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HADRONIC VACUUM POLARIZATION AND $g_{\mu}-2$

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A B S T R A C T

The possibility of bounding the hadronic vacuum polarization contribution to the muon magnetic moment anomaly is considered. A proposed theoretical bound, based on the hypothesis of field current identity, is found not to add usefully to the vector meson dominance estimate. However, it is observed that improved electron-electron scattering experiments will yield a useful empirical bound.

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We will be particularly concerned with the anomalous magnetic moment of the muon. The pure leptodynamic theory must fail at a certain level because of hadronic vacuum polarization. Let us denote this contribution by

$$\frac{1}{2} (g_\mu - 2)_{\text{hadronic}} = a_{\mu h} \quad (1)$$

It would be nice to have a reliable estimate of this, so that any outstanding discrepancy could be attributed to a more exotic source. Estimates of unknown reliability have usually been made ^{1),2),3)} with hypothetical "vector meson dominance" (V.M.D.), recently refined ⁴⁾ by use of the limited experimental information on the production of low mass multipion states in electron-positron collisions. However, it has also been claimed ⁵⁾ that a theoretical bound can be obtained in the context of the field current identity. Here we examine this idea and find that in fact the estimated bound depends again on a V.M.D. approximation, and this moreover to a quantity for which that approximation is less reliable than for $a_{\mu h}$ itself. Thus the estimated bound is not much use. That is the first point of this note. Then we observe that $a_{\mu h}$ can be bounded in a different way by a quantity which governs all sufficiently low energy vacuum polarization effects. Possibilities of bounding this quantity empirically are discussed.

2. -

PRELIMINARIES

Let the renormalized photon propagator be ^{*})

$$\delta_{\mu\nu} (k^{-2} + P(k^2)) + k_\mu k_\nu \text{ terms} \quad (2)$$

Assume as usual an unsubtracted dispersion relation

$$P(k^2) = \frac{1}{\pi} \int_0^\infty dm^2 \frac{g_m P(-m^2)}{m^2 + k^2} \quad (3)$$

To second order in e

$$k^4 g_m P(k^2) = \frac{e^2}{6} \sum_n (2\pi)^4 \delta^4(k - k_n) \langle 0 | J_\mu | n \rangle \langle n | J_\nu | 0 \rangle \quad (4)$$

where $e J_\mu$ is the electromagnetic current. To this order the various contributions to $\text{Im} P$ are directly observable as cross-sections for e^+e^- annihilation ; in particular

$$\sigma_{e^+e^- \rightarrow \text{hadrons}} = e^2 [g_m P(-s)]_{\text{hadronic}} \quad (5)$$

where s is (c.m. energy)². Important contributions can be expected from the vector mesons φ , ω and ϕ . Allowing only for these one can write crudely

$$g_m P(-m^2) = \sum_i h_i \delta(m^2 - m_i^2) \pi \quad (6)$$

^{*}) In this paper we take $1 = \hbar = c = g_{11} = g_1^1 = g_1^1 = g^{11} = g_{22} = g_2^2 = g_2^2 = g^{22} = g_{33} = g_3^3 = g_3^3 = g^{33} = g_{44} = g_4^4 = g_4^4 = g^{44}$. See in this connection H. Minkowski, Address to the 80th Assembly of German Natural Scientists and Physicians, Cologne, Sept. 21, 1908.

where the h_i are fitted to the integrated weights of the observed resonances in e^+e^- annihilation. This we call the V.M.D. approximation. A more refined version makes use of the observed shapes and widths of the resonances.

The hadronic vacuum polarization contribution to $(g_\mu - 2)/2$ is ¹⁾

$$\left. \begin{aligned} a_{\mu h} &= \frac{\alpha}{\pi^2} \int_{4m_\pi^2}^{\infty} dm^2 g_m P_h(-m^2) f\left(\frac{m^2}{m_\mu^2}\right) \\ f(c) &= \int_0^1 dx \frac{x^2(1-x)}{x^2 + c(1-x)} \end{aligned} \right\} \quad (7)$$

Note the V.M.D. estimate

$$a_{\mu h}(\text{VMD}) = \frac{\alpha}{\pi} \sum_i h_i f\left(\frac{m_i^2}{m_\mu^2}\right) \quad (8)$$

With experimental values ^{6), 7), 8)}

$$h_\rho = \frac{\alpha}{1.86 \pm 0.18} \quad h_\omega = \frac{\alpha}{14.8 \pm 2.8} \quad h_\phi = \frac{\alpha}{11.0 \pm 1.6}$$

we have ^{*)}

$$a_{\mu h}(\text{VMD}) = (6.1 \pm 0.5) 10^{-8} \quad (9)$$

The more refined version gives ⁴⁾

$$a_{\mu h} = (6.5 \pm 0.5) 10^{-8} \quad (10)$$

where the error reflects the uncertainty in the experimental data used as input.

