Pions in Nuclei and Manifestations of Supersymmetry in Neutrinoless Double Beta Decay.

Amand Faessler, Sergey Kovalenko

Institute für Theoretische Physik der Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Fedor Šimkovic

Department of Nuclear Physics, Comenius University, Mlynská dolina F1, 84215 Bratislava, Slovakia

Abstract

We examine the pion realization of the short ranged supersymmetric (SUSY) mechanism of neutrinoless double beta decay ($0\nu\beta\beta$-decay). It originates from the $R$-parity violating quark-lepton interactions of the SUSY extensions of the standard model of the electroweak interactions. We argue that pions are dominant SUSY mediators in $0\nu\beta\beta$-decay. The corresponding nuclear matrix elements for potentially $0\nu\beta\beta$-decaying isotopes are calculated within the proton-neutron renormalized quasiparticle random phase approximation (pn-RQRPA). We define those isotopes which are most sensitive to the SUSY signal and outlook the present experimental situation with the $0\nu\beta\beta$-decay searches for the SUSY. Upper limits on the $R$-parity violating 1st generation Yukawa coupling $\lambda_{111}$ are derived from various $0\nu\beta\beta$-experiments.

The observation of neutrinoless nuclear double beta decay $(A, Z) \rightarrow (A, Z + 2) + 2e^-$ ($0\nu\beta\beta$) would undoubtedly indicate the presence of the new physics beyond the standard model (SM) of electroweak interactions. However, as yet there is no any experimental evidence for this lepton-number violating ($\Delta L = 2$) exotic process. On the other hand non-observation of $0\nu\beta\beta$-decay at certain experimental sensitivity allows one to set limits on some parameters of the new physics. An unprecedented accuracy and precision of the modern $0\nu\beta\beta$-decay experiments allows one in certain cases to push these limits far away of the reach of the other accelerator and non-accelerator experiments.

A well known example is given by the upper limit on the light effective Majorana neutrino mass $\langle m^2_{\nu}\rangle$. From the $0\nu\beta\beta$-decay experiments [1] it was found $\langle m^2_{\nu}\rangle \lesssim O(1.1\text{ eV})$ [2]. Recall that the Majorana neutrino mass term violates the lepton number $\Delta L = 2$. This is exactly that is necessary for $0\nu\beta\beta$-decay to proceed via the virtual neutrino exchange between the two neutrons. In this case the $0\nu\beta\beta$-decay amplitude is proportional to $\langle m^2_{\nu}\rangle$.

The Majorana neutrino exchange is not the only possible mechanism of $0\nu\beta\beta$-decay. The lepton-number violating quark-lepton interactions of the R-parity non-conserving supersymmetric extensions of the SM ($R_p$-SUSY) can also induce this process [3]-[6]. $R$-parity is a discrete multiplicative $Z_2$ symmetry defined as $R_p = (-1)^{3B+L+2S}$, where $S$, $B$ and $L$ are the spin, the baryon and the lepton quantum number. The $0\nu\beta\beta$-decay proved to be very sensitive probe of the new interactions predicted in the $R_p$-SUSY [5]-[7].

Searching for tiny effects of the physics beyond the SM in $0\nu\beta\beta$-decay requires a reliable treatment of the nuclear structure. In this note we present the results of our calculations within

[1] On leave of absence from the Joint Institute for Nuclear Research, Dubna, Russia.
the proton-neutron renormalized Quasiparticle Random Phase Approximation (pm-RQRPA) [8]. We are listing the nuclear matrix elements for the $R_p$ SUSY mechanism of the $0\nu\beta\beta$-decay and their specific values for the experimentally interesting isotopes.

An important point of our approach is the hadronization of the underlying $\Delta L = 2$ quark-level $0\nu\beta\beta$-transition $d+d \rightarrow u+u+2e^-$. In our previous paper Ref. [6] we had considered the two-pion realization of this underlying subprocess and shown that the corresponding contribution to $0\nu\beta\beta$-decay absolutely dominates over the conventional two nucleon mode realization. In this note we generalize the previous treatment and take into account the last possible option of the hadronization in the form of the one-pion mode. We will show that in this case the two-pion mode is again dominant.

