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ELECTROMAGNETIC MASS DIFFERENCE OF KAONS *

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A B S T R A C T

The K^+-K^0 mass difference is examined, using Weinberg's first sum rule and soft pion technique but the second sum rule is violated in which case the mass difference becomes logarithmically divergent. It is suggested that depending on the value of the coupling constants that appear in the spectral functions the correct sign of the K^+-K^0 mass difference can be obtained without the $\mathcal{O}$ terms.

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1. INTRODUCTION

Das et al., ¹⁾ have calculated the electromagnetic splitting of pions using Weinberg's sum rules ²⁾ for the spectral functions of vector and axial vector currents that result from chiral $SU(2) \times SU(2)$ current algebra and the soft pion technique. If the same technique is extended to chiral $SU(3) \times SU(3)$ to calculate the kaon mass difference, one obtains

$$M_{K^+}^2 - M_{K^0}^2 \simeq M_{\pi^+}^2 - M_{\pi^0}^2$$

which is in disagreement with the experimental result.

It is evident that one must take into account additional terms or breaking of chiral symmetry in order to obtain the right sign for the $K^+ - K^0$ mass difference. One such method is to attribute the mass difference to the δ terms which are expectation values of equal time commutators of current with their divergence. This term appears in the reduction of the kaons but not in the reduction of the pions and corresponds to a tadpole graph, in which a member of a scalar meson octet makes a virtual transition to vacuum ³⁾. Recently, the correct $K^+ - K^0$ mass difference was obtained using tadpoles ^{4), 5)}, but in the absence of a scalar meson octet tadpoles must be regarded as a convenient way of introducing a free parameter, and their contributions are assumed to be finite.

The explanation of the $K^+ - K^0$ mass difference may well rest with the δ terms, but before accepting this, it is of interest to examine an alternative method of introducing symmetry breaking into the soft pion technique of Ref. ¹⁾. One such approach pursued here is to ignore the δ terms and break the $SU(3) \times SU(3)$ symmetry by not using Weinberg's generalized second sum rule ⁶⁾. This second sum rule yields a relation among the coupling constants that is inconsistent with experiments ⁷⁾. One must then pay a price because the integral over the photon momentum q^2 becomes logarithmically divergent, resulting in the appearance of a free parameter, the cut-off.

2.

In Section 2, the consequences of not using Weinberg's second sum rule are studied, one of which is the possibility of obtaining the correct sign of the K^+-K^0 mass difference, and remarks are made in Section 3.

2. K^+-K^0 MASS DIFFERENCE CALCULATIONS

A straightforward application of the method of Ref. ¹⁾ to the kaon leads to ^{4),5)}

$$\begin{aligned}\Delta(M_K^2) &= M_{K^+}^2 - M_{K^0}^2 \\ &= \frac{e^2}{(2\pi)^4} \frac{2}{f_K^2} \int \frac{d^4 q}{q^2} (\delta_{\mu\nu} - 4 \frac{q_\mu q_\nu}{q^2}) [\Delta_{\mu\nu}^{3V}(q) + \Delta_{\mu\nu}^{8V}(q) \\ &\quad - \Delta_{\mu\nu}^{4A}(q) - \Delta_{\mu\nu}^{5A}(q) + \text{6}^3 \text{term}] \quad (1)\end{aligned}$$

where the propagation function of the vector current is

$$\Delta_{\mu\nu}^{iV}(q) = \int d^4 x e^{iq \cdot x} \langle 0 | T[V_\mu^i(x) V_\nu^i(0)] | 0 \rangle,$$

and a similar expression for $\Delta_{\mu\nu}^{iA}(q)$ with axial vector currents, and the superscript i is the unitary spin index.

Using the spectral representations of the propagator functions given in Ref. ¹⁾ and the generalization of Weinberg's first spectral function sum rule to chiral $SU(3) \times SU(3)$ in the form ⁶⁾

$$\int dm^2 \left[\frac{\rho_V^i(m^2) - \rho_A^j(m^2)}{m^2} \right] = f_K^2, \quad i=3,8, \quad j=4,5 \quad (2)$$

or a weaker assumption

$$\int dm^2 \left[\frac{\rho_V^3 + \rho_V^8 - \rho_A^4 - \rho_A^5}{m^2} \right] = 2 f_K^2, \quad (3)$$

we obtain

$$\Delta(M_K^2) = -\frac{3i\alpha}{8\pi^3} \frac{1}{f_K^2} \int \frac{d^4q}{q^2} \int dm^2 \left[\frac{(P_V^3 + P_V^8 - P_A^4 - P_A^5)}{q^2 + m^2} + \sigma^3 \text{ term} \right]_{(4)}$$

where $\alpha = e^2/4\pi$.

