Pulsar Signal of Deconfinement†

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A solitary millisecond pulsar, if near the mass limit, and undergoing a phase transition, either first or second order, provided the transition is to a substantially more compressible phase, will emit a blatantly obvious signal—spontaneous spin-up. Normally a pulsar spins down by angular momentum loss to radiation. The signal is trivial to detect and is estimated to be “on” for 1/50 of the spin-down era of millisecond pulsars. Presently about 25 solitary millisecond pulsars are known. The phenomenon is analogous to “backbending” observed in high spin nuclei in the 1970’s.

1. THE LIVES OF NEUTRON STARS

The formation of a new phase of matter, a softer one than nuclear matter, may cause a rapidly rotating pulsar to produce a prolonged signal that is dramatic, easy to detect and easy to understand [1]. The most plausible high density phase transition is deconfinement as predicted by QCD [2]. The signal I will describe will occur for either a first or second order transition so long as it is accompanied by a sufficient softening of the equation of state. (Cf. Fig. 1.)

Strictly speaking we do not even know that quarks can be deconfined under extreme conditions or otherwise. It is an ‘expectation’ based on the QCD property of asymptotic freedom [2]. We would like to prove that this phase is a possible phase of matter. If so, it would have pervaded the very early universe, but quark confinement in hadrons occurred at an early time and the thermal equilibrium that existed then leaves no signal today.

From the balance of gravitational and centrifugal forces on a particle at the surface of rapidly rotating stars such as the millisecond pulsars, we know that the central density is a few times nuclear, the same range of energy densities as are expected to be produced in relativistic nuclear collisions. Let us assume that the critical deconfinement density occurs in the density range spanned by spherical stars and therefore in the population of slow pulsars to which the Crab belongs and of which there are about 800 presently known. In this case newly born neutron stars have a quark core essentially from birth. But we have no way to tell if this is indeed the case. Models of neutron stars having different composition generally differ in the range of masses and radii permitted, and their density

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Figure 1. The equation of state (labeled ‘Hybrid’) of neutron star matter with a first order deconfinement phase transition. The normal phase contains nucleons, hyperons and leptons in equilibrium. The mixed phase contains as well, the three light flavor quarks. Comparison is made with a case in which deconfinement is not taken account of (labeled ‘n+p+H’). (Nuclear properties include the observed binding, saturation density, symmetry energy and $K = 300$ MeV, $m^*_\text{sat}/m = 0.7$).

Figure 2. Density profiles of two stars of the same mass $M = 1.42M_\odot$ but differing composition; (1) Hyperon star (neutron-proton-hyperon-lepton), (2) Hybrid (a pure quark matter core surrounded by mixed phase and outer pure hadronic confined phase). Equations of state as in Fig. 1. Interior differences are dramatic but not directly measurable. (For a description of neutron star matter and relativistic stars see Ref. [3].)

profiles may be very different (cf. Fig. 2). However as far as measurable properties are concerned, ordinary neutron stars and hybrid stars (neutron stars with a quark core) are practically indistinguishable. Cooling rates are presumably sensitive to the internal composition, but theoretical estimates are very uncertain.

I will sketch the generally accepted evolutionary life of pulsars [4]. There are two distinct populations of pulsars (see Figs. 3 and 4), the canonical pulsars (about 800 of them now known) with periods between 1/50 sec and 8 sec, and the millisecond pulsars (about 50), which are believed to be more evolved than the former. Stars are born into the first of these populations, and may, on a very long time-scale, evolve into the second. (See Fig. 4.)

As the stellar core of a luminous star collapses to form a neutron star, it is spun up by conservation of angular momentum and acquires an enormous magnetic field of
Figure 3. Distribution of pulsar periods. The lower group consists of ‘recycled’ millisecond pulsars, the higher to the canonical pulsars in the first stage of their evolution (see Fig. 4).

$10^{12}$ to $10^{13}$ gauss because of flux conservation. The star is born as a rotating magnetic dipole. It has a tremendous store of rotational energy that will keep it spinning for 10 million years. The electromagnetic radiation beamed along the spinning dipole is what we see as pulsed radio emission once each rotation, if as observers, we lie on the cone swept out by the beam.