Within the minimal $R_p$ SUSY extension of the SM ($R_p$MSSM) the afore-mentioned quark-level $0\nu\beta\beta$-transition is described by the following low-energy effective Lagrangian [5]

$$\mathcal{L}_{\nu e} = \frac{G_F}{2m_\nu} \varepsilon(1+\gamma_5)\varepsilon^* \left[ (\eta_{\bar{\nu}} + \eta_{\nu})(J_{\nu} J_{\bar{\nu}} + J_S J_S) - \frac{1}{4} \eta_{\nu} B_{\mu\nu} J_{\mu\nu} \right].$$

These $\Delta L_e = 2$ lepton-number violating effective interactions are induced by the heavy SUSY particles exchange. The color-singlet hadronic currents in Eq. (1) are $J_{\nu} = \bar{u}^\alpha d_{\alpha}$, $J_S = \bar{u}^\alpha d_{\alpha}$, $J_{\mu\nu} = \bar{u}^\alpha d_{\alpha} (1+\gamma_5) d_{\alpha}$ where $\alpha$ is the color index. The effective lepton-number violating parameters $\eta$ depend on the fundamental parameters of the $R_p$MSSM and can be written in the form

$$\eta_{\bar{\nu}} = \Lambda^2 \left( 2\alpha_2 \frac{m_{\bar{\nu}}^2}{m_{\bar{\gamma}}} + \frac{3}{4} \alpha_2 \frac{m_{\bar{\nu}}}{m_\chi} (\epsilon_R d + \epsilon_L u) \right),$$

$$\eta_{\nu} = \Lambda^2 \left( 2\alpha_2 \frac{m_\nu^2}{m_{\gamma}} + \frac{3}{2} \alpha_2 \frac{m_\nu}{m_\chi} (\frac{m_{\gamma}}{m_\zeta})^2 \mathcal{C} \right).$$

Here $\Lambda = (\sqrt{2}\pi/3)^{1/11} m_\chi^{-1} m_\gamma^2$ and $\mathcal{C} = 6 (m_{\gamma}/m_\zeta)^2 \epsilon_L^2 - \epsilon_R \epsilon_L - \epsilon_L \epsilon_{Rd} (m_{\gamma}/m_\zeta)^2 - \epsilon_L \epsilon_{Ru}$. $\alpha_2 = g_2^2/(4\pi)$ and $\alpha_s = g_2^2/(4\pi)$ are $SU(2)_L$ and $SU(3)_c$ gauge coupling constants; $m_{\bar{\gamma}}$ and $m_\chi$ are masses of the gluino $\gamma$ and of the lightest neutralino $\chi$. The latter is a linear combination of the gaugino and higgsino fields $\chi = \alpha_3 B + \beta_3 W^3 + \delta_3 H^u_1 + \gamma_3 H^d_2$. Here $W^3$ and $B$ are the neutral $SU(2)_L$ and $U(1)$ gauginos while $H^u_1$, $H^d_2$ are the higgsinos which are superpartners of the two neutral Higgs boson fields $H^u_1$ and $H^d_2$ with weak hypercharges $Y = -1, +1$, respectively. The mixing coefficients $\alpha_3, \beta_3, \gamma_3, \delta_3$ can be obtained from diagonalization of the $4 \times 4$ neutralino mass matrix [9]. Neutralino couplings are defined as [9] $\epsilon_{\nu e} = -T_3(\bar{\psi})\beta_3 + \tan \theta_W (T_3(\bar{\psi}) - Q(\bar{\psi}) \alpha_3)$. $\epsilon_{Rd} = Q(\bar{\psi}) \tan \theta_W \alpha_3$. Here $Q$ and $T_3$ are the electric charge and the weak isospin of the fields $\bar{\psi} = u, d, e$. In Eqs. (2) and (3) we applied a widely used ansatz of the universal squark $\tilde{q}$ mass $m_{\tilde{q}}$ at the weak scale $m_\tilde{g} \approx m_\tilde{\gamma} \approx m_{\tilde{q}}$. This approximation is well motivated by the constraints from the flavor changing neutral currents.

The next step deals with reformulation of the quark-lepton interactions in Eq. (1) in terms of the effective hadron-lepton interactions. This is necessary for the subsequent nuclear structure calculations. There are the two possibilities of hadronization of the effective Lagrangian $\mathcal{L}_{\nu e}$ in Eq. (1). One can place the four quark fields in the two initial neutrons and two final protons separately. This is the conventional 2N-mode of $0\nu\beta\beta$-decay shown in Fig.1(a). Then $rm \rightarrow pp+2e^-$-transition is directly induced by the underlying quark subprocess $dd \rightarrow uu+2e^-$. In this case the nucleon transition is mediated by the exchange of a heavy supersymmetric
particle like the gluino $\tilde{g}$ with the mass $m_{\tilde{g}} \gtrsim 100\text{GeV}$. Therefore, the two decaying neutrons are required to come up very closely to each other what is suppressed by the nucleon repulsion. Another possibility is to incorporate quarks involved in the underlying particle like the gluino $\tilde{g}$ or into two virtual pions [6] or into one pion as well as into one initial neutron and one initial proton. Now $nn \to pp + 2e^-$ transition is mediated by the charged pion-exchange between the decaying neutrons, as shown in Fig.1(b,c). This is what we call the one- and two-pion modes of $0\nu\beta\beta$-decay. Since the interaction region extends to the distances $\sim 1/m_p$ this mode is not suppressed by the nucleon repulsion. An additional enhancement of the $\pi$-modes comes from the hadronization of the $R_p$ SUSY quark-lepton vertex operator in Eq. (1) as discussed below. In Ref. [6] it was shown that the two-pion mode absolutely dominates over the 2N-mode. In what follows, we are arguing that it dominates over the one-pion mode as well.