^{*)} Note that with accurate experimental information on (5) over some range, integration of (7) over the corresponding range gives a lower bound for $a_{\mu h}$. It seems reasonable to regard (9) as such a lower bound to a good approximation.

3. -

THEORETICAL BOUND

Terazawa ⁵⁾ proposed to bound the high mass part of the integration (7) by a procedure applied to the charge renormalization constant by Kroll, Lee and Zumino (K.L.Z.) ⁹⁾. From the asymptotic behaviour of (2) this renormalization constant is

$$Z_3^{-1} = 1 + \frac{1}{\pi} \int dm^2 g_m P(-m^2) \quad (11)$$

whence

$$(Z_3^{-1} - 1)_{\text{hadronic}} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dm^2 g_m P_h(-m^2) \quad (12)$$

$$\leq \frac{1}{4m_\pi^2} \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dm^2 m^2 g_m P_h(-m^2) \quad (13)$$

We will consider in a moment how K.L.Z. estimate this last integral.

Terazawa first sets aside for separate consideration the very low mass region in the integral (7), $4m_\pi^2 < m^2 < 16m_\pi^2$, (which contains no resonances). The remainder is

$$a'_{\mu h} = \frac{\alpha}{\pi^2} \int_{16m_\pi^2}^{\infty} dm^2 g_m P_h(-m^2) f\left(\frac{m^2}{m_\mu^2}\right) \quad (14)$$

Because f is a positive decreasing function,

$$a'_{\mu h} \leq \frac{\alpha}{\pi^2} \frac{1}{16m_\pi^2} f\left(\frac{16m_\pi^2}{m_\mu^2}\right) \int_{4m_\pi^2}^{\infty} dm^2 m^2 g_m P_h(-m^2) \quad (15)$$

which involves again the K.L.Z. integral of (13). Actually Terazawa quotes this same bound for a smaller quantity obtained from $a'_{\mu h}$ by subtracting the pion pair contributions.

What then does the hypothesis of field current identity say about the integral in (13) and (15) ? The electromagnetic current (we will write only the isovector part for simplicity) is written as a multiple of a renormalized meson field :

$$J_\mu = - \frac{m_g^2}{g_g} \varphi_\mu$$

For the renormalized propagator K.L.Z. [Eqs. (4.10), (4.12)] assume a representation

$$F(q^2) = \int dm^2 \frac{\sigma(m^2)}{m^2 + q^2} \quad (16)$$

whence [K.L.Z. Eq. (6.9)]

$$\frac{1}{\pi} g_m P_h(-m^2) = e^2 m^{-4} \left(\frac{m_g^2}{g_g} \right)^2 \sigma(m^2) \quad (17)$$

- the extra factors $(m^{-2})^2$ coming from the two photon propagators. The integral (16) being supposed to converge at all q^2 , it converges in particular at $q^2 = 0$. Thus the integral of interest

$$\frac{1}{\pi} \int dm^2 m^2 g_m P_h(-m^2) = e^2 \left(\frac{m_g^2}{g_g} \right)^2 \int \frac{dm^2}{m^2} \sigma(m^2) \quad (18)$$

at least converges ^{*)}. This is the main point of K.L.Z. They also give an approximate evaluation of the integral, in terms of observable quantities, which we have now to study.

A superficial reading could leave the impression that the evaluation depended on one of the canonical commutation rules (C.C.R.) which are the characteristic feature of the theory. One of these is indeed supposed to give a sum rule for the integral of interest [K.L.Z. Eq. (4.6)] :

^{*)} It follows that $a_{\mu h}$ is at least finite : this has perhaps less surprise value than the corresponding statement for the renormalization factor (11) with which K.L.Z. were concerned.