In a plot of magnetic field $B$ vs period $P$ (Fig. 4), stars move from top left to right because of loss of angular momentum to radiation. They disappear as active pulsars when a combination of angular velocity and field strength is insufficient to produce radiation. It takes about $10^7$ years to complete this first phase. However, either from birth, or afterward, the star may have or capture a lower density companion as is often evidenced by the presence of an orbiting white dwarf. During an accretion era, the compact star is spun up by infalling matter that it tears off from its less dense companion. It loses some magnetic field during accretion, perhaps by ohmic resistance during the long radio silent era.

In this ‘silent’ era the the star moves diagonally from top right to bottom left in the $B−P$ diagram. The neutron star becomes centrifugally flattened as it approaches millisecond periods. The central density falls. The core of quark matter shrinks as quarks recombine to form baryons. The quark core may disappear altogether.

The silent neutron star, having completed a part of its life cycle, turns on again as a millisecond pulsar of low magnetic field when the lower field but higher angular velocity can once again produce radiation. Presently 50 such pulsars have been discovered, half of
Figure 4. Pulsars are born with field $B \sim 10^{12}$ to $10^{13}$ gauss and evolve toward the right. Periods change with time as $\sim 1/P$ and so pulsars accumulate at large $P$. They become radio silent in about $10^7$ years and remain stagnant until they capture a companion star, or unless they had one all along. Accretion from the less dense companion spins them up along a line like ‘silent evolution’. A combination of the now weaker field but higher frequency turns them on again as ‘recycled’ millisecond pulsars. They then again evolve toward shorter period, but now on a very long time-scale because of the weaker field.

which still have a binary companion. The number of millisecond pulsars presently known is believed to be a fraction of the total population because of search selection effects.

Millisecond pulsars, which have weaker fields ($10^8$ to $10^9$ gauss), spin down very slowly since the deceleration is proportional to $B^2$. Their characteristic age is $P/2\dot{P} \sim 10^9$ years. The central density is initially centrifugally diluted but as it spins down, the central density will rise again and the critical density will be reached, first at the center, and then in an expanding region. The growth of the central region of deconfined matter is paced by the slow spin-down, slow because of the coupling of rotation of the stellar magnetic dipole to electromagnetic processes.

Stiff nuclear matter is being replaced in the core by highly compressible quark matter. The weight of the overlaying layers of nuclear matter weigh down on the core and compress it. Its density rises. The star shrinks—mass is redistributed with growing concentration at the center. The by-now more massive central region gravitationally compresses the outer nuclear matter even further, amplifying the effect. The density profile for a star at three angular velocities, (1) the limiting Kepler velocity which is stretched in the equatorial plane and centrally diluted, (2) an intermediate angular velocity, and (3) a non-rotating star, are shown in Fig. 5. We see that the central density rises with decreasing angular
velocity by a factor of three and the equatorial radius decreases by 30 percent. In contrast, for a model for which the phase transition did not take place, the central density would change by only a few percent [5]. The phase boundaries are shown in Fig. 6 from the highest rotational frequency to zero rotation. The redistribution of mass and shrinkage of

Figure 5. Mass profiles as a function of equatorial radius of a star rotating at three different frequencies, as marked. At low frequency the star is very dense in its core, having a 4 km central region of highly compressible pure quark matter. At intermediate frequency, the pure quark matter phase is absent and the central 8 km is occupied by the mixed phase. At higher frequency (nearer $\Omega_K$) the star is relatively dilute in the center and centrifugally stretched. Inflections at $\epsilon = 220$ and 950 are the boundaries of the mixed phase.

Figure 6. Radial boundaries at various rotational frequencies separating (1) pure quark matter, (2) mixed phase, (3) pure hadronic phase, (4) ionic crust of neutron rich nuclei and surface of star. The pure quark phase appears only when the frequency is below $\Omega \sim 1370 \text{ rad/s}$. Note the decreasing radius as the frequency falls. The frequencies of two pulsars, the Crab and PSR 1937+21 are marked for reference.

the star change its moment of inertia and hence the characteristics of its spin behavior. The star must spin up to conserve angular momentum which is being carried off only slowly by the weak electromagnetic dipole radiation. The star behaves like an ice skater who goes into a spin with arms outstretched, is slowly spun down by friction, temporarily spins up by pulling the arms inward, after which friction takes over again.

