CM-P00057897

Archives

Ref. TH.983

THE X^0 WIDTH

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ABSTRACT

Representing the $X\pi \rightarrow \eta\pi$ reaction amplitude with a Veneziano formula one calculates the X^0 width in terms of f_π . A possible connection with the small width of the δ and the influence of the soft pion limits is also discussed.

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1. INTRODUCTION

The formula recently proposed by Veneziano ¹⁾ for the invariant two-body amplitude combines the presence of resonances with Regge behaviour in all channels. As pointed out by Lovelace ²⁾, it can be fruitfully used to reach decay amplitudes where three Mandelstam variables take positive values greater than the corresponding thresholds. In the cases so far studied, however, only three-body final state interactions among decay pions were explicitly introduced. The purpose of this note is to study a strong interaction three-body decay process which, when related through a Veneziano formula to a two-body scattering amplitude, is fully determined in terms of a residue at a meson pole term. The X^0 (or η') decay into $\eta\pi\pi$ is particularly interesting. Even though it is a strong process, it has a relatively small width and the $\pi^+\pi^-\chi$ decay mode retains an important branching ratio ³⁾. All the particles involved being spinless, the decay amplitude can be written in terms of the ρ meson width, say, provided one uses SU_3 symmetry and introduces mixing angles for tensor and pseudoscalar mesons. The value then obtained for the width is indeed relatively small and in agreement with present experimental information.

It should be stressed though that the value thus reached is very sensitive to the partial decay width of the A_2 into $\eta\pi$. Even though a value for this is available ⁴⁾, the recently observed splitting of the A_2 ^{5),6)} makes it now highly questionable. If one retains a value of 90 MeV for the A_2 width and takes 0.1 for the $\eta\pi$ branching ratio, the calculated η' width, with an $\eta\pi\pi$ branching ratio of 0.7, is found to be 1.1 MeV, when there exists an experimental upper limit of 4 MeV ^{3),7),8)}. Decreasing the $\eta\pi$ partial width by a factor 1.5 will raise this width to 2.1 MeV. We can therefore propose but an order of magnitude which turns out to be acceptable. Any particular value has only little meaning in the presently uncertain situation concerning the A_2 . It is nevertheless interesting to see how the various couplings involved get related when an explicitly crossing symmetric amplitude is written according to the Veneziano prescription ¹⁾ as a first approximation to the $\eta\eta'\pi\pi$ amplitude and this is indeed the main purpose of this paper.

Our calculation assumes the decoupling of a scalar particle to the $\eta\pi$ channel. This particle is expected on the first daughter trajectory in the same way as a 0^+ state which could be identified with the ε^0 is found as a manifestation of the first ρf^0 daughter trajectory in $\pi\pi$ scattering. Here, however, no such state seems to exist and it has to be eliminated. It could possibly be identified with the δ^9 which is in effect weakly coupled to the $\eta\pi$ decay channel. Writing again the simplest form for the $\eta\eta'\pi\pi$ amplitude, where the scalar pole is eliminated, one can at the same time impose the Adler condition ¹⁰⁾ which is a priori not satisfied for an arbitrary target mass with a Veneziano formula limited to its first term ²⁾. This leads to expect a 1^- particle in the neighbourhood of the A_2 with isospin one and negative G parity, a still open possibility even though its quantum numbers would not agree with the usual quark model assignments ^{8),11),12)}.

2. THE X^0 WIDTH

We first consider $\pi\eta_8 \rightarrow \pi\eta_8$ "scattering", where η_8 stands for the octet member isospin zero pseudoscalar meson. It involves the f^0 (or P') trajectory in one channel and the A_2 (or R) trajectory in the other two. These trajectories are assumed to coincide as a consequence of duality ^{13),14)}. Therefore we write a fully symmetrical Veneziano formula in order to impose positive signature in all three channels.

$$A(\pi\eta_8 \rightarrow \pi\eta_8) = \beta_{88} (V(s,t) + V(u,t) + V(s,u)) \quad (1)$$

with

$$V(x,y) = \frac{\Gamma(1-\alpha(x)) \Gamma(1-\alpha(y))}{\Gamma(1-\alpha(x)-\alpha(y))} \quad (2)$$

where

$$\alpha(x) = \frac{1}{2} + \alpha'(x - m_\pi^2) \quad (3)$$

The slope is given as

$$\alpha' = [2(m_\rho^2 - m_\pi^2)]^{-1} = 0.88 \text{ Gev}^{-2}$$

when we have imposed $\alpha(m_\pi^2) = \frac{1}{2}$ in order that the Adler condition is satisfied for $\pi\pi$ scattering²⁾. With πA_1 degeneracy, implied by the postulated absence of resonances in the $\rho^+\pi^+$ channel, this further leads to $m_{A_1}^2 \simeq 2m_\rho^2$ ¹⁵⁾.