$$\int \frac{dm^2}{m^2} \sigma(m^2) = \frac{1}{Z} \left(\frac{1}{m_\gamma^0} \right)^2 \quad (19)$$

where m_γ^0 is the bare mass, and Z the field renormalization factor. One can choose [K.L.Z. Eq. (4.22)] $Z = Z_0 = (m_\gamma / m_\gamma^0)^2$, so that

$$\int \frac{dm^2}{m^2} \sigma(m^2) = \frac{1}{m_\gamma^2} \quad (20)$$

as quoted by Terazawa. Then

$$\frac{1}{\pi} \int dm^2 m^2 g_m P_h(-m^2) = e^2 \left(\frac{m_\gamma^2}{g_\gamma} \right)^2 \frac{1}{m_\gamma^2} \quad (21)$$

- which would be fine if g_γ were now measurable. It is indeed one of the factors contributing to the observable weight h_γ (adopting the small width approximation for simplicity) of Eq. (6). If the γ contribution to the weight function $\sigma(m^2)$ is written

$$(Z_1/Z_0) \delta(m^2 - m_\gamma^2) \quad (22)$$

where Z_1 is the residue of the pole in the unrenormalized propagator, then the corresponding contribution to $\text{Im} P(-m^2)$ is

$$\pi h_\gamma \delta(m^2 - m_\gamma^2) \quad (23)$$

with [from (17)]

$$h_\gamma = \left(\frac{e}{g_\gamma} \right)^2 \frac{Z_1}{Z_0} \quad (24)$$

Then (21) can be rewritten

$$\frac{1}{\pi} \int dm^2 m^2 g_m P_h(-m^2) = h_\gamma m_\gamma^2 (Z_0/Z_1) \quad (25)$$

and it only remains to evaluate (Z_0/Z_1) . To do this K.L.Z. ^{*)} assume V.M.D. for the propagator (16) near $q^2 = 0$. From (22) then

$$\int dm^2 m^{-2} \sigma(m^2) = M_\pi^{-2} (Z_1/Z_0)$$

and by comparison with (20), $Z_1 = Z_0$. So finally the K.L.Z. estimate is

$$\frac{1}{\pi} \int dm^2 m^2 g_m P_h(-m^2) = h_\pi M_\pi^2 \quad (26)$$

But we note that once V.M.D. is assumed for the integral on the right-hand side of (18), and therefore for the integral on the left-hand side, the result (26) follows immediately from (23), without any reference to the C.C.R. sum rule (20). Thus it is only a redundant loop in the argument which invokes this sum rule ; only with some independent argument for $Z_1 \approx Z_0$ would the C.C.R. sum rule add credibility to the vector meson dominance on which (26) directly depends.

So in our opinion, (26) is no more than a V.M.D. estimate of the integral in question. Moreover since this integral converges less well than the integrals (7) and (12) of primary interest, it would seem that V.M.D. is now being applied in an even more unreliable way. So we do not have in the end anything better than the V.M.D. estimates of the original integrals. To make this point quite clear, suppose we have some version of V.M.D. which adequately represents $\text{Im } P(-m^2)$ up to some value of m^2 , and let a'_μ (V.M.D.) denote the corresponding part of the integral (14). Then

*) K.L.Z. do this in assuming that the pole term $(Z_1/Z_0)(q^2+m_\pi^2-i\sqrt{m_\pi})^{-1}$ [their Eq. (4.18)] gives not only $\text{Im } F$ near $-q^2=m_\pi^2$ (which they use implicitly in relating $g_\pi^2 Z_0/Z_1$ to $g \approx e^+e^-$ on p. 1394) but also $\text{Re } F^{-1}$ at $q^2=0$ [K.L.Z., between Eqs. (7.8) and (7.9)]. In this connection they quote an empirical check [Eq. (7.8)] of the constancy of the product of $(m_\pi^2+q^2)\text{Re } F^{-1}$ with another function of q^2 but, as they clearly imply, the separate constancy of the two functions is quite hypothetical.

$$a'_{\mu h} = a'_{\mu}(\text{VMD}) + \epsilon \quad (27)$$

where $\epsilon (>0)$ denotes the residual integral. Evaluating the bounding integral (15) in the same way we learn only that

$$a'_{\mu h} < a'_{\mu B}(\text{VMD}) + \epsilon_B \quad (28)$$

where both known and unknown quantities are greater than in (27) :

$$a'_{\mu B}(\text{VMD}) > a'_{\mu}(\text{VMD}) \quad \epsilon_B > \epsilon$$

4. -

EMPIRICAL BOUNDS

From (7), we may write

$$\begin{aligned} f(c) &= \int_0^1 dx \left\{ \frac{x^2}{c} - \frac{x^2}{c} \frac{x^2}{x^2 + c(1-x)} \right\} \\ &= \frac{1}{3c} \left\{ 1 - \bar{f}(c) \right\} \end{aligned} \quad (29)$$