The effective hadronic Lagrangian taking into account both the nucleon $(p, n)$ and $\pi$-meson degrees of freedom in a nucleus can be written as follows:

$$
\mathcal{L}_{he} = \mathcal{L}_{2N} + \mathcal{L}_{2\pi} + \mathcal{L}_{1\pi} + \mathcal{L}_s = \frac{G_F^2}{2m_p} \bar{\mathcal{P}}^{(i)} n \cdot \mathcal{P}^{(i)} n \cdot \bar{e}(1 + \gamma_5) e^c \\
- \frac{G_F^2}{2m_p} m_{\pi}^2 \left[m_{\pi}^2 a_{2\pi} (\pi^{-})^2 - a_{1\pi} \bar{p} i\gamma_5 n \cdot \pi^- \right] \cdot \bar{e}(1 + \gamma_5) e^c + \\
g \bar{p} i\gamma_5 n \pi^+.
$$

(4)

Here $\mathcal{L}_{2N}, \mathcal{L}_{2\pi}, \mathcal{L}_{1\pi}$ describe the conventional two-nucleon mode, the two and one pion-exchange modes respectively. The last term $\mathcal{L}_s$ stays for the standard pion-nucleon interaction.

The basic parameters $a_{2\pi}$ and $a_{1\pi}$ of the Lagrangian $\mathcal{L}_{he}$ in Eq. (4) can be approximately related to the parameters of the quark-lepton Lagrangian $\mathcal{L}_{e}$ using the on-mass-shell "matching conditions" [6]

$$
<\pi^+, 2e^-|\mathcal{L}_{2N}|\pi^-> = <\pi^+, 2e^-|\mathcal{L}_{2\pi}|\pi^->, \quad <\pi^+, p, 2e^-|\mathcal{L}_{1\pi}|n> = <\pi^+, p, 2e^-|\mathcal{L}_{1\pi}|n> (5)
$$

In order to solve these equations we apply the widely used factorization and vacuum dominance approximations [10] for the matrix elements of the products of the two quark currents. Then we obtain, taking properly into account the combinatorial and color factors:

$$
\langle \pi^+ | J_{\mu} J_{\mu} | \pi^- \rangle \approx \frac{5}{3} \langle \pi^+ | J_{\mu} | 0 \rangle \langle 0 | J_{\mu} | \pi^- \rangle, \quad \langle \pi^+ | J_{\mu}^\mu J_{\nu}^\nu | \pi^- \rangle \approx -4 \langle \pi^+ | J_{\mu} | 0 \rangle \langle 0 | J_{\mu} | \pi^- \rangle, \\
\langle p | J_{\mu} J_{\mu} | n \pi^- \rangle \approx \frac{5}{3} \langle p | J_{\mu} | n \rangle \langle 0 | J_{\mu} | \pi^- \rangle, \quad \langle p | J_{\mu}^\mu J_{\nu}^\nu | n \pi^- \rangle \approx -4 \langle p | J_{\mu} | n \rangle \langle 0 | J_{\mu} | \pi^- \rangle, (6)
$$

While $<0 | J_{\beta} | \pi > = <0 | J_{\beta}^{\mu\nu} | \pi (p_\pi) > = 0$. The scalar matrix element vanishes due to the parity arguments, the tensor one vanishes due to $J_{\mu}^\mu = -J_{\nu}^\nu$ and impossibility of constructing an antisymmetric object having only one external 4-vector $p_\pi$. We also use the relationships

$$
\langle 0 | \bar{u} \gamma_5 d | \pi^- \rangle = i \sqrt{2} f_{\pi} \frac{m_{\pi}^2}{m_u + m_d} \equiv im_{\pi} h_{\pi}, \quad \langle p | \bar{u} \gamma_5 d | n \rangle = F_p \langle p | \bar{u} \gamma_5 n | n \rangle. (7)
$$
where \( f_\pi = 0.668 \, m_\pi \). For the nucleon pseudoscalar constant \( F_P \) we take its bag model value \( F_P \approx 4.41 \) from Ref. [11]. In this approximation we solve the "matching conditions" in Eq. (5) and determine the coefficients in Eq. (4)

\[
\alpha_{k\pi} = c_{k\pi} \left[ \eta_i + \frac{5}{8} \eta_f \right].
\]

with \( c_{1\pi} = (8/3) h_n F_P \approx 132.4, c_{2\pi} = (4/3) h_n^2 \approx 170.3 \). Here we used the conventional values of the current quark masses \( m_u = 4.2 \text{ MeV}, m_d = 7.4 \text{ MeV} \). The large ratio of the pion mass to the small current quark masses provide an additional enhancement factor of the pion mechanism as mentioned before the Eq. (4). Thus, we have obtained the approximate hadronic "image" \( \mathcal{L}_{he} \) of the fundamental quark-lepton Lagrangian \( \mathcal{L}_{qe} \) given in Eq. (1).

