_{KP}^{\omega}$	$-\beta_{KP}^{\varphi}$	$-\beta_{KP}^{A_2}$	β_{KP}^f	$\beta_{KP}^{f'}$	β_{KP}^F
(pp)	β_{pp}^{ρ}	β_{pp}^{ω}	β_{pp}^{φ}	$\beta_{pp}^{A_2}$	β_{pp}^f	$\beta_{pp}^{f'}$	β_{pp}^F
(np)	$-\beta_{pp}^{\rho}$	β_{pp}^{ω}	β_{pp}^{φ}	$-\beta_{pp}^{A_2}$	β_{pp}^f	$\beta_{pp}^{f'}$	β_{pp}^F
$(\bar{p}p)$	$-\beta_{pp}^{\rho}$	$-\beta_{pp}^{\omega}$	$-\beta_{pp}^{\varphi}$	$\beta_{pp}^{A_2}$	β_{pp}^f	$\beta_{pp}^{f'}$	β_{pp}^F
$(\bar{n}n)$	β_{pp}^{ρ}	$-\beta_{pp}^{\omega}$	$-\beta_{pp}^{\varphi}$	$-\beta_{pp}^{A_2}$	β_{pp}^f	$\beta_{pp}^{f'}$	β_{pp}^F

- Table II -

	(1,1)	(35,8)	
$G_1^{(\alpha)A}$	$G_1^{(T_1)}(0)$	$G_1^{(T_8)}(0)$	$G_1^{(A_2)}(0)$
P	$1316 \frac{1}{M_0}$	$-153 \frac{1}{M_0}$	$-89 \frac{1}{M_0}$
π^+	$743 \frac{1}{M_0}$	$-106 \frac{1}{M_0}$	0
K^+	$743 \frac{1}{M_0}$	$53 \frac{1}{M_0}$	$-91 \frac{1}{M_0}$

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FIG.1

FIG.2