where *)

$$\bar{f}(c) = 3 \int_0^1 dx \frac{x^4}{x^2 + c(1-x)} < \frac{3}{c} \log(1+c) \quad (30)$$

is a positive decreasing function. Then remembering from (3)

$$\frac{1}{\pi} \int \frac{dm^2}{m^2} \text{Im } P(-m^2) = P(0) \quad (31)$$

we have from (7) and (29)

$$P_h(0) \frac{\alpha}{\pi} \frac{m_\mu^2}{3} \left\{ 1 - \bar{f}\left(\frac{4m_\pi^2}{m_\mu^2}\right) \right\} < a_{\mu h} < \frac{\alpha}{\pi} \frac{m_\mu^2}{3} P_h(0) \quad (32)$$

Various phenomena involving low q^2 vacuum polarization can be analyzed for information about $P_h(0)$, if the hadronic vacuum polarization is regarded as the only non-calculable element in the situation.

For comparison we list the contributions to $P(0)$ from a single electron loop, a single muon loop, and from hadrons in V.M.D. approximation :

*) Numerically $\bar{f}(4m_\pi^2/m_\mu^2) = 0.37$, while the quoted bound gives $\bar{f}(4m_\pi^2/m_\mu^2) < 0.87$.

$$P_{e\bar{e}}(0) = \frac{1}{15} \frac{\alpha}{\pi} \frac{1}{m_e^2} = 2.3 \cdot 10^{-25} \text{ cm}^2 \quad (33)$$

$$P_{\mu\bar{\mu}}(0) = \frac{1}{15} \frac{\alpha}{\pi} \frac{1}{m_\mu^2} = 5.4 \cdot 10^{-30} \text{ cm}^2 \quad (34)$$

$$P_{\text{VMD}}(0) = \sum \frac{h_i}{m_i^2} = 3.1 \cdot 10^{-30} \text{ cm}^2 \quad (35)$$

a) - Atomic Physics

Leptonic vacuum polarization contributes significantly to the Lamb shift in hydrogen ^{*)} and to the energies of μ mesic X-rays. In both cases the experimental precision is at a level of a few per cent of these contributions. One could then be tempted to say that $P_h(0)$ cannot be more than a few per cent of (33).

In fact it is not possible to separate the effect of P_h from that of the finite size of the hadron - indeed $P_h(0)$ can be regarded as giving an increment in the mean square charge radius

$$\delta \langle r^2 \rangle = -6 P_h(0) \quad (36)$$

In the V.M.D. estimate (35), this amounts to

$$\delta \langle r^2 \rangle \approx -1.9 \cdot 10^{-29} \text{ cm}^2 \quad (37)$$

which is a very small part of the total mean square radius of the proton ¹¹⁾

^{*)} In μ mesic hydrogen vacuum polarization is even the dominant contribution to the Lamb shift (Di Giacomo ¹⁰⁾).

$$\langle r^2 \rangle = (.64 \pm .03) 10^{-26} \text{ cm}^2 \quad (38)$$

as measured in electron-proton scattering (leptonic vacuum polarization having been allowed for as a radiative correction). But even if it were bigger it would not be possible to separate off the $P_h(0)$ contribution so long as the source of the photon is a hadron.

We note in passing that even (38), which is much bigger than (37), is at present only marginally significant ^{*)} for the Lamb shift ($2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ level difference) in hydrogen. It would give an increment

$$\delta \Delta E = \frac{1}{12} (Z\alpha)^4 m_e^3 \langle r^2 \rangle$$

$$\approx .14 \text{ MHz}$$

Including such a contribution in the theory one has at present ^{**))}

$$\Delta E^{\text{exp}} - \Delta E^{\text{theory}} = .24 \pm .14 \text{ MHz}$$

The finite size effect will, however, be important in μ mesic hydrogen ¹⁰⁾

b) - Electron Magnetic Moment Anomaly

At present experiment ¹⁷⁾ and theory differ by ¹⁶⁾

$$a_e^{\text{exp}} - a_e^{\text{theory}} = -(90 \pm 30) 10^{-9}$$

^{*)} See, however, Barrett et al, ¹²⁾ and Yennie ¹³⁾.

^{**))} For a comparison between experiments ¹⁴⁾ and theory ¹⁵⁾ see Farley ¹⁶⁾.

