THE HYBRID MODEL: FURTHER RESULTS

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ABSTRACT

In a previous paper we proposed a "hybrid" model for the large-energy, small-angle elastic scattering of hadrons, and used it to understand the structure in the angular distributions of proton-proton and proton-antiproton differential cross-sections. This model describes the scattering amplitude as the sum of an optical diffractive piece and a piece arising from the exchange of "absorbed" Regge poles; alternatively, it can be viewed as a specific prescription for calculating the effects of Regge cuts. In the present paper, we present some further results obtained from this hybrid model: we extrapolate our solutions for the pp amplitude to higher energies. We show how our model may be extended to deal with inelastic (or backward elastic) scattering, and consider processes which cannot be described by the single exchange of any known Regge trajectory, such as K⁻p backward elastic scattering. We explain why Regge cuts do not affect the presence or location of the dip in the π⁺p near-backward differential cross-section which is thought to arise from a nonsense zero of the nucleon trajectory. We predict that the differential cross-sections of elastic and of inelastic reactions should have the same t dependence at large |t|, whether or not they do at small |t|.

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1. **INTRODUCTION**

There has been a great deal of interest recently in the subject of possible unitarity corrections to strong interaction scattering amplitudes. Some results of this interest have been the investigations of the properties of Regge cuts, the attempts to apply absorptive corrections to Regge poles, and the calculation of multiple scattering terms in composite models.

Multiple scattering corrections have been discussed within the framework of the Chou-Yang model \(^1\), as well as within the quark model \(^2\), \(^3\). These corrections can be calculated by methods analogous to those developed by Glauber \(^4\) for scattering by nuclei. The absorption model has been applied to Regge pole amplitudes by several groups \(^5\), especially in attempts to explain the polarization observed in \(\pi N\) charge exchange scattering. The justifications advanced for this procedure differ, but the prescription used is always equivalent to multiplying the partial wave projection of the Regge pole by the elastic \(S\) matrix. It has also been suggested that Regge cuts might provide corrections to Regge pole amplitudes. From models \(^6\), we can learn several things about Regge cuts: their position and signature, the types of singularity, and in some cases the signs of their discontinuities. Although this information is not enough to enable us to explicitly compute the contributions of these cuts to the scattering amplitude, it has proven sufficient to permit the prediction of certain features of the \(\pi N\) charge exchange polarization \(^7\) and of the \(pp\) elastic differential cross-section \(^8\).

In a previous paper \(^9\) (hereafter called I), we suggested a "hybrid" model which could relate these three approaches to unitarity corrections \(^10\); in the present paper we review, elaborate upon, and extend this model. In the hybrid model we identify the eikonal function with the sum of Regge pole contributions. Formal expansion in powers of
2.

the eikonal then yields the Glauber multiple scattering series; the single scattering term contains the Regge poles which were used as input, and the multiple scattering terms are Regge cuts. Furthermore, in I we took the Pomeron to be a flat trajectory, whose residue function in proton-proton scattering was proportional to the square of the proton charge form factor, so that we recovered the Chou-Yang model \(^1\) in the infinite energy limit; however, most of the work presented in the present paper is independent of any specific model for the Pomeron, and so would be equally valid in the eikonal formulation proposed earlier by Arnold \(^2\).

In the next Section we elaborate further on the hybrid model. We also review the analysis of pp elastic scattering presented in I, and extrapolate the solution obtained there to higher energies. In Section 3 we discuss the extension of this model to inelastic processes, where to first order in the inelastic transition it corresponds exactly to the exchange of absorbed Regge poles. We point out that in this picture the differential cross-sections of elastic and of inelastic reactions should have the same t dependence at large \(|t|\) whether or not they do at small \(|t|\). We argue that the application of absorptive corrections does not alter the successful predictions of Regge pole theory; in particular, we show that absorption does not affect the prediction of a dip in the near-backward \(\pi^+ p\) differential cross-section at the value of \(u\) at which the nucleon trajectory becomes nonsense. In Section 4 we present the further extension of our model to second order in inelastic transitions, which allows us to discuss reactions, such as \(K^- p \rightarrow K^+ \Sigma\), which require exchange of more than one Regge pole.