Relation (1) implies equality between the $A_2 \eta_8 \pi$ coupling constant squared and the product of the $f^0 \eta_8 \eta_8$ and $f^0 \pi\pi$ coupling constants. They are respectively proportional to the residues of the A_2 and f^0 poles which are readily obtained from (1) in terms of β_{88} . This relation would not be affected by the presence of satellite terms¹⁾ added to (1) since they would not contribute to a leading pole term. We assume that only one tensor meson A_2 pole term is relevant as an SU_3 partner of the f^0 . The ratio of the octet to singlet tensor meson couplings follows from the observation that the f' meson does not decay into two pions. The $f^0 \eta_8 \eta_8$ and $f^0 \pi\pi$ couplings are then related through the tensor meson mixing angle θ_T

$$\frac{g(f^0 \eta_8 \eta_8)}{g(f^0 \pi\pi)} = -\frac{1}{\sqrt{3}} \frac{1 - \tan^2 \theta_T}{1 + \tan^2 \theta_T} \quad (4)$$

The $A_2 \eta_8 \pi$ and $f^0 \eta_8 \eta_8$ coupling constants being related through SU_3 symmetry, the compatibility of (1) and unitary symmetry, together with factorization at the 2^+ meson pole, imposes a value for the tensor meson mixing angle. One finds

$$\tan^2 \theta_T = \frac{1}{2} \quad (5)$$

This is a value already obtained by Chiu and Finkelstein ¹⁶⁾ as a result of duality and the absence of resonances corresponding to higher SU_3 representation. It is still relatively close to the value of 0.31 obtained through a phenomenological analysis of the mass values of the 9 tensor mesons ⁸⁾. Using (4) and (5), we get

$$g(A_2 \eta \pi) = 0.57 g(f^0 \pi^0 \pi^0) \quad (6)$$

This leads with SU_3 symmetry to a value of 7 MeV for the $A_2 K\bar{K}$ partial width. This is compatible with present experimental information ¹⁷⁾.

The $A_2 \eta \pi$ coupling constant can, in principle, be obtained in terms of the $A_2 \eta \pi$ partial width. We write

$$\left| \frac{g(A_2 \eta \pi)}{g(f^0 \pi^0 \pi^0)} \right| = \sqrt{\frac{3 \Gamma(A_2 \rightarrow \eta \pi)}{2 \Gamma(f^0 \rightarrow \pi \pi)}} \frac{m_{A_2}}{m_{f^0}} \left(\frac{q_{f^0}}{q_{A_2}} \right)^{5/2} \quad (7)$$

This is of course the difficult step to make, having to take care of some SU_3 breaking and in particular in view of the present uncertainty concerning the A_2 partial width ¹⁸⁾. With A_2 splitting, a determination of the $A_2 \eta \pi$ coupling from the Regge pole analysis of production off nucleons, used together with ρ A_2 exchange degeneracy in K nucleon scattering, would not be more reliable. Taking 9 MeV and 6 MeV as tentative values for this width, we get :

$$\left| \frac{g(A_2 \eta \pi)}{g(f^0 \pi^0 \pi^0)} \right| = \begin{cases} 0.46 & (9 \text{ MeV}) \\ 0.37 & (6 \text{ MeV}) \end{cases}$$

The $A_2 X \pi$ coupling constant is then obtained in terms of the pseudo-scalar meson mixing angle θ_P ^{19), 20)} with $\tan \theta_P = -0.19$ ⁸⁾ :

$$g(A_2 X \pi) = \frac{\cos \theta_p g(A_2 \eta \pi) - g(A_2 \eta_8 \pi)}{\sin \theta_p} \quad (8)$$

With the sign ambiguity on $g(A_2 \eta \pi)$ this gives

$$\frac{g(A_2 \eta \pi) g(A_2 X \pi)}{g(f^0 \pi^0 \pi^0)} = \gamma = \begin{cases} 0.31 & \text{or } -2.5 & (9 \text{ MeV}) \\ 0.43 & \text{or } -1.5 & (6 \text{ MeV}) \end{cases}$$

