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WEINBERG SUM RULES AND THE RATIO F_K/F_π

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A B S T R A C T

The ratio F_K/F_π is derived using Weinberg's first spectral function sum rule in $SU(3) \times SU(3)$ while the second sum rule is used only in $SU(2) \times SU(2)$.

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In recent papers ¹⁾, the ratio between the decay constants F_K and F_π , the decay constants in the leptonic decays of K and π mesons respectively, has been evaluated using the two spectral function sum rules derived by Weinberg ²⁾ and their $SU(3)$ generalizations given by Das, Mathur and Okubo ³⁾. The derivations in Ref. ¹⁾ make use of both of the sum rules and thus necessarily imply, in the pole approximation considered, the exact $SU(3) \times SU(3)$ result $m_\rho = m_{K^*}$, $m_{A_1} \simeq m_{K_A}$. We derive here the above-mentioned ratio by using the first sum rule ⁴⁾ in $SU(3) \times SU(3)$ while the second sum rule is used only in the $SU(2) \times SU(2)$ subgroup. The argument ⁵⁾ for the restriction on the use of the second sum rule derives from the successful predictions ^{2),3)} $m_{A_1} \simeq \sqrt{2} m_\rho$, $m_{K_A} \simeq \sqrt{2} m_{K^*}$ and the disappearance of the unpleasant feature mentioned above.

The first spectral function sum rule gives rise to the following relations ^{2),3)} :

$$\begin{aligned} \frac{G_\rho^2}{m_\rho^2} - \frac{G_{A_1}^2}{m_{A_1}^2} &= F_\pi^2 \\ \frac{G_{K^*}^2}{m_{K^*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2} &= F_K^2 \\ \frac{G_\rho^2}{m_\rho^2} &= \frac{G_{K^*}^2}{m_{K^*}^2} \end{aligned}$$

The second spectral function sum rule applied in the subgroup $SU(2) \times SU(2)$ leads to the relations :

$$G_\rho = G_{A_1} \quad \text{and} \quad G_{K^*} = G_{K_A}$$

These relations then lead to

$$\frac{F_K}{F_\pi} = \frac{m_{A_1}}{m_{K_A}} \sqrt{\frac{m_{K_A}^2 - m_{K^*}^2}{m_{A_1}^2 - m_\rho^2}}$$

2.

which ⁶⁾ gives $(F_K/F_\pi) \simeq 1.07$ against the value ~ 1.16 found in Ref. ¹⁾. Also it follows :

$$\frac{G_{A_1}^2}{m_{A_1}^2} \simeq \frac{G_{K_A}^2}{m_{K_A}^2}$$

The Cabibbo angles θ_A and θ are related through F_K/F_π by the relation

$$\frac{F_K}{F_\pi} = \frac{\tan \theta_A}{\tan \theta}$$

which can be rewritten ⁷⁾ as

$$\frac{1}{F_+(0)} \left(\frac{F_K}{F_\pi} \right) \simeq \frac{\sqrt{1 - F_+^2(0) \sin^2 \theta}}{F_+(0) \sin \theta} \cdot \tan \theta_A$$

Here $F_+(0)$ is the $K-\pi$ form factor normalized to unity in exact $SU(3)$ limit. Now θ_A can be obtained from the branching ratio of $K \mu_2$ and $\pi \mu_2$ decays while $[F_+(0) \sin \theta]$ is determined from $K l_3$ decays. Using for the right-hand side the value ⁸⁾ ~ 1.28 , we obtain for the form factor, $F_+(0) \simeq 0.84$. On the other hand, if we assume $F_+(0) = 1$, the calculated value of the Cabibbo angle turns out to be $\tan \theta \simeq 0.26$ which is about 18% higher than the value quoted in Ref. ⁸⁾.