Starting from the Lagrangian \( \mathcal{L}_{he} \) in Eq. (4) it is straightforward to calculate \( nn \rightarrow pp + 2e^- \) transition amplitude and then the corresponding \( 0\nu\beta\beta \)-nuclear matrix element and the half-life formula. The final result for the half-life of \( 0\nu\beta\beta \)-decay in \( 0^+ \rightarrow 0^+ \) channel with the two outgoing electrons in the S-state, regarding all the three above-described possibilities of hadronization, reads

\[
[T^{0\nu}_{1/2}(0^+ \rightarrow 0^+)]^{-1} = G_{01} \left| \eta_i \cdot \mathcal{M}^{2N}_{ii} + \eta_f \cdot \mathcal{M}^{2N}_{ij} + \left( \frac{4}{3} M^{1\pi} + M^{2\pi} \right) \right|^2.
\]

Here \( G_{01} \) is the standard phase space factor tabulated for various nuclei in Ref. [12]. The nuclear matrix elements \( \mathcal{M}^{2N}_{ii,j} \) governing the sub-dominant two-nucleon mode were presented in Ref.[5]. As was already mentioned its contribution can be safely neglected. The one- and the two-pion modes nuclear matrix element \( M_{1\pi} \) and \( M_{2\pi} \) we write down in the form

\[
\mathcal{M}^{k\pi} = \left( \frac{m_A}{m_A} \right)^2 \frac{m_e}{m_e} \alpha^{k\pi} \left( M_{GT}^{k\pi} + M_{FT}^{k\pi} \right)
\]

Here, \( m_A = 850 \text{ MeV} \) is the mass scale of the nucleon form factor.

The structure coefficients in Eq. (10) are related to the coefficients \( c_{k\pi} \) introduced in Eq. (8). Their numerical values are \( c_{1\pi} = -4.4 \times 10^{-2} \) and \( c_{2\pi} = 0.2 \) (for more details see [6]). The two types of the Gamow-Teller and tensor nucleon matrix elements are given by the expressions

\[
M_{GT}^{k\pi} = \langle 0^+_f | \sum_{ij} \tau^+_i \tau^+_j \vec{\sigma}_i \cdot \vec{\sigma}_j F^{(k)}_1(x_\pi) \frac{R}{r_{ij}} | 0^+_i \rangle, \quad \text{with} \quad k = 1, 2
\]

\[
M_{FT}^{k\pi} = \langle 0^+_f | \sum_{ij} \tau^+_i \tau^+_j \left[ 3(\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] F^{(k)}_2(x_\pi) \frac{R}{r_{ij}} | 0^+_i \rangle,
\]

where \( x_\pi = m_\pi r_{ij}, r_{ij} = |\vec{r}_i - \vec{r}_j| \) and \( \vec{r}_{ij} = (\vec{r}_i - \vec{r}_j)/r_{ij} \). \( R = r_0 A^{1/3} \) is the mean nuclear radius, with \( r_0 = 1.1 fm \).

The structure functions \( F_1^{(k)}(x_\pi) \) and \( F_2^{(k)}(x_\pi) \) have their origin in the integration of the pi-meson propagators and take the following form:

\[
F_1^{(1)}(x) = e^{-x}, \quad F_1^{(2)}(x) = (3 + 3x + x^2) e^{-x},
\]

\[
F_1^{(2)}(x) = (x - 2)e^{-x}, \quad F_2^{(2)}(x) = (x + 1)e^{-x}.
\]
We calculate the nuclear matrix elements within the proton-neutron renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) [8]. This nuclear structure method has been developed from the proton-neutron QRPA (pn-QRPA) approach, which has been frequently used in the $0\nu\beta\beta$-decay calculations. The pn-RQRPA is an extension of the pn-QRPA by incorporating the Pauli exclusion principle for the fermion pairs. The limitation of the conventional pn-QRPA is traced to the quasi-boson approximation (QBA), which violates the expectation value in the uncorrelated BCS ground state. In this way the QBA implies the Pauli exclusion principle has to be incorporated in order to limit the number of quasiparticle pairs in the correlated ground state. The commutator is not anymore boson like, but obtains corrections to its bosonic behavior due to the fermionic constituents. The pn-RQRPA goes beyond the QBA. The Pauli effect of fermion pairs is included in the pn-RQRPA via the renormalized QBA (RQBA) [8], i.e. by calculating the commutator of two bifermion operators in the correlated QRPA ground state. Now it is widely recognized that the pn-RQRPA provides significantly more reliable treatment of the nuclear many-body problem for the description of the $0\nu\beta\beta$ decay.