This is, however, only a poor substitute for a direct determination of the $X \pi$ branching ratio in A_2 decay. The value obtained is directly related to the residue at the A_2 pole in (1) since the same relation is also suitable for $X \pi \rightarrow \eta \pi$ scattering, but with a different factor β . Writing also the $\pi^+ \pi^-$ amplitude as ²⁾:

$$A(\pi^+ \pi^- \rightarrow \pi^+ \pi^-) = \beta_{\pi\pi}^{\pi\pi} V(s, t) \quad (9)$$

We obtain

$$\beta_{X\pi}^{\eta\pi} = \frac{\gamma}{2} \beta_{\pi\pi}^{\pi\pi} \quad (10)$$

$\beta_{\pi\pi}^{\pi\pi}$ can be obtained identifying (9) in its linear small s, t approximation to the Weinberg $\pi\pi$ amplitude ²¹⁾. One obtains

$$\beta_{\pi\pi}^{\pi\pi} = -(\pi f_\pi^2 \alpha')^{-1} = -52.5$$

With this value, the ρ width obtained from the residue at the ρ pole is found to be

$$\Gamma = \frac{1}{12 m_\rho^2} \frac{\beta_{\pi\pi}^{\pi\pi}}{8\pi} (m_\rho^2 - 4m_\pi^2)^{3/2} = 105 \text{ MeV}$$

in reasonable agreement with experiment.

From (1) and (10) one gets the $X \pi \eta \pi$ amplitude in principle everywhere and one can calculate the $X \eta \pi^+ \pi^-$ decay rate

$$\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8 m_X} \int |A(\omega_1, \omega_2)|^2 d\omega_1 d\omega_2 \quad (11)$$

where ω_1 and ω_2 are the two pion energies and the integral extends over the physical region for X decay. In so doing, however, one has to face a problem. Relation (1) includes a daughter trajectory which leads to a scalar particle at the mass of the ρ . It corresponds to the welcome Σ^0 in $\pi\pi$ scattering. No such particle however exists in the $\eta\pi$ channel. At most, it could be associated with the δ which is weakly coupled to the $\eta\pi$ channel but with a large error in mass. This means that the simplest approximation to the $X \pi \eta \pi$ amplitude written in (1) is not suitable as it is, and that a satellite ¹⁾ should be added. Taking the next simplest form, we write

$$\begin{aligned} A(X\pi \rightarrow \eta\pi) = & \beta (V(s,t) + V(u,t) + V(s,u)) \\ & + \gamma (V'(s,t) + V'(u,t)) + \delta V'(s,u) \end{aligned} \quad (12)$$

with

$$V'(x,y) = (1 - \alpha(x) - \alpha(y))^{-1} V(x,y) \quad (13)$$

We have not imposed full s, t, u symmetry on the satellite term since there is neither an a priori reason to forbid negative signature at the satellite level, nor to eliminate the Σ^0 pole. As already

mentioned, adding a satellite does not modify the relations already obtained between the coupling constants and from which a value for β can be obtained. Both terms in the right-hand side have single poles at $\alpha(s)=1$ and $\alpha(u)=1$. They should cancel one another so that no $0^+ \eta\pi$ resonance appears. This imposes a relation

$$\gamma + \delta = \beta \left(2\alpha(0) + \alpha'(m_X^2 + m_\eta^2 + 2m_\pi^2 - m_\rho^2) \right) \quad (14)$$

Noting that $s=u=m_\rho^2$ happens to be inside the small physical region, we keep only the value of (12) at this point, since - once the pole term has been cancelled as a consequence of (14) - the amplitude can be approximated by its value in the central physical region for X decay. This gives :

$$A \simeq 2 \beta_{X\pi\pi}^{\eta\pi} \quad (15)$$

Combining (10), (11) and (15), we get

$$\begin{aligned} \Gamma &= 2.15 \Gamma(X \rightarrow \eta\pi^+\pi^-) = 11.6 \gamma^2 \text{ MeV} \\ &= 1.1 \text{ MeV} (9 \text{ MeV}) \text{ or } 2.1 \text{ MeV} (6 \text{ MeV}) \end{aligned} \quad (16)$$

The lowest values thus obtained (same sign for the $A_2 \eta\pi$ and the $A_2 \eta_8\pi$ couplings) are compatible with present experimental information. Relation (15) follows from the elimination of a scalar isospin one particle as a possible intermediate state in X decay. This should be done since even the δ could not contribute. It does not depend, however, on the particular way it has been achieved, (12), provided we can identify the average value with the decay amplitude at $s=u=m_\rho^2$. A more complicated expression, involving higher order satellites, could of course be introduced in (12). The constraints to be imposed are that no ancestors ²²⁾ should occur. They could come at $\alpha(s)$ or $\alpha(u)=1$ from the expansion in powers of t of $(N-\alpha(t))$ denominators which could be present on the pole terms. In any case it is possible, as done here, to relate two a priori remote physical quantities, the $X \eta\pi\pi$ decay width and the residue at the A_2 pole in the same amplitude, calculated in terms of f_π through SU_3 symmetry.