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In conclusion, it is of interest to note the implications of our results on the branching ratio $\Gamma_{\rho \rightarrow \pi\pi} / \Gamma_{K^* \rightarrow K\pi}$. Assuming pole dominance in the pion form factor we can derive

$$G_{\rho} g_{\rho\pi\pi} = \sqrt{2} m_{\rho}^2$$

The K_{l3} form factor F_+ under the assumption of K^* pole dominance leads to analogous relation :

$$G_{K^*} g_{K^*K\pi} = \frac{F_+(0)}{\sqrt{2}} m_{K^*}^2$$

We obtain, thereby, (using $G_{K^*}/m_{K^*} = G_{\rho}/m_{\rho}$) :

$$\frac{g_{\rho\pi\pi}}{g_{K^*K\pi}} = 2 \left(\frac{m_{\rho}}{m_{K^*}} \right) \frac{1}{F_+(0)}$$

and

$$\frac{\Gamma_{\rho^+ \rightarrow \pi\pi}}{\Gamma_{K^{*+} \rightarrow K\pi}} = \frac{4}{3} \left(\frac{k_{c.m.}^{\rho}}{k_{c.m.}^{K^*}} \right)^3 \frac{1}{(F_+(0))^2} \approx \frac{2.455}{(F_+(0))^2}$$

For $F_+(0) \simeq 1$ this implies $\Gamma_{\rho \rightarrow \pi\pi} = 122$ MeV for $\Gamma_{K^* \rightarrow K\pi} = 49.6$ MeV. For $F_+(0) \simeq 0.84$ it implies $\Gamma_{\rho \rightarrow \pi\pi} \simeq 173$ MeV. However, due to great uncertainties in the experimental decay widths of the particles involved it is not possible to draw definite information on $F_+(0)$ from this piece of experimental data.

R E F E R E N C E S

- 1) S.L. Glashow, H.J. Schnitzer and S. Weinberg - Phys.Rev.Letters 19, 139 (1967) ;
H.T. Nieh - Phys.Rev.Letters 19, 43 (1967).
- 2) S. Weinberg - Phys.Rev.Letters 18, 507 (1967).
- 3) T. Das, V.S. Mathur and S. Okubo - Phys.Rev.Letters 18, 761 (1967) ;
Phys.Rev.Letters 19, 470 (1967). We use the notation of their papers.

- 4) The first spectral function sum rule reads : [i, j being the SU(3) indices]

$$\int_0^{\infty} [\rho_V^i(\mu^2) - \rho_A^j(\mu^2)] \frac{d\mu^2}{\mu^2} = F_j^2$$

while the second sum rule is :

$$\int_0^{\infty} [\rho_V^i(\mu^2) - \rho_A^j(\mu^2)] d\mu^2 = 0$$

etc.

- 5) Similar doubts about the applicability of the second sum rule in SU(3) $\times$ SU(3) have also been raised by J. Sakurai - Phys.Rev. Letters 19, 803 (1967).
- 6) For $m_{A_1} = \sqrt{2} m_\rho$ and $m_{K_A} = \sqrt{2} m_{K^*}$ it gives $F_K = F_\pi$ implying no renormalization effect due to SU(3) breaking. We have used $m_{K_A} = 1309$, $m_{K^*} = 892.4$, $m_{A_1} = 1058$ and $m_\rho = 774$.
- 7) We make use of the fact that $F_+(0) \simeq 1$ and θ is a small angle. The author is indebted to Dr. A. Sirlin and Dr. N. Brene for a discussion on this point.

- 8) N. Brene, L. Veje, M. Roos and C. Cronstrom - Phys.Rev. 149, 1288 (1966) ;

L.B. Auerbach, J. Mac G. Dobbs, A.K. Mann, W.K. Mc Farlane,
D.H. White, R. Cester, P.T. Eschstruth, G.K. O'Neill and
D. Yount - Phys.Rev. 155, 1505 (1967).

See, however, the footnote (9) in Ref. ¹⁾ (Glashow et al.), where they mention that the value quoted for θ should be $\sim 10\%$ higher.