2. THE HYBRID MODEL FOR ELASTIC SCATTERING

For the moment, we ignore the presence of spin. Suppose we write the S matrix in the impact parameter representation as:

\[ S(a, b) = \exp[2iB(a, b)] \]  

(1)
or, equivalently, the full amplitude as

\[ A(\omega, t) = B(\omega, t) + \frac{i}{4\pi} \int dt_1 dt_2 B(s,t_1) B(\omega, t_2) C(t, t_1, t_2) \Theta(C) \]

\[ + (\text{terms involving more than two } B's) \quad (2) \]

Here \( C \) is the triangle function; see the Appendix of Ref. 12). The amplitude \( A \) is normalized so that \( \sigma^T = 4\pi \text{ Im } A(s,0) \). Eq. (1) or (2) may be considered to be a definition of \( B \), which corresponds to the single-scattering term of Glauber theory. In the hybrid model, we write

\[ B(\omega, t) = P(t) + \sum_i R_i(s, t), \quad (3) \]

where \( P \) is the "Pomeranchuk" contribution; we recover the Chou-Yang model \(^1\) for \( pp \) scattering by letting \( P \) be proportional to the square of the proton electric form factor: \( P(t) = i k F_1(t) \). The summation over \( R_i = \beta_i(t)s \alpha_i(t)^{-1} \) in Eq. (3) then extends over Regge poles other than the Pomeron; we refer to these as "proper" Regge poles.

Computations are particularly simple if, at fixed energy, \( B \) is expressed as a sum of exponentials in \( t \). For example, if \( B \) contains two terms \( \lambda_i e^{t/a_i} \) and \( \lambda_j e^{t/a_j} \), there will be a contribution in the double scattering [i.e., the second term on the right-hand side of Eq. (2)]. This is given simply by the rule:

\[ \text{Regge poles } \lambda_i e^{t/a_i} \text{ and } \lambda_j e^{t/a_j} \Rightarrow \text{Regge cut } (1 - \frac{1}{2} \delta_{ij}) \frac{\lambda_i \lambda_j a_i a_j}{2(a_i + a_j)} e^{\frac{t}{a_i + a_j}} \]

\[ (4) \]

A Regge pole \( R_i = \gamma e^{-i \pi (\alpha(1/2)/(s/s_0))^2} \), with \( \alpha(t) = \alpha_0 + \alpha' t \), can be written in the form \( \lambda e^{t/a} \), with \( \lambda = \gamma e^{-i \pi (\alpha(1/2)/(s/s_0))^2} \), and \( a^{-1} = \alpha'(ln(s/s_0) - (i\pi/2)) \). The cut expression in (4) has precisely
the same form as we used in Ref. 7) as the most simple parametrization consistent with the supposed 6) properties of Regge cuts. We note that the phase (as well as the magnitude) of the cut contribution is specified by (4); this phase is controlled by the signature factors of the pole terms, which have been incorporated into the parameters \( a \) and \( \gamma \).

The hybrid model may be thought of as a Reggeization of an optical model, or, alternatively, as an optical prescription for calculating the effects of cuts in a Regge model. Looked at in either way, it constitutes a guess, which we certainly cannot prove to be correct. To some, this guess may be plausible because of the fact that it corresponds to what one would obtain from a composite picture of hadrons - for example, the quark model, in either the strong or the weak binding limit - in which the components interact by exchange of Regge poles. To others, it may be plausible because the prescription for calculating cuts is so simple, and yet all of the intricate features of the cuts - signature, type of singularity, etc. - that we expect on the basis of more sophisticated models 6) are reproduced. The test of this guess is, of course, by comparison with experimental data.