It is that simple to describe, and that is the blatant signal I mentioned—the spontaneous spin-up of an isolated millisecond pulsar that is radiating angular momentum and ought
otherwise to be slowing down.

2. BACKBENDING IN PULSARS: Signal of Phase Transition

The mathematical description of the intuitive process described above is difficult but well defined. We must compute in General Relativity a sequence of rotating stellar objects of the same baryon number, but varying in angular velocity based on an equation of state that describes a phase transition as in Fig. 1. The expression for the moment of inertia of a non-rotating star was derived years ago by Hartle and Thorne but it is inadequate [6,7]. It ignores the centrifugal stretching of the star, changes in its composition that result, changes in the metric of space time as a result of rotation, and it ignores the dragging of local inertial frames by the rotating star. Fortunately, in a different connection, we derived the General Relativistic expression for a rotating star that incorporates all of the above effects [8,9].

Figure 7. A star rotating with angular velocity $\Omega$ as measured by a distant observer, sets the local inertial frames into rotation by a position dependent angular velocity $\omega(r)$. A particle dropped on the star in the equatorial plane falls not toward the center, but is swept to the side as illustrated. Outside the star $\omega$ has a simple form.

Frame dragging by a rotating star is as inseparable from a description of space-time as mass is [3]. A particle dropped in the equatorial plane onto a rotating star falls not toward the star’s center, but is swept ever more in the sense of the stars rotation, as illustrated in Fig. 7. The centrifugal force acting on a fluid element of the star is governed, not by its angular velocity $\Omega$, but by the angular velocity relative to that of the local inertial frames $\omega(r)$. The latter is a function of position as shown in Fig. 8.

For a sequence of stars near the mass limit, we show in Fig. 9 the moment of inertia in a small band of angular velocity in the vicinity where the quark matter core radius grows rapidly with respect to changing $\Omega$ (cf. Fig. 6). Normally, the moment of inertia would decrease as a smooth function of $\Omega$ as the centrifugal force allows the star to relax to a more nearly spherical shape. Instead, we see the behavior just described for an ice skater. The neutron star actually spins up during the growth of the quark core. The reason, as explained, is that as stiff nuclear matter is replaced by compressible quark matter, the
Figure 8. Angular velocity \( \omega(r, \theta) \) of local inertial frames as a fraction of the star’s angular velocity \( \Omega \) is shown as a function of radial coordinate in the equatorial plane of rotation. The centrifugal forces are determined not by \( \Omega \) but by \( \Omega - \omega(r, \theta) \). Inside the star, \( \omega(r, \theta) \) influences and in turn is influenced by the distribution of matter.

moment of inertia decreases, not solely because of the diminishing centrifugal forces, but because of the change of state; the star must spin up to conserve angular momentum. A very similar phenomenon was predicted for rotating nuclei in 1960 by Mottelson and Valatin and observed in the 1970’s [10–12]. A nucleus undergoes a phase change from a phase at high angular velocity in which the nucleus carries its angular momentum through aligned nucleon spins, to a phase at low angular momentum which is superfluid. The change takes place over a few units of angular velocity and the “backbending” of the moment of inertia is shown in Fig. 10.

It is trivial to measure the rate at which a pulsar’s period changes. It is especially trivial for millisecond pulsars where the period is sometimes known to 14 significant figures. For example, PSR1937+21 has a period (measured on 29 November 1982 at 1903 UT)

\[
P = 1.5578064487275(3) \text{ ms}.
\]

Its rate of change of period is a mere \( \dot{P} \sim 10^{-19} \). But because of the high accuracy of the period measurement, it takes only two measurements spaced 0.3 hours apart to detect a unit change in the last significant figure, and hence to detect in which direction the period is changing. So in fact we have uncovered a signal of a phase transition in pulsars that is trivial to observe if it occurs during the time of observation. Let us now compute the length of the epoch over which the pulsar will be spinning up because of a change of phase and the slow envelopment of the central region by the new phase.

3. DURATION OF THE SPIN-UP ERA

Having the moment of inertia as a function of angular velocity for a model star, as in Fig. 9, that is to say, for a model of the equation of state which describes a phase
Figure 9. Moment of inertia of a neutron star at angular velocities for which the central density rises from below to above critical density for the pure quark matter phase as the centrifugal force decreases. Time flows from large to small $I$. The most arresting signal of the phase change is the spontaneous spin-up that an isolated pulsar would undergo during the growth in the region of pure quark matter.