The spectral functions ρ are approximated by the corresponding $\rho(770)$, $\omega(783)$, $\phi(1019)$, and $K_A(1320)$ meson poles

$$\begin{aligned} \rho_V^3 &= g_\rho^2 \delta(m^2 - m_\rho^2), \\ \rho_V^8 &= g_\omega^2 \delta(m^2 - m_\omega^2) + g_\phi^2 \delta(m^2 - m_\phi^2), \\ \rho_A^4 + \rho_A^5 &= 2 g_{KA}^2 \delta(m^2 - m_{KA}^2). \end{aligned} \quad (5)$$

We shall drop the contribution from the σ^3 term on the right-hand side of Eq. (4), because we are interested in an alternative possible way of obtaining the correct sign of $\Delta(M_K^2)$. After putting (5) into (4) and integrating with respect to dm^2 , we obtain with the aid of $g_\rho^2 = 2m_\rho^2 f_\pi^2$,

$$\Delta(M_K^2) = -\frac{3i\alpha}{8\pi^3} \left(\frac{f_\pi}{f_K}\right)^2 \int \frac{d^4q}{q^2} 2m_\rho^2 \left[\frac{1}{q^2 + m_\rho^2} + a_\omega \frac{1}{q^2 + m_\omega^2} + a_\phi \frac{1}{q^2 + m_\phi^2} - 2a_{KA} \frac{1}{q^2 + m_{KA}^2} \right] \quad (6)$$

where

$$a_\omega = g_\omega^2/g_\rho^2, \quad a_\phi = g_\phi^2/g_\rho^2, \quad \text{and} \quad a_{KA} = g_{KA}^2/g_\rho^2. \quad (7)$$

4.

We do not assume the validity of Weinberg's generalized second sum rule

$$\int dm^2 [\rho_V^j(m^2) - \rho_A^j(m^2)] = 0 \quad j = 1, 2, \dots, 8. \quad (8)$$

or

$$a_\omega + a_\phi = 1, \quad a_{KA} = 1. \quad (9)$$

However, if Eq. (9) is substituted into (6), the integral converges resulting in

$$\Delta(M_K^2) = \frac{3\alpha}{4\pi} \left(\frac{f_I}{f_K} \right)^2 m_\rho^2 \left[\ln \frac{m_{KA}^2}{m_\rho^2} + \ln \frac{m_{KA}^2}{m_\phi^2} + a_\omega \ln \frac{m_\phi^2}{m_\omega^2} \right] > 0 \quad (10)$$

which yields the wrong sign of $\Delta(M_K^2)$.

Returning to Eq. (6), in the absence of Eq. (8) or a weaker assumption ⁵⁾

$$\int dm^2 [\rho_V^3 + \rho_V^8 - \rho_A^4 - \rho_A^5] = 0, \quad (11)$$

which also leads to a convergent $\Delta(M_K^2)$, one obtains $\Delta(M_K^2)$ that is logarithmically divergent. If one accepts logarithmically divergent models, this opens up a new possibility because the sign of the mass difference can depend on the sign of the coefficient of the divergent term.

From Eqs. (5) and (11), this coefficient that we define as $(1-a)$ is

$$1-a = 1 - (2a_{KA} - a_\phi - a_\omega). \quad (12)$$

In order to get $\Delta M_K^2 < 0$, it is necessary that $a > 1$. When Eq. (8) holds $a=1$, so that a is a convenient parameter to express symmetry breaking. The size of the deviation of a from $a=1$ would give information about the extent Weinberg's generalized second sum rule is violated.