In I, we used an extremely simple form of this model to try to understand the general features of pp scattering. We took the dipole expression for the electric form factor which was used by Durand and Lifes 13) to produce, by means of the multiple scattering series, an asymptotic amplitude which has zeros at values of momentum transfer \((t \approx -1.3 \text{ GeV}^2 \text{ and } -6 \text{ GeV}^2)\) close to those at which the measured differential cross-sections show some indication of having structure 14). We retained two proper trajectories, the ones on which lie the \( \omega \) and the \( f^0 \), and approximated them by one exchange degenerate trajectory, with \( \alpha(t) = \frac{1}{2} + t \), and exponential residue 15). Thus we wrote

\[
B(\kappa, t) = \frac{\kappa}{(\mu^2 - t)^4} + \frac{t}{\kappa} \left( \frac{s}{s_0} \right)^t.
\]

(5)
Following Durand and Lipes, we set $N = 1 \text{ GeV}^2$, and adjusted $K$ so that we fit the total cross-section; that left us with two parameters, $\Gamma$ and $s_0$, with which to reproduce the real part in the forward direction, the structure in the angular distribution, and all of the energy dependence.

Figure 1 shows the differential cross-sections predicted by this calculation together with data at $P_{\text{lab}} = 12.4 \text{ GeV/c}$ \cite{14,16} and $P_{\text{lab}} = 19.2 \text{ GeV/c}$ \cite{17}, for the values $\Gamma = 22 \text{ GeV}^{-1}$, $s_0 = 4.5 \text{ GeV}^2$. The data at 19.2 GeV/c are new, and were not used in determining the values of the parameters. Although our parametrization is too crude to give a very good fit, it does reproduce the general features of the energy dependence, as well as the structure around $t = -1.3 \text{ GeV}^2$; there is also some slight structure, hard to see from the Figure, in the calculated curves near $t = -6 \text{ GeV}^2$. In II we stressed that, in our calculation, the real part of the amplitude was largely responsible for this structure, although at much higher energies the structure is primarily due to zeros of the imaginary part, as in the calculation by Durand and Lipes \cite{13}.

In Figure 2 we display the differential cross-sections predicted by our calculation at several higher values of the energy. These predictions are influenced by the fact that we have assumed the Pomeron to be flat. However, a small slope for the Pomeron probably would not make much difference, especially since the existence of multiple scattering corrections tends to obscure more than ever the phenomenological differences that would be caused by a possible small slope for the Pomeron. One very interesting feature of this extrapolation is that the angular structure does not monotonically get sharper with increasing energy, as one might expect it would if the real part of the amplitude merely served to fill up the dips in the asymptotically constant imaginary part. Instead, there is an intermediate energy region in which the structure shifts from the real to the imaginary part, and only after that do the dips get sharper.
By changing the sign of the $\omega$ contribution, we were able, in I, to predict the $\bar{p}p$ differential cross-section with no free parameters. We found the cross-over between the $pp$ and $\bar{p}p$ differential cross-sections, and observed that the structure seen \(^{18}\) at $-t = 0.5$ GeV\(^2\) at $P_{lab} = 5.9$ GeV/c should move slowly outward in $t$ as $P_{lab}$ increased. This last prediction has since then been confirmed by new $\bar{p}p$ data at $P_{lab} = 8$ and 16 GeV/c \(^{19}\).

3. THE HYBRID MODEL FOR INELASTIC SCATTERING

3.1 The absorption model

It has been pointed out by Arnold \(^{11}\) that the eikonal formulation for elastic scattering leads naturally to the absorption model for inelastic scattering. In Eqs. (1) and (3) we have written

$$S = \exp\left[2iP + 2iR\right],$$

where $R$ now stands for the proper Regge poles; to first order in $R$, this is $S = \exp\left[2iP\right] [1 + 2iR]$. From this model for elastic scattering, we could compute, for example, the difference of the elastic $\pi^+p$ and $\pi^-p$ amplitudes, and so conclude that the $\pi^-p$ charge exchange amplitude is

$$A_{\pi^-p \rightarrow n^p} = \exp\left[2iP\right] R_\rho = R_\rho + 2i A_{\text{elastic}} \cdot R_\rho.$$