Figure 10. Nuclear moment of inertia as a function of squared frequency for $^{158}$Er, showing backbending in the nuclear case. Quantization of spin yields the unsmooth curve compared to the one in Fig. 9. There the spin at the center of the spin-up is $J \sim 10^{41}$.

transition, we are in a position to compute the time evolution of the angular velocity. The rate at which energy is radiated by a rotating magnetic dipole is given by

$$\frac{dE}{dt} = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = -C \Omega^4. \tag{1}$$

Here, $I$ is the moment of inertia, $\Omega$ is the angular velocity of the star and $C$ is a constant that depends on the square of the magnetic field strength. Therefore, the rate of change of frequency is governed by

$$\dot{\Omega} = -\frac{C}{I(\Omega)} \left[ 1 + \frac{I'(\Omega)}{2I(\Omega)} \right]^{-1} \Omega^3 \tag{2}$$

where $I' = dI/d\Omega$. From the behavior of the moment of inertia, we see that its derivative is infinite at the two frequencies that mark the beginning and end of the spin-up era; consequently $\dot{\Omega}$ vanishes at the boundaries of this era.

By integrating the equation we can find the length of the epoch of spin-up. It is $2 \times 10^7$ years. The characteristic time for spin-down of millisecond pulsars is $10^9$ years. So if
the conditions for spin-up are fulfilled, namely (i) the equation of state describes a phase transition involving a substantial softening of the equation of state, and (ii) the critical density is attained in stars very near the mass limit, we can expect a spontaneous spin-up of millisecond pulsars to occur in the above ratio, namely $1/50$. This is a very attractive “event rate” given that 25 of the presently known millisecond pulsars are isolated.

There is another observable which endures for an even longer time. It is the so-called dimensionless braking index $\Omega \ddot{\Omega} / \dot{\Omega}^2$. It would be equal to the magnetic dipole value $n = 3$ of the energy-loss mechanism (1) if the frequency were small or if the moment of inertia were a constant. However these conditions are not usually fulfilled and the measurable quantity is not constant. Rather it has the value

$$n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{3I''\Omega + I''\Omega^2}{2I + I'\Omega}$$

(3)

where $I'' = dI^2/d\Omega^2$. The progression of the new phase through the central region of the star will be signaled by an anomalous value of the braking index, far removed from the canonical value of 3. Indeed, at the two turning points of $I$ that mark the boundaries of the spin-up era, $\dot{\Omega}$ vanishes so that $n(\Omega)$ is infinite and of opposite sign. We show the behavior of $n(\Omega)$ in Fig. 11. The epoch over which it is anomalous is $10^8$ years. Note that one cannot, and does not need to measure the shape of the curve. A single anomalous value
that differed significantly from the dipole value of 3 would suffice. How could one reconcile a value of the multipolarity of say 10, or $-5$ with observed electromagnetic processes? However it is hard to measure the second time derivative and therefore the braking index, especially for millisecond pulsars. So the practical signal is spontaneous spin-up of an isolated pulsar. We specify ‘isolated’ to ensure that spin-up is not attributable to accretion from a companion.

4. SUMMARY

An isolated millisecond pulsar will spin up over an epoch of $2 \times 10^7$ years out of a spin-down life of $10^9$ years if it undergoes a phase transition obeying the two conditions (i) the transition causes a substantial softening of the equation of state, and (ii) the critical density is attained in stars very near the mass limit. The spin-up epoch, compared to the spin-down life of the pulsar, corresponds to an ‘event rate’ of 1/50. The determination of whether a pulsar is spinning up or down is trivial. Of the presently known millisecond pulsars, about 25 are isolated. We are approaching the moment of truth for this observable signal of a phase transition.

We have emphasized that the transition need not be of first order so long as it is accompanied by a sufficient softening of the equation of state. We do not have a measure of what we mean by this. Of course our model does possess the requisite softening, or our results would not have exhibited backbending.

If no pulsar is observed to produce the signal, little is learned. Just as in the search for deconfinement in high energy nuclear collisions, failure to observe a signal does not inform us that the deconfined phase does not exist.

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