We appeal to leptonic decay modes $\rho \rightarrow e^+e^-$ (8), $\omega \rightarrow e^+e^-$ (9) and $\phi \rightarrow e^+e^-$ (10) to get bounds on a_ω and a_ϕ . Using the experimental values

$$\begin{aligned}\Gamma(\rho \rightarrow e^+e^-)/\Gamma_\rho &= (6.5 \pm 1.4) \times 10^{-5} \\ \Gamma(\omega \rightarrow e^+e^-)/\Gamma_\omega &= (20 \pm 12) \times 10^{-5} \\ \Gamma(\phi \rightarrow e^+e^-)/\Gamma_\phi &\leq 74 \times 10^{-5}\end{aligned}$$

we find

$$\begin{aligned}g_\rho^2 &= (1.70 \pm 0.38) \times 10^{10} (\text{MeV})^4, \\ g_\omega^2 &= (1.58 \pm 0.95) \times 10^{10} (\text{MeV})^4, \\ g_\phi^2 &\leq 3.78 \times 10^{10} (\text{MeV})^4,\end{aligned}$$

or

$$\begin{aligned}a_\omega &= (0.93 \pm 0.59), \\ a_\phi &< 2.2.\end{aligned}\tag{13}$$

With the aid of $g_\rho^2 = 2m_\rho^2 f_\pi^2$, one finds $f_\pi = 120$ MeV in contrast with $f_\pi = 2M_N g_A / G_{NN\pi} = 164$ MeV.

The result of combining Eqs. (2), (5) and (13) is

$$a_\phi = \frac{m_\phi^2}{m_\rho^2} \left(1 - a_\omega \frac{m_\rho^2}{m_\omega^2} \right) = (0.17 \pm 0.10), \quad (14)$$

$$a_{KA} = \frac{m_{KA}^2}{m_\rho^2} \left[1 - \frac{1}{2} \left(\frac{f_K}{f_\pi} \right)^2 \right] = 0.9 - 1.4, \quad (15)$$

where for the value of f_K/f_π in Eq. (15), recent theoretical estimates obtained on the basis of only Weinberg's generalized first sum rule were taken, as this corresponds to the present approach¹¹⁾. The combinations of Eqs. (13) to (15) yield $a = 0.7 - 1.7$. Another way to estimate a is to use the calculated values of Das et al.,¹²⁾ i.e., $a_\omega = 0.43$, $a_\phi = 1.03$ in which case $a = 0.34 - 1.34$.

On the other hand, if we use the weaker Weinberg's second sum rule, Eq. (3) rather than Eq. (2), we obtain with the aid of Eq. (13)

$$a_{KA} = \frac{m_{KA}^2}{2m_\rho^2} \left[1 + \frac{m_\rho^2}{m_\omega^2} a_\omega + \frac{m_\rho^2}{m_\phi^2} a_\phi - \left(\frac{f_K}{f_\pi} \right)^2 \right] \leq 1.47. \quad (16)$$

Let us keep an open mind and proceed from Eq. (6) to study the relation between the parameter a and the required cut-off to obtain the experimental $K^+ - K^0$ mass difference $\Delta(M_K^2)/2M_K = -4.05$ MeV. The integration of Eq. (6) with a cut-off M yields

$$\begin{aligned} \Delta(M_K^2) &= \frac{3\alpha}{4\pi} \left(\frac{f_\pi}{f_K} \right)^2 m_\rho^2 \left[\ln \left(1 + \frac{M^2}{m_\rho^2} \right) + a_\omega \ln \left(1 + \frac{M^2}{m_\omega^2} \right) + a_\phi \ln \left(1 + \frac{M^2}{m_\phi^2} \right) \right. \\ &\quad \left. - 2a_{KA} \ln \left(1 + \frac{M^2}{m_{KA}^2} \right) \right] \\ &\simeq \frac{3\alpha}{4\pi} m_\rho^2 (1-a) \ln \Lambda^2 \end{aligned} \quad (17)$$

where $\Lambda \approx M/m_\rho$.

As an illustration, if chiral $SU(3) \times SU(3)$ symmetry is broken such that $a = 1.7$, the most favourable estimate, then $\ell_n \Lambda = 2.76$, a cut-off which is too large for any comfort. In other words, for the present model to make any sense, the symmetry must be badly broken, i.e., a must be very large.