Equation (5) coincides with the expression that would be obtained by applying the absorption model to the $\rho$ Regge pole. It has been pointed out \(^{12}\) that this model will give the correct sign for the polarization in $\pi^-N$ charge exchange; more detailed calculations \(^5\) have shown that it can also give approximately the correct magnitude for the polarization, as well as a good fit to the differential cross-section. These successful calculations can now be regarded also as successes for the hybrid model.
Strictly speaking, our model only allows us to compute those inelastic amplitudes which can be expressed as differences of elastic amplitudes, but we may easily generalize to obtain the absorptive prescription for other processes. In our formalism, the simplest way is to treat Eqs. (1), (2) and (3) as equations for matrix amplitudes which can connect different states. As long as we work to first order in inelastic transitions\textsuperscript{20}, this procedure is unambiguous; to this order, there is no question as to what states to include in the calculation, since any "intermediate" state must be connected by an elastic transition to either the initial or the final state. For charge exchange scattering, this formulation clearly coincides with the one given previously, but it is sufficiently general to allow us to compute absorptive corrections in cases in which elastic scattering may be different in the initial and the final states.

The hybrid model can enable us to understand some general features of the angular distributions of inelastic processes. It is known that the slopes in momentum transfer of both elastic and inelastic differential cross-sections tend to decrease as the momentum transfer increases; in our model, this comes about mainly through multiple scattering. However, there is no reason to believe that the single scattering term (i.e., the Regge pole) does not continue to fall rapidly with momentum transfer, especially if the Regge trajectory function continues to decrease. Thus, it is likely that, at large $|t|$, the (inelastic) single scattering term is falling much more rapidly than is the elastic amplitude. In this case, if we assume that the initial and final elastic amplitudes have more or less the same slope, it follows easily that the absorbed Regge pole has the same slope as does the elastic amplitude. Put another way, at large $|t|$, the dominant mechanism is the exchange of one proper trajectory together with many Pomeron, in which case the slope of that one trajectory becomes irrelevant.
Thus we are led to predict that at large $|t|$ (but always small angle!) inelastic and elastic differential cross-sections have the same slope, whether or not they do at small $|t|$ \textsuperscript{(21)}. This behaviour is seen in the reactions $pp \rightarrow pN^*(1520)$ and $pp \rightarrow pN^*(1690)$: for $|t|$ larger than about 2 GeV$^2$, the differential cross-sections for these reactions and for $pp \rightarrow pN^*$ elastic scattering are strikingly parallel \textsuperscript{(17,22)}. In these examples the inelastic process is itself presumably due to Pomeron exchange (diffraction dissociation); nevertheless, at small $|t|$ the slopes are markedly different from the elastic slope. We would expect the differential cross-section for production of the $N^*(1238)$ also to have the same slope (not the same energy dependence) at large $|t|$. At large $|t|$, the details of the production process are irrelevant; the slope is determined by absorption, and so is the same as the elastic slope.

3.2 What about the Regge pole model?

It is clear that absorption can eliminate many of the difficulties of the Regge pole model; for example, it can produce polarization where otherwise there would be none, and provide conspiracy without the need for conspiring trajectories. However, since the corrections we compute tend to be fairly large - typically around 25% of the amplitude at $t=0$, and larger than that for $t \neq 0$ - one might worry that this would also eliminate many of the triumphs of the Regge pole model. We would like to argue that this is not so.

Probably the most important aspect of Regge theory is its prediction of energy dependence, in particular its prediction that the amplitude in the physical scattering region is bounded by $s^{2(0)}$; this bound is in no way disturbed by multiple scattering corrections. Furthermore, since near the forward direction the tips of the cuts lie near the pole, their energy dependence will be quite similar. Thus we expect
the energy dependence predicted by Regge theory to be maintained \(^{23}\); this works in the case of \(\pi N\) charge exchange \(^{5}\), and as we shall see it is also true in the example discussed below.

Multiple scattering corrections will affect the \(t\) dependence of amplitudes, but in most cases Regge theory does not specify \(t\) dependence, since there is always an unknown residue function. Regge theory does predict dips when trajectories pass through nonsense points, and as we shall now show, at least in the example of \(\pi^+ p\) near-backward elastic scattering, absorption does not affect this prediction.

The nucleon trajectory is thought to pass through \(\alpha = -\frac{1}{2}\) at \(-u \approx 0.2\) GeV\(^2\); at this value of \(u\), the \(\pi^+ p\) differential cross-section is expected \(^{24}\), and indeed observed \(^{18},^{25},^{26}\) to have a dip. Because of the expected zero in the nucleon pole term, the absorptive correction, when evaluated near the backward direction, will be anomalously small, and this in turn will mean that the dip is not disturbed. The zero in the pole term means that the backward peak is extremely steep; this makes the absorption small. Furthermore, since the pole term changes sign, there will be extensive cancellations within the integral defining the absorption, and so the absorption will be even smaller.