For numerical treatment of the $0\nu\beta\beta$-decay matrix elements given in Eqs. (11) and (12) within the pn-RQRPA we transform them by using the second quantization formalism to the form containing the two-body matrix elements in the relative coordinate. One obtains [2]:

$$< O_{ij} > = \sum_{j_0, J_0} \frac{(-1)^{j_0 + j_0' + J_0 + J + (2J + 1)}}{2J + 1} \left\{ \frac{j_0}{j_0'} \frac{j_0}{J_0} \frac{j_0}{J} \right\} \times$$

$$< p, p'; J | f(r_{ij}) \tau_i^+ \tau_j^- | O_{ij} | n, n'; J > \times$$

$$< 0^+_i || [\gamma_i^+ \gamma_i] J || J^p m_f > < J^p m_f | J^p m_i > < J^p m_i || [\gamma_i^+ \gamma_i] J || 0^+_i > .$$  (15)

$O_{ij}$ represents the coordinate and spin dependent part of the two body transition operators of the $0\nu\beta\beta$-decay nuclear matrix elements in Eqs. (11) and (12). The short-range correlations between the two interacting nucleons are taken into account by the correlation function

$$f(r) = 1 - e^{-ar^2} (1 - br^2) \quad \text{with} \quad \alpha = 1.1 \text{ fm}^2 \quad \text{and} \quad b = 0.68 \text{ fm}^2.$$

The one-body transition densities and the other details of the nuclear structure model are given in [2, 8].

The calculated nuclear matrix elements for the $0\nu\beta\beta$-decay of various isotopes within the pn-RQRPA are presented in Table 1. The considered single-particle model spaces both for protons and neutrons have been as follows: i) For $A=76$, 82 the model space consists of the full $2 - 4\hbar \omega$ major oscillator shells. ii) For $A=96$, 100, 116 we added to the previous model space
Table 1: Nuclear matrix elements for the pion-exchange R-parity violating SUSY mode of 0νββ-decay for the experimentally most interesting isotopes calculated within the renormalized pn-QRPA. \( G_{01} \) is the integrated kinematical factors for \( 0^+ \rightarrow 0^+ \) transition [11]. \( \zeta(Y) \) denotes according to Eq. (18) the sensitivity of a given nucleus \( Y \) to the SUSY signal.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( M_{GT}^{1\nu} )</th>
<th>( M_{GT}^{2\nu} )</th>
<th>( M_{GT}^{3\nu} )</th>
<th>( M_{GT}^{1\nu} )</th>
<th>( M_{GT}^{2\nu} )</th>
<th>( M_{GT}^{3\nu} )</th>
<th>( G_{01} \times 10^{15} )</th>
<th>( \zeta(Y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{76}\text{Ge})</td>
<td>1.30</td>
<td>-1.02</td>
<td>-1.34</td>
<td>-0.65</td>
<td>-18.2</td>
<td>-001</td>
<td>7.93</td>
<td>5.5</td>
</tr>
<tr>
<td>(^{82}\text{Se})</td>
<td>1.23</td>
<td>-0.87</td>
<td>-1.26</td>
<td>-0.57</td>
<td>-23.9</td>
<td>-551</td>
<td>35.2</td>
<td>10.8</td>
</tr>
<tr>
<td>(^{96}\text{Zr})</td>
<td>0.77</td>
<td>-1.11</td>
<td>-0.85</td>
<td>-0.67</td>
<td>22.1</td>
<td>-458</td>
<td>73.6</td>
<td>11.8</td>
</tr>
<tr>
<td>(^{109}\text{Mo})</td>
<td>1.43</td>
<td>-1.73</td>
<td>-1.52</td>
<td>-1.05</td>
<td>19.4</td>
<td>-776</td>
<td>57.3</td>
<td>18.1</td>
</tr>
<tr>
<td>(^{116}\text{Cd})</td>
<td>0.92</td>
<td>-0.78</td>
<td>-0.94</td>
<td>-0.47</td>
<td>-9.3</td>
<td>-423</td>
<td>62.3</td>
<td>10.8</td>
</tr>
<tr>
<td>(^{128}\text{Te})</td>
<td>1.25</td>
<td>-1.57</td>
<td>-1.40</td>
<td>-0.99</td>
<td>21.4</td>
<td>-720</td>
<td>22.1</td>
<td>3.3</td>
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<tr>
<td>(^{130}\text{Te})</td>
<td>1.10</td>
<td>-1.48</td>
<td>-1.26</td>
<td>-0.93</td>
<td>25.1</td>
<td>-660</td>
<td>55.4</td>
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<tr>
<td>(^{136}\text{Xe})</td>
<td>0.61</td>
<td>-0.84</td>
<td>-0.74</td>
<td>-0.54</td>
<td>15.5</td>
<td>-387</td>
<td>59.1</td>
<td>9.0</td>
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<tr>
<td>(^{150}\text{Nd})</td>
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<td>-2.07</td>
<td>-1.68</td>
<td>56.4</td>
<td>-1129</td>
<td>269</td>
<td>55.6</td>
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</table>