3. REMARKS

It was found in the framework of the soft kaon method, when Weinberg's generalized second sum rule is violated, i.e., $a = 2a_{KA} - a_{\omega} - a_{\phi} \neq 1$, the $K^+ - K^0$ mass difference is logarithmically divergent and it can have the correct sign if $a > 1$. The larger the symmetry breaking is, (the larger a is) the smaller the required cut-off (that gives the correct $K^+ - K^0$ mass difference) is. For $a = 1.7$, a large cut-off $\ell_n \Lambda = 2.76$ was necessary. Estimates of the value of a were inclusive. Reliable information on a would determine the validity of this possible explanation of the $K^+ - K^0$ mass difference.

It is difficult to assess what influence the finite mass ¹³⁾ of the kaon has on the cut-off value in the soft kaon method. A rough estimate based on the conventional method in which the contribution from the intermediate kaons is considered indicates that the required cut-off is almost unchanged.

In the soft kaon method, the K^0 meson has no electromagnetic mass so that a negative $\Delta(M_K^2)$ corresponds to a negative self-mass of the K^+ meson. One recalls that in the conventional method the contributions that come from the intermediate vector meson states are negative ¹⁴⁾. The same situation occurs for the soft kaon method provided $a > 1$, and the cut-off is sufficiently large.

When the K^+-K^0 mass difference becomes logarithmically divergent, because Weinberg's generalized second sum rule does not hold, it is indicated that there is a possibility that the correct sign of the mass difference can be obtained. Also one can understand the mass difference in terms of σ terms. It would be desirable to have an explanation of the sign of the K^+-K^0 mass difference that is free from such arbitrariness.

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REFERENCES

- 1) T. Das, G.S. Guralnik, V.S. Mathur, F.E. Low and J.E. Young, Phys.Rev.Letters 18, 759 (1967).
- 2) S. Weinberg, Phys.Rev.Letters 18, 507 (1967).
- 3) The author benefitted from a discussion with M. Jacob about σ terms.
- 4) C.L. Cook, L.E. Evans, M.Y. Han, N.R. Lipshutz and M. Straumann, Phys.Rev.Letters 20, 295 (1968).
- 5) F. Capra, to be published.
- 6) S.L. Glashow, H.J. Schnitzer and S. Weinberg, Phys.Rev.Letters 19, 139 (1967).
- 7) T. Das, V.S. Mathur and S. Okubo, Phys.Rev.Letters 19, 470 (1967);
J.J. Sakurai, Phys.Rev.Letters 19, 803 (1967);
S.P. DeAlwis, Nuovo Cimento 51A, 846 (1967).

The weaker form of Weinberg's second sum rule does not seem to be inconsistent with experiments, and leads to a prediction of $f_K = 130$ MeV with the aid of (2) and (13). The present approach is to pursue to consequences of not assuming this form as well.

- 8) J.G. Asbury, U. Becker, W.K. Bertram, P. Joos, M. Rohde, A.J.S. Smith, C.L. Jordan and C.C. Ting, Phys.Rev.Letters 19, 869 (1967).
- 9) A.H. Rosenfeld et al., University of California, Radiation Laboratory report no. UCRL-8030, Part 1 (Jan. 1968).
- 10) R.C.C. Chase, P. Rothwell and R. Weinstein, Phys.Rev.Letters 18, 710 (1967).

- 11) The experimental result is $(f_K/f_\pi) \approx 1.15$ according to Ref. ⁵).
 Determinations of this ratio without Weinberg's generalized second
 sum are found to be $f_K/f_\pi = 1.07$ by P.P. Srivastava, Phys.Letters
26B, 233 (1968), $f_K/f_\pi = 1$ by R.J. Oakes (to be published), and
 $f_K/f_\pi = 1.08$ by S.L. Glashow and S. Weinberg, Phys.Rev.Letters 20,
 224 (1968).
- 12) T. Das, V.S. Mathur and S. Okubo, Phys.Rev.Letters 19, 470 (1967).
- 13) Finite pion mass effects have been studied by I.S. Gerstein,
 B.W. Lee, H.T. Nieh and H.J. Schnitzer, Phys.Rev.Letters 19,
 1064 (1967);
 J.S. Bell and R.P. Van Royen, Phys.Letters 25B, 354 (1967);
 G.C. Wick and B. Zumino, Phys.Letters 25B, 479 (1967);
 M.B. Halpern and G. Segrè, Phys.Rev.Letters 19, 611 (1967).
- 14) R.H. Socolow, Phys.Rev. 137, B1221 (1965).