This point can easily be verified by direct calculation. The solid curve in Fig. 3 represents a nucleon-exchange Regge pole term which was constructed so that it, by itself, fits the \(\pi^+ p\) differential cross-section at \(P_{\text{lab}} = 5.9\) GeV/c. When we apply absorptive corrections to this pole term - the dull details of this calculation, including the spinology, are presented in Appendix A - the result is the dashed curve shown in the same Figure. For small \(|u|\) the two curves almost coincide; the position of the dip is not changed.
\[ G_+ (s,t) = (1 - \frac{1}{2} \delta_{ij}) i k^2 \cos \frac{Q}{2} \int_0^\infty b \, db \, J_0 (b \sqrt{t}) \left[ \chi_o^i \chi_o^j + \chi_f^i \chi_f^j \right] \]

\[ G_- (s,t) = (1 - \frac{1}{2} \delta_{ij}) i k^2 \int_0^\infty b \, db \, J_0 (b \sqrt{t}) \left[ \chi_o^i \chi_f^j + \chi_f^i \chi_o^j \right] \]

(A.4)

If the Regge pole terms \( \tilde{G}_\pm \) are chosen to be exponentials in \( t \), then all the integrals in (A.3) and (A.4) can be done analytically. For the absorption calculation, it is convenient to imagine that the elastic amplitude is given by a single pole term (i.e., ignore multiple scattering corrections to the elastic amplitude), and then to use (A.4). We have used the \( \pi p \) elastic data of Ref. 29) to fit an exponential form to the elastic amplitude, which we assume to be purely imaginary and spin independent (\( \chi_{t=0} \)).

We have assumed the nucleon trajectory to be given by

\[ \chi_N (u) = -0.34 + u. \]

We parametrize the near backward amplitude due to exchange of this trajectory as

\[ \overline{G}_+ = \frac{C_6 D_0}{\sqrt{\alpha}} \left( 1 + e^{-i \pi (\chi - \frac{1}{2})} \right) \left( \frac{s}{\alpha} \right)^{\chi - \frac{1}{2}} \]

\[ \overline{G}_- = \frac{C_6 \sqrt{\alpha}}{\sqrt{\alpha}} \left( 1 + e^{-i \pi (\chi - \frac{1}{2})} \right) \left( \frac{s}{\alpha} \right)^{\chi - \frac{1}{2}} \]

(A.5)

The parameter \( D_0 \) corresponds to \( 1/\sqrt{u_0} \) in Ref. 24), where it was determined to be 1.18 GeV\(^{-1}\). If we neglect the \( \pi N \) mass difference, we can construct an equivalent forward problem by interchanging \( \overline{G}_+ \) and \( \overline{G}_- \), and writing \( t \) for \( u \); we can then directly use (A.4).
With the following values of the parameters: $C_0 = 4.45$, and $s_0 = 0.5$ GeV$^2$, the pole term alone fits the data at $P_{\text{lab}} = 5.9$ GeV/c; this is the solid curve in Fig. 3. To make the absorbed pole better fit the data, we change only the value of $s_0$ - to 0.9 GeV$^2$ - and compute the curves shown in Fig. 4.
APPENDIX B

CALCULATION OF K⁻p→K⁺π⁻

Figure 5 is intended as a mnemonic for the following procedure: from the K* contribution to the amplitude for K⁻p→π⁰Σ⁰, and to π⁰Σ⁰→K⁺Ξ⁻, calculate (two sets of) eikonals according to Eq. (A.3), and then calculate a contribution to the amplitude for K⁻p→K⁺Ξ⁻, according to Eq. (A.4). The δᵢⱼ in (A.4) is equal to one, since there is only one kind of trajectory involved.