1\( f_{5/2} \), 1\( f_{7/2} \), 0\( h_{9/2} \) and 0\( h_{11/2} \) levels. iii) For \( A=128, 130, 136 \) the model space comprises the full \( 2-5h\omega \) major shells. iv) For \( A=150 \) the model space extends over the full \( 2-5h\omega \) shells plus \( 0\ h_{11/2} \) and \( 0\ h_{13/2} \) levels. The single particle energies were obtained by using Coulomb corrected Woods Saxon potential. The interaction employed was the Brueckner G-matrix which is a solution of the Bethe-Goldstone equation with the Bonn one-boson exchange potential. Since the considered model space is finite, we have renormalized pairing interactions by the strength parameters \( d_{pp} \) and \( d_{nn} \) for the empirical gaps denoted by Moeller and Nix [14]. The particle-particle and particle-hole channels of the G-matrix interaction of nuclear Hamiltonian \( H \) have been renormalized with parameters \( g_{pp} \) and \( g_{ph} \), respectively. The nuclear matrix elements listed in the Table 1 have been obtained for the \( g_{ph} = 0.80 \) and \( g_{pp} = 1.0 \) The following note is in order. According to our numerical analysis, variations of the nuclear matrix elements presented in Table 1 do not exceed 20% within the physical region of the nuclear structure parameter \( g_{pp} \) (0.8 \( \leq g_{pp} \leq 1.2 \)).

As seen from the Table 1 \( M_{GT}^{2\nu} \) is significantly larger than \( M_{GT}^{1\nu} \). It is partially due to the mutual cancellation of the \( M_{GT}^{1\nu} \) and \( M_{GT}^{2\nu} \) in the construction of \( M_{GT}^{1\nu} \) in Eq. (10) (see Table 1) and due to the suppression of the structure coefficients \( a^{1\nu} \) in comparison with \( a^{2\nu} \). Thus, we conclude that the two-pion mode contribution to 0νββ-decay (Fig.1(b)) dominates both over the one-pion (Fig.1(c)) and the two-nucleon contributions (Fig.1(a)). The dominance of the two-pion mode over the two-nucleon one was previously proven in Ref. [6].

Having all the quantities in the 0νββ-decay half-life formula (9) specified we are ready to extract the limits on the \( R_p \) parameters from non-observation of the 0νββ-decay.

The experimental lower bound \( T_{1/2}^{exp} \) for the half-life of certain isotope \( Y \) provides the following constraint on the effective \( R_p \) SUSY parameters

\[
\frac{\eta_1}{8} + \frac{5}{8} \eta_2 \equiv \eta_{susy} \leq \eta_{susy}^{exp} = \frac{10^{-7}}{\zeta(Y)} \sqrt{\frac{\tau_{1/2}^{exp}}{10^{24} \text{years}}},
\]

(17)
Table 2: The present state of the $R_p$-SUSY searches in $\beta\beta$-decay experiments. $T^{\text{exp}}_{1/2}$(present) is the best presently available lower limit on the half-life of $0\nu\beta\beta$-decay for a given isotope. $\eta^{\text{exp}}_{\text{SUSY}}$ is the corresponding upper limit on the $R_p$ SUSY parameter. $T^{\text{exp}}_{1/2} (\eta_{\text{SUSY}}^{H-M})$ is the calculated half-life of $0\nu\beta\beta$-decay assuming $\eta_{\text{SUSY}} = \eta_{\text{SUSY}}^{H-M}$ with $\eta_{\text{SUSY}}^{H-M}$ being the best limit deduced from the Heidelberg-Moscow $^{76}$Ge experiment [1].