If we choose to regard this value, with its awkward sign, as not significantly different from zero, we can write

$$|a_e^{\text{exp}} - a_e^{\text{theory}}| \lesssim 120 \cdot 10^{-9}$$

The theory referred to includes second ¹⁸⁾ and fourth ¹⁹⁾ order contributions but only an impression of sixth order radiative corrections ²⁰⁾; we assume that these and higher order terms are together no larger than expected. Then

$$a_{eh} \lesssim 120 \cdot 10^{-9} \quad (39)$$

For the electron the bounds like (32) differ negligibly and we can take

$$a_{eh} = \frac{\alpha}{\pi} \frac{m_e^2}{3} P_h(0) \quad (40)$$

whence, with (39),

$$P_h(0) \lesssim 2.5 \cdot 10^{-25} \text{ cm}^2 \quad (41)$$

c) - Electron Electron Scattering

Large momentum transfer electron-electron scattering has been measured to test quantum electrodynamics. With colliding beams of 556 MeV electrons at SLAC ²¹⁾ no deviation from theory was found, after of course including radiative corrections ²²⁾. In terms of a possible Feynmann cut-off in the photon propagator

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2} - \frac{1}{k^2 + \Lambda^2} \quad (42)$$

the quoted result is

$$\begin{aligned} -\Lambda^{-2} &= .06 \pm .06 \text{ (GeV/c)}^{-2} \\ &\lesssim .12 \text{ (GeV/c)}^{-2} = (430 m_\pi^2)^{-1} \end{aligned} \quad (43)$$

Note that with such large $-\Lambda^2$ and the modest values of k^2 reached [$< 2(0.556 \text{ GeV})^2 \approx 32 m_\pi^2$] the second term in (42) is essentially a constant

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2} - \frac{1}{\Lambda^2} \quad (44)$$

We wish to reinterpret the result as limiting hadron contributions to the photon propagator :

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2} + P_h(k^2)$$

To overcome the difficulty that $P_h(k^2)$ need not be constant, note that

$$P_h(k^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dm^2}{m^2} g_m P_h(-m^2) \frac{m^2}{m^2 + k^2} \geq \frac{4m_\pi^2}{4m_\pi^2 + k^2} P_h(0) \quad (45)$$

Then from (43) and $k^2 < 2(0.556 \text{ GeV})^2 \approx 32 m_\pi^2$,

$$P_h(0) \lesssim \frac{36}{4} \left(\frac{1}{430} \right) \frac{1}{m_\pi^2} = 4.2 \cdot 10^{-28} \text{ cm}^2 \quad (46)$$

One could be more efficient if empirical knowledge of $\sigma(e^+e^- \rightarrow \text{hadrons})$ allowed the low mass part of the dispersion integral (45) to be calculated explicitly. To illustrate this let us assume that V.M.D. is adequate in the mass region up to say $10 m_\pi$. Then one finds that this region contributes negligibly in the experiment in question. Denoting the residual contribution by a prime, (45) and (46) can be replaced by

$$P'_h(k^2) = \frac{1}{\pi} \int_{100 m_\pi^2}^{\infty} \frac{dm^2}{m^2} g_m P'_h(-m^2) \frac{m^2}{m^2 + k^2} \gg \frac{100 m_\pi^2}{100 m_\pi^2 + k^2} P'_h(0) \quad (47)$$

$$P'_h(0) \lesssim \left(\frac{13.2}{100}\right) \left(\frac{1}{430}\right) \frac{1}{m_\pi^2} = 6.2 \cdot 10^{-29} \text{ cm}^2 \quad (48)$$

d) - Muon Magnetic Moment Anomaly

From the upper bound (32), and (46) from electron-electron scattering,

$$a_{\mu h} \lesssim 9 \cdot 10^{-6}$$

Again it is better if possible to calculate separately low mass parts of the dispersion integrals. If for example V.M.D. is good up to $(10 m_\pi)^2$ then from (48)

$$a'_{\mu h} < \frac{\alpha}{\pi} \frac{m_\mu^2}{3} P'_h(0) \lesssim 1.3 \cdot 10^{-6}$$

This is quite large compared with the V.M.D. contribution (9), and even with respect to the present experimental accuracy ²³⁾

$$a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = (0.52 \pm 0.31) \cdot 10^{-6}$$

Thus improved $(g_\mu - 2)$ experiments should be complemented by improved experiments on ee scattering at large momentum transfer, and on e^+e^- annihilation at all available energies.

We have profited from discussion of this subject with J. Bailey, A. Di Giacomo, F. Farley, T.D. Lee, A. Petermann and E. Picasso.

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