3. SOFT PION LIMITS FOR η (AND X) π SCATTERING

The simplest relation already used in order to forbid the decay of a scalar particle (12) otherwise giving a pole term in the $X \eta \pi \pi$ amplitude could be further constrained so that the Adler condition could be satisfied. We have so far obtained a relation giving $(\gamma + \delta)$. Imposing that the amplitude vanishes when either pion has a zero four momentum, one can further get the ratio δ/γ . The Adler condition would not be satisfied without a satellite term when the ρ f^0 and A_2 trajectories have been taken as degenerate ¹⁴⁾ and such that it should be obeyed for $\pi\pi$ scattering ²⁾. We would take the attitude that in $\pi\pi$ (or πK) scattering, $\gamma = \delta = 0$ follows then from relation (3) between the ρ (or K^*) and the πA_1 (or K, K_A) trajectories, a condition which cannot be satisfied in $\eta\pi$ (or $X\pi$) scattering. For an arbitrary target mass, it can result only through a cancellation between the corresponding leading and satellite terms, since the trajectories are assumed to be already determined through independent results. Now, however, the detailed structure of the satellite term introduced matters. Nevertheless, using the simple form already written, (12), we may calculate the values of $(\gamma + \delta)$ and δ/γ which are such that the amplitude (12) vanishes when ²³⁾ :

$$s(u) = m_X^2, \quad u(s) = m_\eta^2, \quad t = m_\pi^2$$

The Adler condition may also be satisfied for $\eta\pi$ and $X\pi$ elastic scattering for which expressions similar to (12), with different values for β , γ and δ , can be written. The corresponding results are given in the following table :

Reaction	$(\gamma + \delta)/\beta$	δ/γ
$X \pi \rightarrow \eta \pi$	-1.56	-11.34
$\eta \pi \rightarrow \eta \pi$	-1.02	- 3.57
$X \pi \rightarrow X \pi$	-2.1	-65.95

(16)

With $\delta / \gamma \neq 1$, as necessary with the simplest Veneziano form (12), if the Adler relation is satisfied, one predicts particles of both even and odd spin on the satellite trajectory when the daughters of the leading trajectory will all correspond to even spin. In particular, a 1^- state appears at the mass of the A_2 and is coupled to the $\eta\pi$ channel. Its residue, as obtained from (12), is proportional to $\beta(\gamma - \delta)$. One checks that with the values obtained from PCAC, (16), factorization at this pole is well satisfied. At the same time, factorization at the ϵ^0 pole is also satisfied within 6% accuracy. $(\gamma - \delta)$ is required to be positive as here found. The decay widths into $\eta\pi$ and $X\pi$ can be calculated to be respectively 46 and 14 MeV, taking 9 MeV for the $A_2 \eta\pi$ partial width. This state should however be considered on a similar basis as the 1^- daughter at the f^0 meson mass, obtained in $\pi\pi$ scattering ²⁾ (9) and the elastic decay width of which is found to be 180 MeV. This state should however actually correspond to a wide highly inelastic resonance since the f^0 which is mainly produced peripherally is found to correspond to a charge neutral branching ratio which leaves but little room for some isospin 1 admixture on the observed f^0 peak. The presence of such particles with so-called "exotic" quantum numbers ¹¹⁾ is a necessity in models based on rising trajectories ²⁴⁾. They could yet be weakly produced in present experiments and could have escaped detection. We would not identify the 1^- particle obtained with one of the A_2 peaks. The elastic widths here mentioned are calculated from pole residues and would change in a more refined approach where complex values for α could be introduced and eventually where unitarity would be satisfied at the same time as crossing symmetry.

It is a pleasure to thank Dr. K. Igi and Dr. C. Lovelace for discussions. J.B. and S.P. wish to express their thanks to Professor W. Thirring and to Professor J. Prentki for the hospitality extended to them in the CERN Theoretical Study Division. J.B. gratefully acknowledges the Bundesministerium für wissenschaftliche Forschung for financial support.

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