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{76}$Ge</th>
<th>$^{82}$Se</th>
<th>$^{90}$Zr</th>
<th>$^{99}$Mo</th>
<th>$^{116}$Cd</th>
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<td>Ref.</td>
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<td>[15]</td>
<td>[16]</td>
<td>[17]</td>
<td>[18]</td>
</tr>
<tr>
<td>$T^{\text{exp}}_{1/2}$(present)</td>
<td>$1.1 \times 10^{24}$</td>
<td>$2.7 \times 10^{24}$</td>
<td>$3.9 \times 10^{20}$</td>
<td>$5.2 \times 10^{22}$</td>
<td>$2.9 \times 10^{22}$</td>
</tr>
<tr>
<td>$\eta^{\text{exp}}_{\text{SUSY}}$</td>
<td>$5.5 \times 10^{-9}$</td>
<td>$5.6 \times 10^{-8}$</td>
<td>$1.4 \times 10^{-6}$</td>
<td>$2.4 \times 10^{-8}$</td>
<td>$5.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T^{\text{exp}}<em>{1/2} (\eta</em>{\text{SUSY}}^{H-M})$</td>
<td>$1.1 \times 10^{25}$</td>
<td>$2.9 \times 10^{24}$</td>
<td>$2.4 \times 10^{24}$</td>
<td>$1.0 \times 10^{24}$</td>
<td>$2.9 \times 10^{24}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{128}$Te</th>
<th>$^{130}$Te</th>
<th>$^{136}$Xe</th>
<th>$^{150}$Nd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref.</td>
<td>[19]</td>
<td>[20]</td>
<td>[21]</td>
<td>[22]</td>
</tr>
<tr>
<td>$T^{\text{exp}}_{1/2}$(present)</td>
<td>$7.7 \times 10^{24}$</td>
<td>$8.2 \times 10^{24}$</td>
<td>$4.2 \times 10^{23}$</td>
<td>$1.2 \times 10^{21}$</td>
</tr>
<tr>
<td>$\eta^{\text{exp}}_{\text{SUSY}}$</td>
<td>$1.1 \times 10^{-8}$</td>
<td>$7.4 \times 10^{-8}$</td>
<td>$1.7 \times 10^{-8}$</td>
<td>$5.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T^{\text{exp}}<em>{1/2} (\eta</em>{\text{SUSY}}^{H-M})$</td>
<td>$3.1 \times 10^{25}$</td>
<td>$1.5 \times 10^{24}$</td>
<td>$4.1 \times 10^{24}$</td>
<td>$1.1 \times 10^{23}$</td>
</tr>
</tbody>
</table>

Here $\zeta(Y)$ is the SUSY sensitivity of a $0\nu\beta\beta$-decaying isotope $Y$ [23]:

$$\zeta(Y) = 10^5 |M_x| \sqrt{\alpha_0}. \tag{18}$$

The isotope sensitivity $\zeta(Y)$ is an intrinsic characteristic of an isotope $Y$ depending only on the nuclear matrix element $M_x = \frac{4}{3}M^{1\pi} + M^{2\pi}$ and the phase space factor. The large numerical values of the SUSY sensitivity $\zeta$ in (18) correspond to those isotopes within the group of $\beta\beta$-decaying nuclei which are most promising candidates for searching for SUSY in $0\nu\beta\beta$-decay experiments.

The numerical values of $\zeta(Y)$ calculated in the pn-RQRPA are presented in the Table 1 and displayed in Fig 2 in the form of histogram. By glancing the Fig 2 we see that the most sensitive isotope is $^{150}$Nd followed by $^{100}$Mo.

It is understood that the SUSY sensitivity $\zeta$ cannot be the only criterion for selecting an isotope for the $0\nu\beta\beta$-experiment. Other microscopic and macroscopic properties of the isotope are also important for building a $0\nu\beta\beta$-detector. The current experimental situation in terms of the accessible half-life and the corresponding upper limit on the effective SUSY parameter $\eta_{\text{SUSY}}$ is presented in Table 2. As seen, the best upper limit on the $R_p$ SUSY parameter $\eta_{\text{SUSY}}$ has been established by the Heidelberg-Moscow experiment [1]. We denote this limit as $\eta^{\text{exp}}_{\text{SUSY}} (H - M)$. For comparison in the bottom raw of the Table 2 we show the half-life lower limits $T^{\text{exp}}_{1/2} (\eta_{\text{SUSY}}^{H-M})$ to be reached by the other $0\nu\beta\beta$-experiments to reach this presently best constraint $\eta_{\text{SUSY}} \leq \eta^{\text{exp}}_{\text{SUSY}} (H - M)$ on the $R_p$ SUSY. The result of this comparison is illustrated in Fig 2.