In addition to the picture shown in Fig. 5, we should also consider the contributions of the intermediate states (π⁰Λ⁰), (η⁰Λ⁰), and (η⁰Σ⁰), calculated in the same way. Furthermore, we should include contributions from exchange of the trajectory of the K**(1400). Double exchange of the K** can clearly be treated in the same way as double exchange of the K*. In the SU(3) limit, there is no contribution from exchange of a K* together with a K**, since the amplitude is pure [27] in the t channel, and so must have positive signature. (In the formalism given here, this is guaranteed by the fact that, in addition to the picture shown in Fig. 5, there must be another kind of picture with the Regge poles crossed, and these two kinds of pictures would cancel if a K* and a K** were exchanged together. We can forget these crossed pictures if we remember not to exchange a K* and a K** together.) In principle, we should also consider absorptive effects, that is, there should be Pomerons floating all around Fig. 5.
FOOTNOTES AND REFERENCES


3) N.W. Dean, CERN preprint, TH. 881 (1968).


10) Many of the ideas in I were proposed simultaneously - and independently - by R.C. Arnold as "The next step in high-energy phenomenology", ANL preprint (1968).


15) S. Frautschi and B. Margolis (private communication) have recently analyzed \( pp \) scattering in an eikonal model with a Pomeron of non-zero slope, thus achieving an energy dependent amplitude without invoking secondary trajectories. Since much of the structure can be understood in terms of the general properties of the multiple scattering series (see for example, Ref. 8), their calculation is somewhat similar to ours. Such a model, without secondary trajectories, predicts that the real part of the amplitude is positive in the forward direction and that the pp total cross-section is slowly increasing as the energy increases, as discussed in Ref. 9.
17) Preliminary data from the CERN experiment at \( P_{lab} = 19.2 \text{ GeV/c} \); we thank Dr. A.N. Diddens and Dr. A.M. Wetherell for this private communication.
19) D. Birnbaum et al., paper presented at the Washington Meeting of the APS (1965), and very kindly reported to us by Dr. A.N. Diddens.
20) In the "\(^n\)th order" we include cuts involving exactly \( n \) proper trajectories, so that we are making an expansion in power of \( 1/s \).
21) This argument has previously been given by Arnold (Ref. 10) for the case of small \( |t| \), but extremely large \( s \).
23) For this reason, it is difficult to distinguish cuts from poles by the use of finite-energy cuts alone.
26) This structure was first noticed by H. Brody et al., Phys.Rev.Letters 16, 828 (1966).

27) This application of the hybrid model was first suggested to us by Professor L. Van Hove.

28) We are aware that Fig. 5, if interpreted as a Feynman diagram, does not give a cut. This does not prevent us from speculateing that we might obtain a reasonable approximation to the amplitude by an alternative interpretation of the Figure.

FIGURE CAPTIONS

Figure 1: The pp elastic differential cross-section versus $-t$. Calculated curves are at $P_{lab} = 12.4$ and 19.2 GeV/c. For details on data see Refs. 14, 16 and 17.

Figure 2: Predictions for pp elastic differential cross-section at high energies - $P_{lab} = 25, 70, 200$ and 1700 GeV/c. Note that the $t$ scale begins at $-0.3$ GeV$^2$.

Figure 3: Differential cross-section for $\pi^+ p$ backward elastic scattering at $P_{lab} = 5.9$ GeV/c. The solid curve represents the contribution of a nucleon Regge pole adjusted so that by itself it fits the data. The dashed curve shows the computed cross-section after the absorptive correction is applied. See Appendix A for the parameters used.

Figure 4: Differential cross-section for $\pi^+ p$ backward elastic scattering. The curves at 5.9 GeV/c and 10 GeV/c are obtained with the inclusion of absorptive effect and the readjustment of the pole residue function. Data points: $\circ$ at 5.9 GeV/c, $\Box$ at 9.9 GeV/c. See Refs. 18 and 25 for details.

Figure 5: A contribution to $K^- p \rightarrow K^+ \Xi^-$. See Appendix B for details.
$p + p \rightarrow p + p$

$\frac{d\sigma}{dt}$

$\mu b$ per $GeV^2$

$-t$ vs. $GeV^2$

FIG. 2
\pi^+ p \rightarrow p \pi^+

p_L = 5.9 \text{ GeV/c}

FIG. 3