Using the values of $\eta^{\text{exp}}_{\text{SUSY}}$ from the Table 2 one can easily calculate the corresponding upper limits on the $\lambda_{111}$ parameter. There are two types of the upper limits for each value of $\eta^{\text{exp}}_{\text{SUSY}}$. 


\[
\lambda^\prime_{111} \leq 1.1 \sqrt{\eta_{\text{SUSY}}} \left( \frac{m_\tilde{q}}{100 \text{GeV}} \right)^2 \left( \frac{m_\tilde{g}}{100 \text{GeV}} \right)^{1/2}, \tag{19}
\]
\[
\lambda^\prime_{111} \leq 15.7 \sqrt{\eta_{\text{SUSY}}} \left( \frac{m_\tilde{q}}{100 \text{GeV}} \right)^2 \left( \frac{m_\tilde{g}}{100 \text{GeV}} \right)^{1/2}. \tag{20}
\]

These formulas are derived from Eqs. (2) and (3) assuming no spurious compensations between different terms in the sum \( \eta^{\text{exp}}_{\text{SUSY}} = \eta \tilde{q} + (5/8) \eta \tilde{g} \). We have taken the running QCD coupling constant \( \alpha_s(Q) \) at the scale \( Q = 1 \text{GeV} \) with the normalization on the world average value \( \alpha_s(M_Z) = 0.120 \) [24]. Eq. (20) is derived with the assumptions that the lightest neutralino is B-ino dominant and that \( m_\tilde{q} \geq m_\tilde{g}/2 \). Both these assumptions are phenomenologically reasonable although they are not always true.

The best constraint from the Heidelberg-Moscow experiment [1] is

\[
\lambda^\prime_{111} \leq 1.3 \cdot 10^{-4} \left( \frac{m_\tilde{q}}{100 \text{GeV}} \right)^2 \left( \frac{m_\tilde{g}}{100 \text{GeV}} \right)^{1/2} \tag{21}
\]

\[
\lambda^\prime_{111} \leq 1.2 \cdot 10^{-2} \left( \frac{m_\tilde{q}}{100 \text{GeV}} \right)^2 \left( \frac{m_\tilde{g}}{100 \text{GeV}} \right)^{1/2} \tag{22}
\]

These limits are very strong and, as it was already pointed out in Ref. [5]-[7], lie beyond the reach of the near future accelerator experiments (though, accelerator experiments are potentially sensitive to the other couplings than \( \lambda^\prime_{111} \)).

To constrain the size of \( \lambda^\prime_{111} \) itself one needs in additional assumptions on the masses of the SUSY-partners. If the values of these masses would be around their present experimental lower limits \( \sim 100 \text{GeV} \) [24], one could constrain the coupling to \( \lambda^\prime_{111} \leq 1.3 \cdot 10^{-4} \). A conservative bound can be set by assuming all the SUSY-masses being at the "SUSY-naturalness" bound of 1 TeV, leading to \( \lambda^\prime_{111} \leq 4.1 \cdot 10^{-2} \).

In summary, we have analyzed the general case of the pion realization for the short-ranged \( R_p \) SUSY mechanism of \( 0\nu\beta\beta \)-decay taking into account both the one-pion and two-pion modes. We have shown that the two-pion mode \( R_p \) SUSY contribution to \( 0\nu\beta\beta \)-decay dominates over the one-pion mode contribution. Previously [6] we had proven that the two-pion mode dominates over the conventional two-nucleon one. We also pointed out that non-observation of \( 0\nu\beta\beta \)-decay casts severe limitations on the \( R_p \) SUSY extensions of the standard model of electroweak interaction. Although a many-body problem needs to be solved the limits are so stringent that it overcomes the uncertainties in the nuclear and hadronic matrix elements, leading to the limits that are much stronger than those from accelerator and the other non-accelerator experiments.

We gave the list of nuclear matrix elements for all the experimentally interesting isotopes and presented the so-called SUSY sensitivities of these isotopes. These integral characteristic might be helpful for the planning of future searches for the SUSY in \( 0\nu\beta\beta \)-decay. On this basis we compared the present status of the various \( 0\nu\beta\beta \)-experiments and their abilities to detect the SUSY signal.

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References


[10] see for instance L.B. Okun, ???


Figure Captions

**Fig.1.** The hadronic-level diagrams for the short-ranged SUSY mechanism of $0\nu\beta\beta$-decay. (a) the conventional two-nucleon mode, (b) the one-pion exchange mode, (c) the two-pion exchange mode.

**Fig.2.** The SUSY sensitivity $\zeta(Y)$ for the experimentally interesting nuclei (on the left). This histogram displays the corresponding numerical values in the Table 1. The histogram on the left illustrates the Table 2. The best presently available lower limits on the $0\nu\beta\beta$-decay half-life $T^{\exp}_{1/2}$ are denoted by the black bars. The open bars indicate the half-life limits $T^{\exp}_{1/2}(\eta_{SUSY}^{H-M})$ to be reached by a given experiment to reach the presently best limit on the $R_p$ SUSY parameter $\eta_{SUSY}^{H-M}$ established by the $^{76}$Ge experiment [1].